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Time Domain Transfer Function of the Induction Motor

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Abstract

Small signal stability of electrical machines at frequency domain has been shown by toque coefficients and eigenvalue of motional impedance matrix in state space form. The relation of damping, synchronizing and total synchronizing torque coefficients with the eigenvalue or the roots of the characteristic equation of the perturbed machine shows that the instability occurs at 2 different modes. Static mode represented by real root at over load condition, and dynamic mode represented by complex root at the condition when the total synchronizing coefficient exhibits zero value within the negative range of the damping torque coefficient. However, small signal instability details at time domain are not given in the literatures. This paper discusses with figures the time domain signals of the induction motor perturbation variables under hunting condition, and presents the differences observed between inverse Laplace transform and Fourier transform in time domain response, based on the transform of the transfer function from the frequency domain, The figures demonstrate and confirm the machine small signal stability performance given in frequency domain.

Keywords: Laplace, Fourier, Induction motor, Impulse torque, Perturbation, Transfer function

1. Introduction

Damping T_D, synchronizing T_S and total synchronizing T_{St} torque coefficients are important for small signal stability of electrical machines because of their indirect relation and direct link to the roots of the characteristic equation of state space form of the perturbed machines [Barsoum N. N 1991]. The theorems given in [Barsoum, N. N. & Harris, M. R 2001] proved that instability may occur in 2 different modes. Static mode represented by real root at over load condition at low speed or low frequency, and dynamic mode represented by complex root at the condition when the total synchronizing coefficient exhibits zero value within the negative range of the damping torque coefficient. This instability mode can occur at both low and high frequency β [Barsoum N. N 1998] depends on machine parameters, particularly stator resistance, and working condition of both constant flux and weakening magnetic field.

This remark about the magnitude of the transfer function in [Barsoum, N. N. & Harris, M. R 2001, Barsoum, N. N. 2001] and its characteristics with the perturbation frequency β in frequency domain, gives a relation to stability characteristics in terms of torque coefficients, damping T_D and total synchronizing T_{St}, in time domain. It shows the time response of the perturbation velocity $p\Delta\theta$ (the derivative of the perturbation displacement) and the relation to stability and instability of both real and complex roots [Barsoum, N. N. & Harris, M. R 2001]. Since the impedance matrix of electrical machines can be represented in three different reference frames, stator, rotor and synchronous reference frames as well as the true reference frame [Barsoum, N. N 1991], the induction motor under hunting condition is represented in synchronous flux wave reference frame in this paper, and the response of $p\Delta\theta$ is observed in this particular frame. Two types of transformations are used in this paper, Laplace and Fourier.

These will show the similarity in the response of $p\Delta\theta$ of the motor in case of stable machine, and the unsimilarity for the unstable machine [Barsoum, N. N. & Harris, M. R 2001, Barsoum, N. N 1998]; where Laplace is available to apply on a single operating condition, given by the eigenvalue, while Fourier is for evaluating the response of the transfer function in frequency domain, by considering all the values of β from 0 to ∞, for the same operating condition [Gibbs, W. J 1962]. This matter is discussed in the following section.

2. Laplace and Fourier Transforms for Torque Coefficients

In this section both transformations are applied to obtain the response of $p\Delta\theta$ for the induction motor, defined by h(t) in time domain, for a given unit impulse of torque ΔT_m. The transfer function in frequency domain is therefore:

$H(j\beta) = \frac{p\Delta\theta}{\Delta T_m}(\beta)$, which transformed into $h(t) = p\Delta\theta(t)$, at $\Delta T_m = 1$ unit step, in time domain.

In Laplace transform, the solution of h(t) is given by equation (4) in the appendix, which contains all the eigenvalues for particular operating condition. This is solved in the appendix by means of partial fraction operation (equation (3)) at $\Delta T_m = 1$. This depends on the numerical solution of the eigenvalue, which are obtained from the characteristic equation, found in the denominator of equation (2) in the appendix.

It can be seen that the solution of h(t) is directly related to the condition for stability. It is of damped oscillation for stable machine, increasing with oscillation if the machine has an unstable complex eigenvalue, and increasing exponentially if it has an unstable real eigenvalue [Chen, W. H. 1963]; while the retain eigenvalue remain stable. These are shown in all of Figs(a), but the behavior is different in case of applying Fourier transform shown in all Figs (b).

Fourier transform has been defined by [Heading, J 1969, Chen, W. H. 1963] as:

$$H(j\beta) = \int_0^\infty h(t)\, e^{-j\beta t}\, dt \quad \text{and} \quad h(t) = \frac{1}{\pi}\int_0^\infty H(j\beta)\, e^{j\beta t}\, d\beta$$

which can be modified by truncating β-range at Ω [Chen, W. H. 1963] as follows:

$$h(t) = \frac{1}{\pi}\int_0^\infty H(j\beta)\, e^{j\beta t}\, \frac{\sin(\frac{\beta\pi}{\Omega})}{(\frac{\beta\pi}{\Omega})}\, d\beta \tag{1}$$

In (1), Ω needs to be sufficiently high $\approx \frac{1}{\Delta t}$, where Δt is the smallest time step of interest, as it discussed in [Heading, J 1969, Chen, W. H. 1963], so that the integration of H(jβ) needs a sufficiently high t-value, for converging $h(t) \to 0$. The way of doing this is to get the Gibbs oscillation problem [Gibbs, W. J 1962] well away from the main swing frequency, as it appears in Fig(8a).

The transformation equation (1) has been established in [Chen, W. H. 1963] for all $0 \le \beta \le \Omega$ and used in this result to apply on:

$$H(j\beta) = \frac{\beta}{T_{St}(\beta) + j\beta T_D(\beta)} = \frac{\beta(T_{St} - j\beta T_D)}{T_{St}^2(\beta) + \beta^2 T_D^2(\beta)}$$

in real and imaginary parts, or:

$$H(j\beta) = \frac{\beta}{\sqrt{T_{St}^2(\beta) + \beta^2 T_D^2(\beta)}} \underline{\Big|\ \tan^{-1} - (\frac{\beta T_D}{T_{St}})}$$

in magnitude and angle,.

For induction motor, it can be appreciated from the characteristics of the magnitude of transfer function TF = |H(jβ)| in [Barsoum, N. N. & Harris, M. R 2001, Barsoum, N. N. 2001, Barsoum, N. N.1991], that if the machine is stable, then Fourier h(t) is similar to Laplace h(t) when the dominant eigenvalue is lightly damped. (It should be noticed that Laplace transform is obtained by the eigenvalue, i.e. for a single value of β, which is the frequency of oscillation γ of the dominant root; while Fourier is concerned with all β-values between 0 and Ω.) If the machine works on the stability boundary, where the real part of the dominant root $\sigma = 0$, then TF = ∞ at $\beta = \gamma$ of the same root of having $\sigma = 0$. Thus, Laplace h(t) will oscillate freely at the stability boundary of the complex root, but it is constant at the boundary of the real root (where $\gamma = 0$). While Fourier h(t) does not exist in both cases, since $|H(j\beta)| = \infty$ at $\beta = \gamma$ (if complex) or at $\beta = 0$ (if real). Similarly, for a machine with heavily damped pair of eigenvalue, that $|H(j\beta)| \to 0$ as $\sigma \to \infty$ and therefore it contributes very little to H and consequently Fourier $h(t) \to 0$. Note that, $|H(j\beta)| \to 0$ usually occurs for induction motor when $\beta \to \infty$, so that h(t) of (1) $\to 0$, but starts to oscillate at high t-values, only in case of having very high Ω-value, and courses Gibbs problem. These purposes are all illustrated in Figs(1 to 8).

If the machine is unstable, whether dynamically (in complex root) or statically (in real root) [Barsoum, N. N. & Harris, M. R 2001] or both together, then according to the existence theorems Fourier h(t) excludes all the unstable eigenvalue and measures T_D and T_{St} as the coefficients of the sum of the remaining stable roots. Obviously, it is true for Fourier transform in the high-order systems, which shows the response for the stable roots only, without measuring what is included on the right hand side of complex plane within the coefficients of the transfer function, as the existence theorems state. Therefore, Fourier h(t) concerns only with the stable roots, but if one root is on the boundary then H(jβ) does not exist. Figs(b) show the differences of this method corresponding to Figs(a).

However, this seems unlikely to happen in the second-order system with a pair of unstable roots. The remaining stable roots are none, and so T_{St} and T_D should be in such a way to give H(jβ) = 0 for all β. This seems to require $|T_{St} + j\beta T_D| = \infty$ for all values of β, which seems impossible.

In Figs(b, 8b, 8c), Ω is chosen to be 4 per unit (pu), and the step of β is 0.001 pu. So the integration (1) is of time step equals 0.25 sec. and includes 4000 value for each T_{St} and T_D. However, in Fig(8a), Ω = 10 pu, which is the reason of finding Gibbs oscillation at high t as shown while it is not found in the other figures (8a, 8b) at the same t = 1000 sec.

Figs(b) show the discrete Fourier transform (DFT, in which the integration (1) is effectively a summation of H(jβ) with an infinitesimal steps in a long range of β [Heading, J 1969, Chen, W. H. 1963]) of h(t) and explain how the characteristics of the coefficients T_D and T_{St} diagnose the stability characteristics in very different way than the characteristics of Figs(a), which are investigated by Laplace transform. This difference appears in h(t) response, when it carries the effects of the harmonics (in amplitude and frequency) in some steps of β if it is obtained by Fourier equation (1), while these harmonics are not involved if it is obtained by Laplace equation (4), appendix.

Harmonic effect appears as some ripples occur with the fundamental wave, and can be seen from the cases of heavily damped and real root on the boundary Figs(3b, 4b). In all cases, the magnitudes and frequencies are not the same, comparing Laplace (fundamental) with Fourier (fundamental plus harmonics), but Fourier also shows the cases of stability, instability and boundary of each eigenvalue in different ways of Laplace. These are shown in all Figs(a, b) and illustrate that Fourier h(t) can define all these conditions, but does not recognize the instability in h(t) according to an unstable root. This in fact indicates that Fourier is only concerning with the remaining stable roots in measuring T_{St} and T_D coefficients from β = 0 to Ω, as shown in Figs(5b ,6b).

Although Fourier h(t) shows the case of stability boundary in Figs(3b, 4b), which are similar to Figs(3a, 4a) of Laplace (but are not realy identical), Fourier function in frequency domain H(jβ) at β = γ of the root on the boundary is infinity, and Fourier h(t) of that root does not exist. But in principle the effect of the root on the boundary dominates the solution of h(t), since the other roots are stable and attractive. Thus, the solution is of constant amplitude of the peak values, as shown in the figures.

Figs. 1 show the lightly damping operation of induction motor. The parameters in per unit are:

$R_S = 0.03$, $R_r = 0.015$, $L_{ls} = 0.1$, $L_{lr} = 0.1$, M = 4, S = 0.0, J = 745.0, v = ω = 0.13

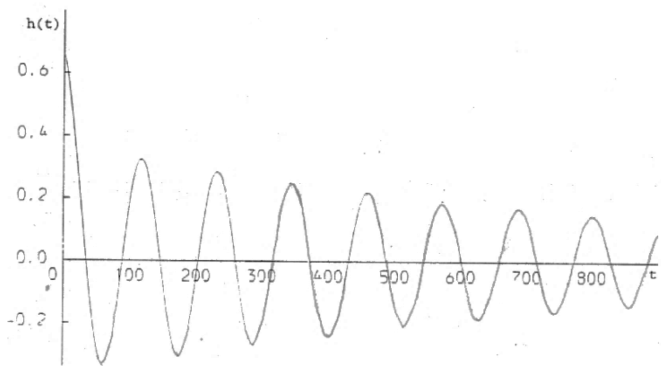

Figure 1a. Laplace transform at unit impuls torque

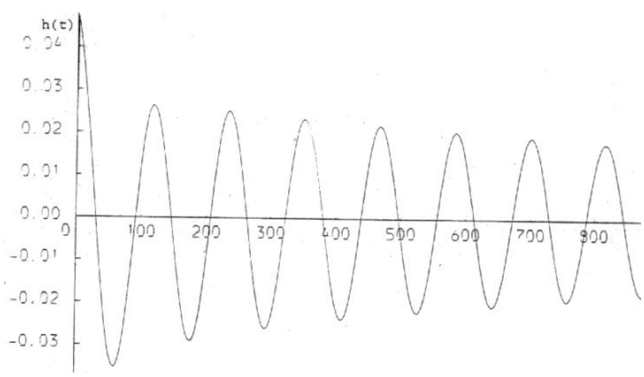

Figure 1b. Fourier transform at unit impulse torque

Figs. 2 show the heavily damping operation of induction motor. The parameters in per unit are:

$R_S = R_r = 0.01$, $L_{lS} = L_{lr} = 0.08$, $M = 3$, $v = 0.09$, $S = 0.0625$, $J = 36$, $\omega = 0.3$

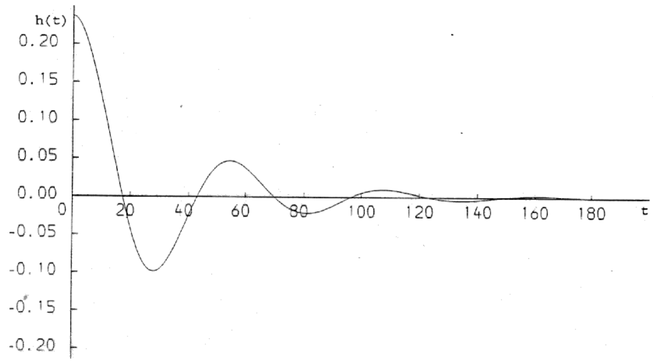

Figure 2a. Laplace transform at unit impulse torque

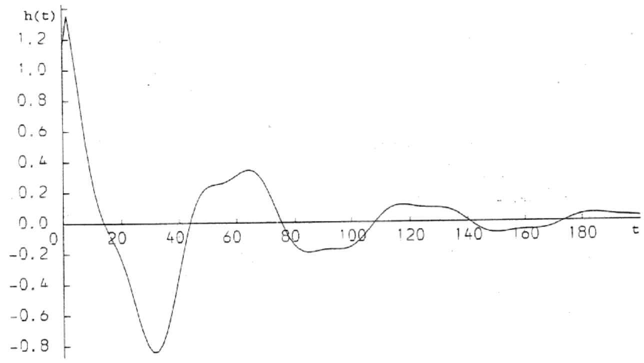

Figure 2b. Fourier transform at unit impulse torque

Figs. 3 show the complex root on stability boundary of induction motor. The Parameters in per unit are:

$R_S = R_r = 0.01$, $L_{lS} = L_{lr} = 0.08$, $M = 3$, $J = 36$, $S = 0.0625$, $v = \omega = 0.3$

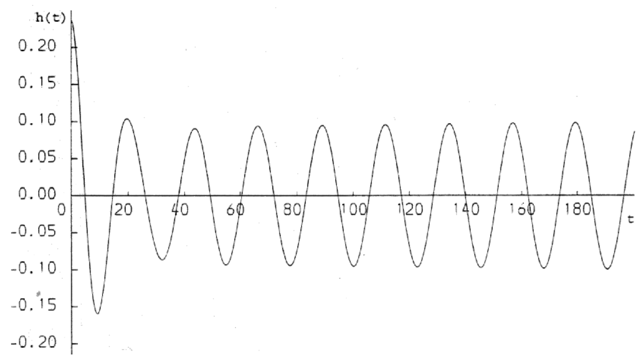

Figure 3a. Laplace transform at unit impuls torque

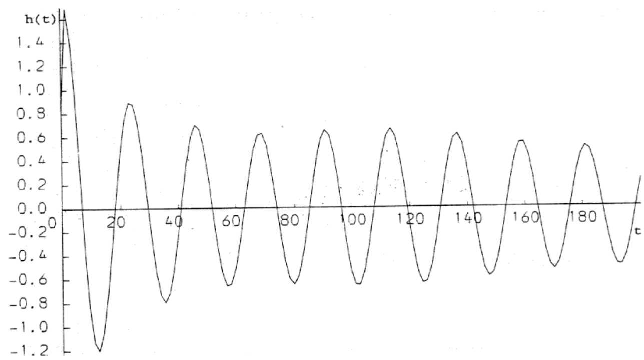

Figure 3b. Fourier transform at unit impulse torque

Figs. 4 show the real root on stability boundary of induction motor The parameters in per unit are:

$R_S = 0.025$, $R_r = 0.015$, $L_{ls} = L_{lr} = 0.1$, $M = 3.5$, $S = 0.075$, $J = 62.8$, $v = \omega = 1.0$

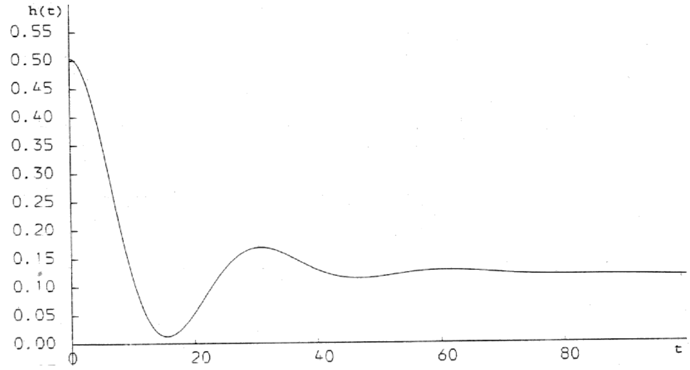

Figure 4a. Laplace transform at unit impulse torque

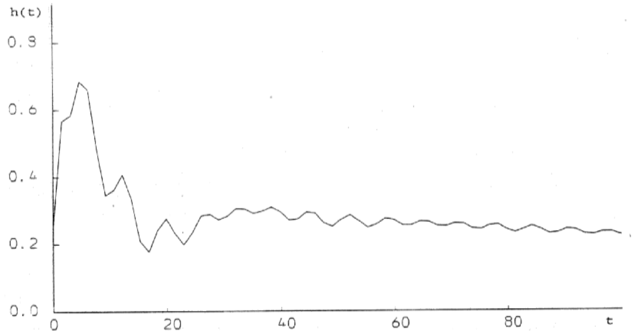

Figure 4b. Fourier transform at unit impulse torque

Figs. 5 show the unstable complex root of induction motor, The parameters in per unit are:

$R_S = 0.03$, $R_r = 0.02$, $L_{ls} = L_{lr} = 0.1$, $M = 5$, $S = 0.005$, $J = 314$, $\omega = 0.1$, $v = 0.08$

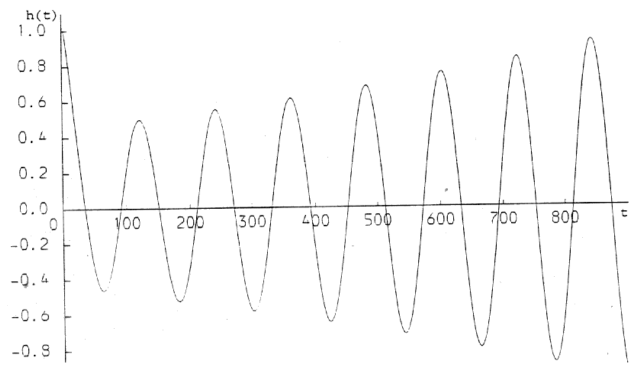

Figure 5a. Laplace transform at unit impuls torque

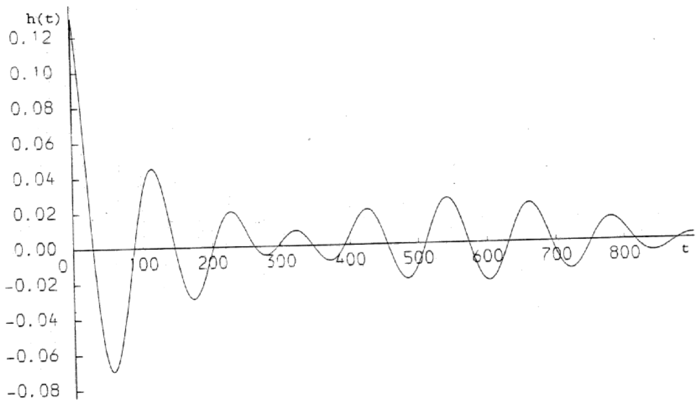

Figure 5b. Fourier transform at unit impulse torque

Figs. 6 show the unstable real root of induction motor. The parameters in per unit are:
$R_S = 0.025$, $R_r = 0.015$, $L_{ls} = L_{lr} = 0.1$, $M = 3.5$, $S = 0.075$, $J = 62.8$, $\omega = 1.3$, $v = 5.6$

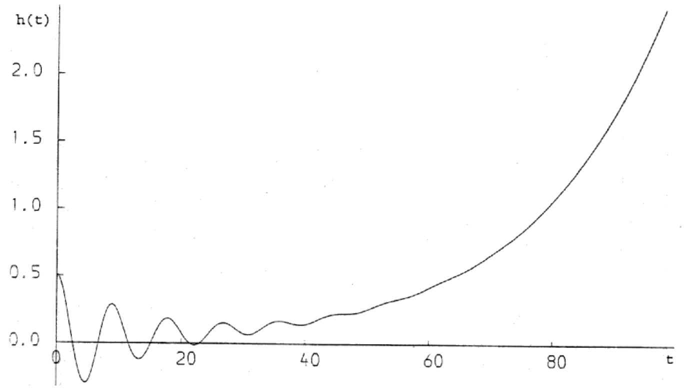

Figure 6a. Laplace transform at unit impulse torque

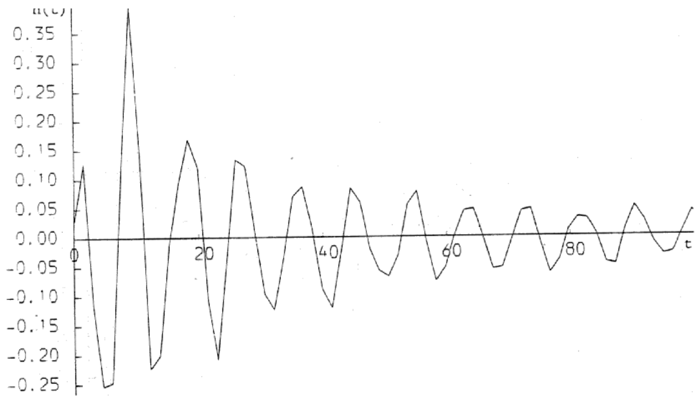

Figure 6b. Fourier transform at unit impulse torque

Figs. 7 show the unstable real and complex roots together of induction motor. The parameters in per unit are:
$R_S = 0.035$, $R_r = 0.015$, $L_{ls} = L_{lr} = 0.1$, $M = 3.5$, $S = 0.085$, $J = 62.8$, $\omega = 1.0$, $v = 5.6$

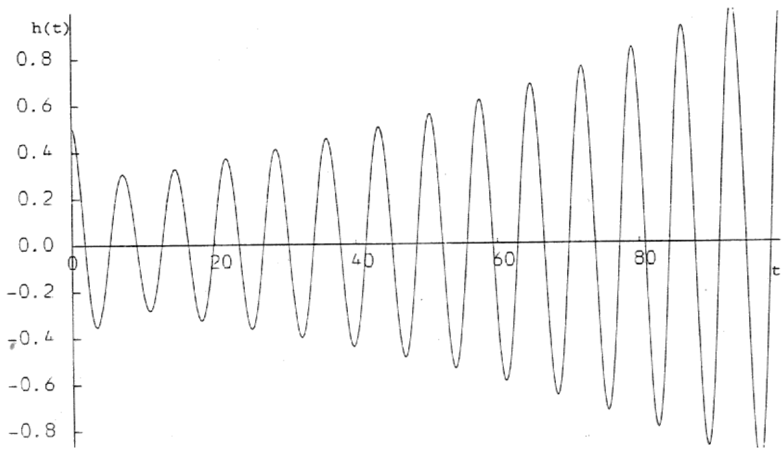

Figure 7a. Laplace transform at unit impulse torque

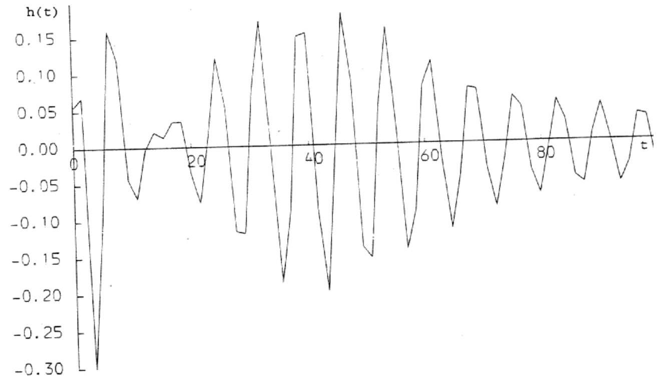

Figure 7b. Fourier transform at unit impulse torque

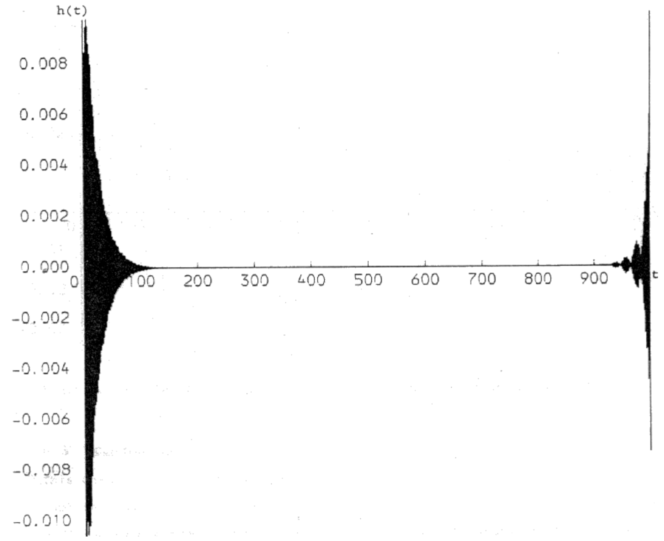

Figure 8a. Fourier transform with Ω = 10 per unit for stable machine

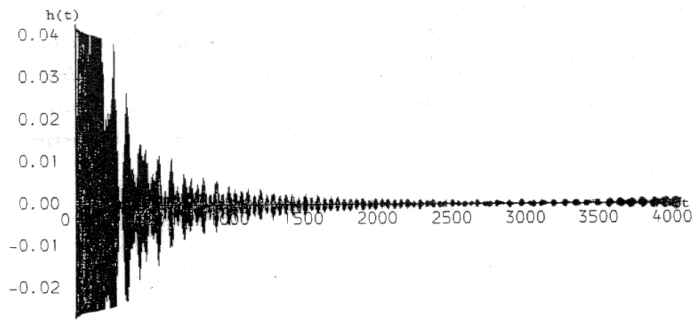

Figure 8b. Fourier transform with Ω = 4 per unit for the same case of Fig(8a)

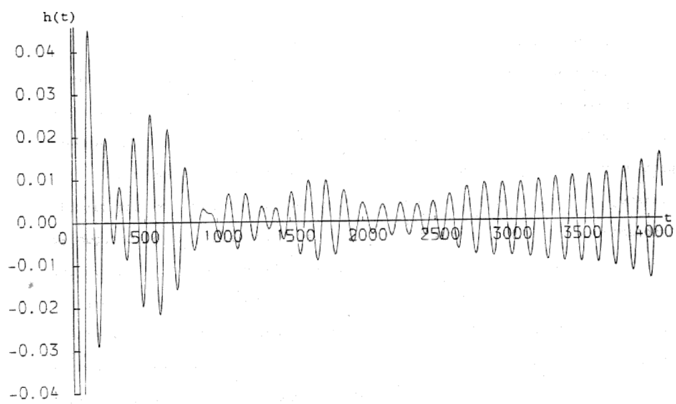

Figure 8c. Fourier transform with Ω = 4 for unstable machine, in complex root

3. Conclusion

The paper presents some figures on time domain response of the output speed $p\Delta\theta$ for an input impulse shaft torque of the induction motor perturbation variables under hunting condition. It shows the similarity of the response between two methods of transformations, Laplace and Fourier in case of small signal stable mode, and the difference on response in two cases, one in finding an unstable root, and one in finding a root on the boundary. This indicates that the response of an unstable machine cannot be truly observed by using Fourier transform. It is, therefore, important to put the machine under test for stability, during the practical work of design, using the method of parameter estimation with Fourier transform. This must be applied when the machine variables are expressed in synchronous flux wave reference frame (not the stator reference frame), since the perturbed variables have a time-invariant coefficients.

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Appendix

The solution of the output perturbation velocity $p\Delta\theta$ in the dynamic machine equation [Barsoum, N. N 1998, Barsoum, N. N 1991] for the induction motor is h(t), given by Laplace transform where h(t) is the response of $p\Delta\theta$ in time-domain to a unit impulse of the torque input ΔT_m.

By applying Laplace transform to the dynamic machine equation [Barsoum, N. N 1991] with $\Delta T_m(\beta) = 1$ and all the initial values of the perturbation currents and speed are zero which can be deduced from the motional impedance matrix at t = 0 where, p = 0 and $\omega = 0$. Therefore, the Laplace transform of $p\Delta\theta$ is denoted by $p\Delta\theta(\rho)$, where:

$$p\Delta\theta(\rho) = \frac{B_5\rho^4 + B_4\rho^3 + B_3\rho^2 + B_2\rho + B_1}{B_5\rho^5 + B_4\rho^4 + (B_3 + C_3)\rho^3 + (B_2 + C_2)\rho^2 + (B_1 + C_1)\rho + C_0}$$

where ρ is the Laplace symbol, and the coefficients B_1 to B_5 and C_0 to C_3 are defined in [Barsoum, N. N 1998, Barsoum, N. N 1991] in terms of machine parameters.

In particular operating condition, the expression of $p\Delta\theta(\rho)$ can be written as follows:

$$p\Delta\theta(\rho) = \frac{B_5\rho^4 + B_4\rho^3 + B_3\rho^2 + B_2\rho + B_1}{(\rho - \sigma_1 - j\gamma_1)(\rho - \sigma_1 + j\gamma_1)(\rho - \sigma_2 - j\gamma_2)(\rho - \sigma_2 + j\gamma_2)(\rho - \sigma_3)} \quad (2)$$

where, $(\sigma_1 \pm j\gamma_1)$, $(\sigma_2 \pm j\gamma_2)$ and σ_3 are the eigenvalue of the induction motor model [Barsoum, N. N 1998] at that operating point.

Equation (2) can be splitted into 5-terms by the method of the partial fraction as follows:

$$p\Delta\theta(\rho) = \frac{K_1}{\rho - \sigma_1 - j\gamma_1} + \frac{K_1^*}{\rho - \sigma_1 + j\gamma_1} + \frac{K_2}{\rho - \sigma_2 - j\gamma_2} + \frac{K_2^*}{\rho - \sigma_2 + j\gamma_2} + \frac{K_3}{\rho - \sigma_3} \quad \text{............(3)}$$

where:

$K_1 = X_1 + jY_1$ $\quad K_2 = X_2 + jY_2$

$K_1^* = X_1 - jY_1$ $\quad K_2^* = X_2 - jY_2$

(* denotes the conjugate), and equation (3) can, therefore, be written as:

$$p\Delta\theta(\rho)=\frac{X_1(\rho-\sigma_1)-Y_1\gamma_1}{(\rho-\sigma_1)^2+\gamma_1^2}+\frac{X_2(\rho-\sigma_2)-Y_2\gamma_2}{(\rho-\sigma_2)^2+\gamma_2^2}+\frac{K_3}{\rho-\sigma_3}$$

where:

$$X_1=\frac{ax_1+by_1}{x_1^2+y_1^2} \qquad X_2=\frac{cx_2+dy_2}{x_2^2+y_2^2}$$

$$Y_1=\frac{bx_1-ay_1}{x_1^2+y_1^2} \qquad Y_2=\frac{dx_2-cy_2}{x_2^2+y_2^2}$$

$$K_3=\frac{B_5\sigma_3^4+B_4\sigma_3^3+B_3\sigma_3^3+B_2\sigma_3+B_1}{[[(\sigma_3-\sigma_1)^2+\gamma_1^2][(\sigma_3-\sigma_2)^2+\gamma_2^2]}$$

$$a=B_5(\sigma_1^4-6\sigma_1^2\gamma_1^2+\gamma_1^4)+B_4\sigma_1(\sigma_1^2-3\gamma_1^2)+B_3(\sigma_1^2-\gamma_1^2)+B_2\sigma_1+B_1$$

$$b=4B_5\sigma_1\gamma_1(\sigma_1^2-\gamma_1^2)+B_4\gamma_1(3\sigma_1^2-\gamma_1^2)+2B_3\sigma_1\gamma_1+B_2\gamma_1$$

$$c=B_5(\sigma_2^4-6\sigma_2^2\gamma_2^2+\gamma_2^4)+B_4\sigma_2(\sigma_2^2-3\gamma_2^2)+B_3(\sigma_2^2-\gamma_2^2)+B_2\sigma_2+B_1$$

$$d=4B_5\sigma_2\gamma_2(\sigma_2^2-\gamma_2^2)+B_4\gamma_2(3\sigma_2^2-\gamma_2^2)+2B_3\sigma_2\gamma_2+B_2\gamma_2$$

$$x_1=-\gamma_1^2[2(\sigma_1-\sigma_2)(\sigma_1-\sigma_3)+(\sigma_1-\sigma_2)^2-(\gamma_1^2-\gamma_2^2)]$$

$$y_1=\gamma_1[(\sigma_1-\sigma_3)\{(\sigma_1-\sigma_2)^2-(\gamma_1^2-\gamma_2^2)\}-2\gamma_1^2(\sigma_1-\sigma_2)]$$

$$x_2=-\gamma_2^2[2(\sigma_2-\sigma_1)(\sigma_2-\sigma_3)+(\sigma_2-\sigma_1)^2-(\gamma_2^2-\gamma_1^2)]$$

$$y_2=\gamma_2[(\sigma_2-\sigma_3)\{(\sigma_2-\sigma_1)^2-(\gamma_2^2-\gamma_1^2)\}-2\gamma_2^2(\sigma_2-\sigma_1)]$$

Thus, the solution of $p\Delta\theta$ in time-domain is the Laplace inverse of $p\Delta\theta(\rho)$ of equation (3), which is equal to h(t), where:

$$h(t)=K_3e^{\sigma_3 t}+e^{\sigma_1 t}(X_1\cos\gamma_1 t-Y_1\sin\gamma_1 t)+e^{\sigma_2 t}(X_2\cos\gamma_2 t-Y_2\sin\gamma_2 t)\cdot \tag{4}$$