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Times Semi-Passive RFID Tags with Double Loop Antennas Arranged as a Shifted Gate Stability Optimization

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Abstract

In this article, Very Crucial subject discussed in Semi-Passive RFID TAGs system stability. Semi-Passive TAGs with double loop antennas arranged as a shifted gate system stability optimization under delayed electromagnetic interferences. The double loop antenna is employed due to the fact that this antenna consists of two parallel loops; i.e., primary and secondary loops. We define $V_{i1}(t)$ and $V_{i2}(t)$ as the voltages in time on double loop antennas. $V_{i1}(t)$ is the voltage in time on the primary loop and $V_{i2}(t)$ is the voltage in time on the secondary loop. The index (i) stand for the first gate (i=1) and second gate (i=2). Due to electromagnetic interferences there are different in time delays respect to gate antenna's first and second loop voltages and voltages derivatives. The delayed voltages are $V_{i1}(t-\tau_1)$ and $V_{i2}(t-\tau_2)$ respectively ($\tau_1 \neq \tau_2$) and delayed voltages derivatives are $dV_{i1}(t-\Delta_1)/dt$, $dV_{i2}(t-\Delta_2)/dt$ respectively $(\Delta_1 \neq \Delta_2\ ;\ \tau_1 \geq 0\ ;\ \tau_2 \geq 0\ ;\ \Delta_1, \Delta_2 \geq 0)$.

Keywords: Double loop antenna, Shifted Gate antennas, Delay Differential Equations (DDEs), Bifurcation, Stability

1. Introduction

In this article, Very Critical and useful subject is discussed: Semi-Passive RFID TAGs system stability.A semi-passive tags operate similarly to passive RFID tags. However, they contain a battery that enables longer reading distance and also enables the tag to operate independently of the reader.Semi-Passive TAGs with double loop antennas arranged as a shifted gate system influence by electromagnetic interferences which effect there stability behavior. The below figure describes the double loop antennas as a shifted gate in x-direction.

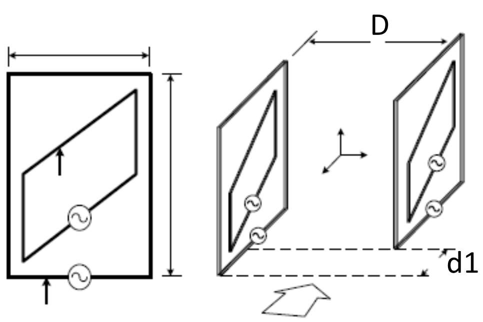

Figure 1. Double loop antennas arranged as a shifted gate in x-direction.

The Semi-Passive RFID TAG with double loop antennas equivalent circuit can be represent as a delayed differential equations which depending on variable parameters and delays.

2. Semi-passive RFID Tag with Double Loop Gate Antenna Equivalent Circuit and Represent Delay Differential Equations

Semi-Passive RFID TAG with double loop antenna can be representing as a two inductors in series (L_{11} and L_{12} for the first double loop gate antenna) with parasitic resistance r_{P1}. The double loop antennas in series are connected in parallel to Semi-Passive RFID TAG. The Equivalent Circuit of Semi-Passive RFID TAG is Capacitor (C_1) and Resistor (R_1) in parallel with voltage generator $V_{s1}(t)$ and parasitic resistance $r_{S1.}$ In case we have Passive RFID TAG switch S_1 is OFF otherwise is ON (Reader/Active RFID system) and long distance is achievable. The second double loop gate antenna is defined as two inductors in series L_{21} and L_{22} with series parasitic resistor r_{P2}. $V_{s2}(t)$ and parasitic resistance r_{S2} are belong to the second gate antenna system with another Semi-Passive RFID TAG (Supakit, et al.) .

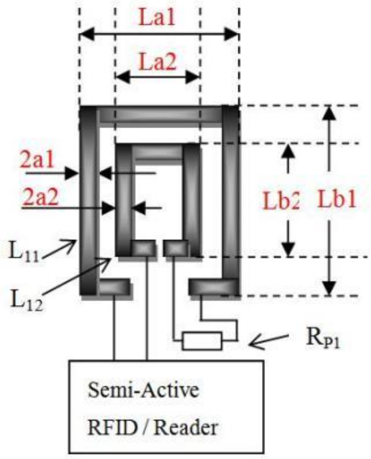

Figure 2.Double loop antennas in series with parasitic resistance and Semi-Passive RFID TAG.

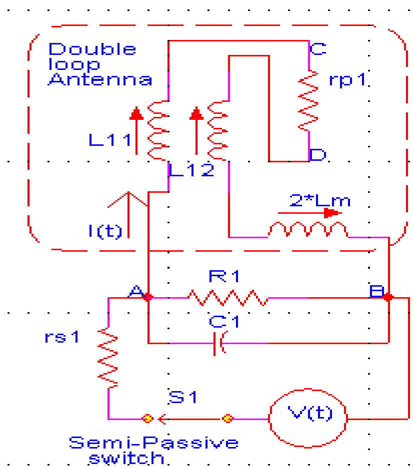

Figure 3. Equivalent circuit of Double loop antennas in series with Semi-Passive RFID TAG.

L_{11} and L_{12} are mostly formed by traces on planar PCB. $2 \cdot L_m$ element represents the mutual inductance between L_{11} and L_{12}. We consider that the double loop antennas parameters values (L_{a1}, L_{a2}, L_{b1}, L_{b2}, a_1, a_2) are the same in the first and second gates. Since two inductors (L_{11}, L_{12}) are in series and there is a mutual inductance between L_{11} and L_{12}, the total antennainductance L_T: $L_T = L_{11} + L_{12} + 2 \cdot L_m$and $L_m = K \cdot \sqrt{L_{11} \cdot L_{12}}$. L_m is the mutual inductance between L_{11} and L_{12}. K is the coupling coefficient of two inductors $0 \le K \le 1$. We start with the case of passive RFID TAG which switch S1 is OFF. I(t) is the current that flow through double loop antenna. V_{11} and V_{12} are the voltages on L_{11} and L_{12} respectively. V_m is the voltage on double loop antenna mutual inductance element.

$$V_{11} = L_{11} \cdot \frac{dI}{dt} \; ; V_{12} = L_{12} \cdot \frac{dI}{dt} \; ; V_{CD} = I \cdot r_{p1} \; ; V_m = 2 \cdot L_m \cdot \frac{dI}{dt} \; ; V_{AB} = V_{R1} = V_{C1} = V_{11} + V_{12} + V_{CD} + V_m \; ; I_{C1} = C_1 \cdot \frac{dV_{C1}}{dt} \quad (1)$$

$$I_{C1} + I_{R1} + I = 0 \Rightarrow C_1 \cdot \frac{dV_{C1}}{dt} + \frac{V_{C1}}{R_1} + I = 0 \; ; L_{11} \neq L_{12} \; ; \frac{dV_{C1}}{dt} = \frac{dV_{11}}{dt} + \frac{dV_{12}}{dt} + \frac{dV_{CD}}{dt} + \frac{dV_m}{dt} \; ; I = \frac{1}{L_{11}} \cdot \int V_{11} \cdot dt = \frac{1}{L_{12}} \cdot \int V_{12} \cdot dt \quad (2)$$

$$\frac{dV_{C1}}{dt} = \frac{dV_{11}}{dt} + \frac{dV_{12}}{dt} + \frac{dV_{CD}}{dt} + \frac{dV_m}{dt} \; ; I = \frac{1}{L_{11}} \cdot \int V_{11} \cdot dt = \frac{1}{L_{12}} \cdot \int V_{12} \cdot dt \; ; V_{CD} = I \cdot r_{p1} = \frac{r_{p1}}{L_{11}} \cdot \int V_{11} \cdot dt = \frac{r_{p1}}{L_{12}} \cdot \int V_{12} \cdot dt \quad (3)$$

$$\frac{dV_{CD}}{dt} = \frac{r_{p1}}{L_{11}} \cdot V_{11} + \frac{r_{p1}}{L_{12}} \cdot V_{12} \; ; \; V_{11} = \frac{L_{11}}{L_{12}} \cdot V_{12} \; ; \; V_{12} = \frac{L_{12}}{L_{11}} \cdot V_{11} \; ; \; I = \frac{1}{L_{11}} \cdot \int V_{11} \cdot dt = \frac{1}{L_{12}} \cdot \int V_{12} \cdot dt \Rightarrow \frac{dI}{dt} = \frac{1}{L_{11}} \cdot V_{11} = \frac{1}{L_{12}} \cdot V_{12} \quad (4)$$

$$V_m = 2 \cdot L_m \cdot \frac{dI}{dt} = 2 \cdot K \cdot \sqrt{L_{11} \cdot L_{12}} \cdot \frac{1}{L_{11}} \cdot V_{11} = 2 \cdot K \cdot \sqrt{\frac{L_{12}}{L_{11}}} \cdot V_{11} \; ; \; \frac{dV_m}{dt} = 2 \cdot K \cdot \sqrt{\frac{L_{12}}{L_{11}}} \cdot \frac{dV_{11}}{dt} \quad (5)$$

We get the following differential equation respect to $V_{11}(t)$ variable, η_1, η_2, η_3 are global parameters.

$$\frac{d^2 V_{11}}{dt^2} \cdot \eta_1 + \frac{dV_{11}}{dt} \cdot \eta_2 + V_{11} \cdot \eta_3 = 0 \; ; \; \eta_1 = C_1 \cdot (1 + \frac{L_{12}}{L_{11}} + 2 \cdot K \cdot \sqrt{\frac{L_{12}}{L_{11}}}) \; ; \; \eta_2 = \frac{C_1 \cdot r_{p1}}{L_{11}} + \frac{1}{R_1} \cdot (1 + \frac{L_{12}}{L_{11}} + 2 \cdot K \cdot \sqrt{\frac{L_{12}}{L_{11}}}) \quad (6)$$

$$\eta_2 = \frac{C_1 \cdot r_{p1}}{L_{11}} + \frac{1}{R_1} \cdot \frac{\eta_1}{C_1} \; ; \; \eta_3 = \frac{1}{L_{11}} \cdot (1 + \frac{r_{p1}}{R_1}) \quad (7)$$

$$\eta_1 = \eta_1(C_1, L_{12}, L_{11}, K) \; ; \; \eta_2 = \eta_2(C_1, r_{p1}, L_{12}, L_{11}, K, R_1) \; ; \; \eta_3 = \eta_3(L_{11}, r_{p1}, R_1) \; ; \; V'_{11} = \frac{dV_{11}}{dt} \; ; \; \frac{dV'_{11}}{dt} = \frac{d^2 V_{11}}{dt^2} \quad (8)$$

$$\frac{dV'_{11}}{dt} = -V'_{11} \cdot \frac{\eta_2}{\eta_1} - V_{11} \cdot \frac{\eta_3}{\eta_1} \; ; \; \frac{dV_{11}}{dt} = V'_{11} \quad (9)$$

In the same manner we find our V_{12} differential equation. We get the following differential equation respect to $V_{12}(t)$ variable, ξ_1, ξ_2, ξ_3 are global parameters.

$$\frac{d^2 V_{12}}{dt^2} \cdot \xi_1 + \frac{dV_{12}}{dt} \cdot \xi_2 + V_{12} \cdot \xi_3 = 0 \; ; \; \xi_1 = C_1 \cdot (1 + \frac{L_{11}}{L_{12}} + 2 \cdot K \cdot \sqrt{\frac{L_{11}}{L_{12}}}) \; ; \; \xi_2 = \frac{C_1 \cdot r_{p1}}{L_{12}} + \frac{1}{R_1} \cdot (1 + \frac{L_{11}}{L_{12}} + 2 \cdot K \cdot \sqrt{\frac{L_{11}}{L_{12}}}) \; ; \; \xi_3 = \frac{1}{L_{12}} \cdot (1 + \frac{r_{p1}}{R_1}) \quad (10)$$

$$\xi_2 = \frac{C_1 \cdot r_{p1}}{L_{12}} + \frac{1}{R_1} \cdot \frac{\xi_1}{C_1} \; ; \; V'_{12} = \frac{dV_{12}}{dt} \; ; \; \frac{dV'_{12}}{dt} = \frac{d^2 V_{12}}{dt^2} \quad (11)$$

$$\xi_1 = \xi_1(C_1, L_{12}, L_{11}, K) \; ; \; \xi_2 = \xi_2(C_1, r_{p1}, L_{12}, L_{11}, K, R_1) \; ; \; \xi_3 = \xi_3(L_{12}, r_{p1}, R_1) \; ; \; \frac{dV'_{12}}{dt} = -V'_{12} \cdot \frac{\xi_2}{\xi_1} - V_{12} \cdot \frac{\xi_3}{\xi_1} \; ; \; \frac{dV_{12}}{dt} = V'_{12} \quad (12)$$

Summary: We get our RFID double loop antennas system's four differential equations.

$$\frac{dV'_{11}}{dt} = -V'_{11} \cdot \frac{\eta_2}{\eta_1} - V_{11} \cdot \frac{\eta_3}{\eta_1} \; ; \; \frac{dV_{11}}{dt} = V'_{11} \; ; \; \frac{dV'_{12}}{dt} = -V'_{12} \cdot \frac{\xi_2}{\xi_1} - V_{12} \cdot \frac{\xi_3}{\xi_1} \; ; \; \frac{dV_{12}}{dt} = V'_{12} \quad (13)$$

$$\begin{pmatrix} \frac{dV'_{11}}{dt} \\ \frac{dV_{11}}{dt} \\ \frac{dV'_{12}}{dt} \\ \frac{dV_{12}}{dt} \end{pmatrix} = \begin{pmatrix} \Gamma_{11} & \cdots & \Gamma_{14} \\ \vdots & \ddots & \vdots \\ \Gamma_{41} & \cdots & \Gamma_{44} \end{pmatrix} \cdot \begin{pmatrix} V'_{11} \\ V_{11} \\ V'_{12} \\ V_{12} \end{pmatrix} \; ; \; \Gamma_{11} = -\frac{\eta_2}{\eta_1} \; ; \; \Gamma_{12} = -\frac{\eta_3}{\eta_1} \; ; \; \Gamma_{33} = -\frac{\xi_2}{\xi_1} \; ; \; \Gamma_{34} = -\frac{\xi_3}{\xi_1} \; ; \; \Gamma_{21} = \Gamma_{43} = 1 \quad (14)$$

$$\Gamma_{13} = \Gamma_{14} = \Gamma_{22} = \Gamma_{23} = \Gamma_{24} = \Gamma_{31} = \Gamma_{32} = \Gamma_{41} = \Gamma_{42} = \Gamma_{44} = 0 \quad (15)$$

The RFID double loop antennas system's primary and secondary loops are composed of a thin wire or a thin plate element (Figure 2). Units are all in cm, and a_1, a_2 are radiuses of the primary and secondary wires in cm. There inductances can be calculated by the following formulas:

$$L_{11} = 4 \cdot \{L_{b1} \cdot \ln[\frac{2 \cdot A_1}{a_1 \cdot (L_{b1} + l_{c1})}] + L_{a1} \cdot \ln[\frac{2 \cdot A_1}{a_1 \cdot (L_{b1} + l_{c1})}] + 2 \cdot [a_1 + l_{c1} - (L_{a1} + L_{b1})]\} \quad (16)$$

$$L_{12} = 4 \cdot \{L_{b2} \cdot \ln[\frac{2 \cdot A_2}{a_2 \cdot (L_{b2} + l_{c2})}] + L_{a2} \cdot \ln[\frac{2 \cdot A_2}{a_2 \cdot (L_{b2} + l_{c2})}] + 2 \cdot [a_2 + l_{c2} - (L_{a2} + L_{b2})]\} ; l_{c1} = \sqrt{L_{a1}^2 + L_{b1}^2} ; A_1 = L_{a1} \cdot L_{b1} ; l_{c2} = \sqrt{L_{a2}^2 + L_{b2}^2} ; A_2 = L_{a2} \cdot L_{b2} \quad (17)$$

Due to electromagnetic interferences we get a shifted gate RFID system's primary and secondary antennas loops voltages with delays τ_1 and τ_2 respectively. Additionally we get antennas loops voltages derivatives with delays Δ_1 and Δ_2 respectively.

$$V_{11}(t) \to V_{11}(t-\tau_1) ; V_{12}(t) \to V_{12}(t-\tau_2) ; V_{11}^{'}(t) \to V_{11}^{'}(t-\Delta_1) ; V_{12}^{'}(t) \to V_{12}^{'}(t-\Delta_2) \quad (18)$$

$V_{12}^{'}(t) \to V_{12}^{'}(t-\Delta_2)$. We consider no delay effect on $\frac{dV_{11}}{dt}$; $\frac{dV_{12}}{dt}$; $\frac{dV_{11}^{'}}{dt}$; $\frac{dV_{12}^{'}}{dt}$. (19)

The RFID shifted gate system differential equations under electromagnetic interferences (delays terms) influence only RFID double loop voltages $V_{11}(t)$, $V_{12}(t)$ and voltages derivatives $V_{11}^{'}(t)$ and $V_{12}^{'}(t)$ respect to time, there is no influence on

$$\frac{dV_{11}(t)}{dt} ; \frac{dV_{12}(t)}{dt} ; \frac{dV_{11}^{'}(t)}{dt} ; \frac{dV_{12}^{'}(t)}{dt} ; \begin{pmatrix} \frac{dV_{11}^{'}}{dt} \\ \frac{dV_{11}}{dt} \\ \frac{dV_{12}^{'}}{dt} \\ \frac{dV_{12}}{dt} \end{pmatrix} = \begin{pmatrix} \Gamma_{11} & \cdots & \Gamma_{14} \\ \vdots & \ddots & \vdots \\ \Gamma_{41} & \cdots & \Gamma_{44} \end{pmatrix} \cdot \begin{pmatrix} V_{11}^{'}(t-\Delta_1) \\ V_{11}(t-\tau_1) \\ V_{12}^{'}(t-\Delta_2) \\ V_{12}(t-\tau_2) \end{pmatrix}. \quad (20)$$

To find equilibrium points (fixed points) of the RFID shifted gate system is by

$$\lim_{t\to\infty} V_{11}(t-\tau_1) = V_{11}(t), \ \lim_{t\to\infty} V_{12}(t-\tau_2) = V_{12}(t), \ \lim_{t\to\infty} V_{11}^{'}(t-\Delta_1) = V_{11}^{'}(t), \quad (21)$$

$$\lim_{t\to\infty} V_{12}^{'}(t-\Delta_2) = V_{12}^{'}(t). \ \frac{dV_{11}(t)}{dt} = 0 ; \frac{dV_{12}(t)}{dt} = 0 ; \frac{dV_{11}^{'}(t)}{dt} = 0 ; \frac{dV_{12}^{'}(t)}{dt} = 0. \ \forall \ t \gg \tau_1 ; t \gg \tau_2 ; t \gg \Delta_1 ; t \gg \Delta_2 \quad (22)$$

$$\exists \ (t-\tau_1) \approx t ; (t-\tau_2) \approx t ; (t-\Delta_1) \approx t ; (t-\Delta_2) \approx t, \ t \to \infty \quad (23)$$

We get four equations and the only fixed point is $E^{(0)}(V_{11}^{'(0)}, V_{11}^{(0)}, V_{12}^{'(0)}, V_{12}^{(0)}) = (0,0,0,0)$ Since (24)

$\eta_3 \neq 0$ & $\eta_1 \neq 0 \Rightarrow \Gamma_{12} \neq 0$; $\xi_3 \neq 0$ & $\xi_1 \neq 0 \Rightarrow \Gamma_{34} \neq 0$. Stability analysis: The standard local stability analysis about any one of the equilibrium points of RFID shifted gate system consists in adding to coordinates $[V_{11}^{'} \ V_{11} \ V_{12}^{'} \ V_{12}]$ arbitrarily small increments of exponential form $[v_{11}^{'} \ v_{11} \ v_{12}^{'} \ v_{12}] \cdot e^{\lambda \cdot t}$, and retaining the first order terms in $V_{11}^{'} \ V_{11} \ V_{12}^{'} \ V_{12}$. The system of four homogeneous equations leads to a polynomial characteristics equation in the eigenvalues λ. The polynomial characteristics equations accept by set the below voltages and voltages derivative respect to time into two RFID shifted gate system equations.

RFID shifted gate system fixed values with arbitrarily small increments of exponential form $[v_{11}^{'} \ v_{11} \ v_{12}^{'} \ v_{12}] \cdot e^{\lambda \cdot t}$ are: i=0 (first fixed point), i=1 (second fixed point), i=2 (third fixed point), etc.,

$$V_{11}^{'}(t) = V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot t} \ ; \ V_{11}(t) = V_{11}^{(i)} + v_{11} \cdot e^{\lambda \cdot t} \ ; \ V_{12}^{'}(t) = V_{12}^{'(i)} + v_{12}^{'} \cdot e^{\lambda \cdot t} \ ; \ V_{12}(t) = V_{12}^{(i)} + v_{12} \cdot e^{\lambda \cdot t} \quad (25)$$

We choose the above expressions for our $V_{11}^{'}(t),\ V_{11}(t)$ and $V_{12}^{'}(t),\ V_{12}(t)$ as small displacement $[v_{11}^{'} \ v_{11} \ v_{12}^{'} \ v_{12}]$ from the system fixed points at time t=0.

$$V_{11}^{'}(t=0) = V_{11}^{'(i)} + v_{11}^{'} \ ; \ V_{11}(t=0) = V_{11}^{(i)} + v_{11} \ ; \ V_{12}^{'}(t=0) = V_{12}^{'(i)} + v_{12}^{'} \ ; \ V_{12}(t=0) = V_{12}^{(i)} + v_{12} \quad (26)$$

For $\lambda < 0,\ t > 0$ the selected fixed point is stable otherwise $\lambda > 0,\ t > 0$ is Unstable. Our system tends to the selected fixed point exponentially for $\lambda < 0,\ t > 0$ otherwise go away from the selected fixed point exponentially. λ is the eigenvalue parameter which establish if the fixed point is stable or Unstable, additionally his absolute value ($|\lambda|$)establish the speed of flow toward or away from the selected fixed point (Yuri&Jack).

Table 1. Semi-passive RFID TAGs with double loop antennas, variables function of λ eigenvalue and time.

	λ<0	λ>0
t=0	$V_{11}^{'}(t=0) = V_{11}^{'(i)} + v_{11}^{'}$; $V_{11}(t=0) = V_{11}^{(i)} + v_{11}$ $V_{12}^{'}(t=0) = V_{12}^{'(i)} + v_{12}^{'}$ $V_{12}(t=0) = V_{12}^{(i)} + v_{12}$	$V_{11}^{'}(t=0) = V_{11}^{'(i)} + v_{11}^{'}$; $V_{11}(t=0) = V_{11}^{(i)} + v_{11}$ $V_{12}^{'}(t=0) = V_{12}^{'(i)} + v_{12}^{'}$ $V_{12}(t=0) = V_{12}^{(i)} + v_{12}$
t>0	$V_{11}^{'}(t) = V_{11}^{'(i)} + v_{11}^{'} \cdot e^{-\lvert\lambda\rvert \cdot t}$; $V_{11}(t) = V_{11}^{(i)} + v_{11} \cdot e^{-\lvert\lambda\rvert \cdot t}$ $V_{12}^{'}(t) = V_{12}^{'(i)} + v_{12}^{'} \cdot e^{-\lvert\lambda\rvert \cdot t}$ $V_{12}(t) = V_{12}^{(i)} + v_{12} \cdot e^{-\lvert\lambda\rvert \cdot t}$	$V_{11}^{'}(t) = V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lvert\lambda\rvert \cdot t}$; $V_{11}(t) = V_{11}^{(i)} + v_{11} \cdot e^{\lvert\lambda\rvert \cdot t}$ $V_{12}^{'}(t) = V_{12}^{'(i)} + v_{12}^{'} \cdot e^{\lvert\lambda\rvert \cdot t}$ $V_{12}(t) = V_{12}^{(i)} + v_{12} \cdot e^{\lvert\lambda\rvert \cdot t}$
t>0 t→∞	$V_{11}^{'}(t \to \infty) = V_{11}^{'(i)}$; $V_{11}(t \to \infty) = V_{11}^{(i)}$ $V_{12}^{'}(t \to \infty) = V_{12}^{'(i)}$ $V_{12}(t \to \infty) = V_{12}^{(i)}$	$V_{11}^{'}(t \to \infty, \lambda > 0) \approx v_{11}^{'} \cdot e^{\lvert\lambda\rvert \cdot t}$ $V_{11}(t \to \infty, \lambda > 0) \approx v_{11} \cdot e^{\lvert\lambda\rvert \cdot t}$; $V_{12}^{'}(t \to \infty, \lambda > 0) \approx v_{12}^{'} \cdot e^{\lvert\lambda\rvert \cdot t}$ $V_{12}(t \to \infty, \lambda > 0) \approx v_{11} \cdot e^{\lvert\lambda\rvert \cdot t}$

The speeds of flow toward or away from the selected fixed point for RFID shifted gate system voltages and voltages derivatives respect to time are

$$\frac{dV_{11}^{'}(t)}{dt} = \lim_{\Delta t \to 0} \frac{V_{11}^{'}(t+\Delta t) - V_{11}^{'}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot (t+\Delta t)} - [V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot t}]}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_{11}^{'} \cdot e^{\lambda \cdot t} \cdot [e^{\lambda \cdot \Delta t} - 1]}{\Delta t} \xrightarrow{e^{\lambda \cdot \Delta t} \approx 1 + \lambda \cdot \Delta t} \lambda \cdot v_{11}^{'} \cdot e^{\lambda \cdot t} \quad (27)$$

$$\frac{dV_{11}(t)}{dt} = \lambda \cdot v_{11} \cdot e^{\lambda \cdot t} \ ; \ \frac{dV_{12}(t)}{dt} = \lambda \cdot v_{12} \cdot e^{\lambda \cdot t} \ ; \ \frac{dV_{12}^{'}(t)}{dt} = \lambda \cdot v_{12}^{'} \cdot e^{\lambda \cdot t} \ ; \ \frac{dV_{11}^{'}(t-\Delta_1)}{dt} = \lambda \cdot v_{11}^{'} \cdot e^{\lambda \cdot t} \cdot e^{-\lambda \cdot \Delta_1} \ ; \ \frac{dV_{11}(t-\tau_1)}{dt} = \lambda \cdot v_{11} \cdot e^{\lambda \cdot t} \cdot e^{-\lambda \cdot \tau_1} \quad (28)$$

$$\frac{dV_{12}(t-\tau_2)}{dt} = \lambda \cdot v_{12} \cdot e^{\lambda \cdot t} \cdot e^{-\lambda \cdot \tau_2} \ ; \ \frac{dV_{12}^{'}(t-\Delta_2)}{dt} = \lambda \cdot v_{12}^{'} \cdot e^{\lambda \cdot t} \cdot e^{-\lambda \cdot \Delta_2} \quad (29)$$

First we take the RFID shifted gate's voltages V_{11}, V_{12} differential equations: $\frac{dV_{11}}{dt} = V_{11}^{'}$; $\frac{dV_{12}}{dt} = V_{12}^{'}$ and adding coordinates $[V_{11}^{'}\ V_{11}\ V_{12}^{'}\ V_{12}]$ arbitrarily small increments of exponential terms $[v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}] \cdot e^{\lambda \cdot t}$ and retaining the first order terms in $v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}$.

$$\lambda \cdot v_{11} \cdot e^{\lambda \cdot t} = V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot t} \text{ ; } V_{11}^{'(i=0)} = 0 \Rightarrow \lambda_1 = \frac{v_{11}^{'}}{v_{11}} \approx 1 > 0 \text{ ; } \lambda \cdot v_{12} \cdot e^{\lambda \cdot t} = V_{12}^{'(i)} + v_{12}^{'} \cdot e^{\lambda \cdot t} \text{ ; } V_{12}^{'(i=0)} = 0 \Rightarrow \lambda_2 = \frac{v_{12}^{'}}{v_{12}} \approx 1 > 0 \quad (30)$$

Second we take the RFID shifted gate's voltages derivatives $V_{11}^{'}, V_{12}^{'}$ differential equations:

$$\frac{dV_{11}^{'}}{dt} = \Gamma_{11} \cdot V_{11}^{'} + \Gamma_{12} \cdot V_{11} \text{ ; } \frac{dV_{12}^{'}}{dt} = \Gamma_{33} \cdot V_{12}^{'} + \Gamma_{34} \cdot V_{12} \quad (31)$$

and adding coordinates $[V_{11}^{'}\ V_{11}\ V_{12}^{'}\ V_{12}]$ arbitrarily small increments of exponential terms $[v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}] \cdot e^{\lambda \cdot t}$ and retaining the first order terms in $v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}$.

$$\lambda \cdot v_{11}^{'} \cdot e^{\lambda \cdot t} = \Gamma_{11} \cdot [V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot t}] + \Gamma_{12} \cdot [V_{11}^{(i)} + v_{11} \cdot e^{\lambda \cdot t}] \text{ ; } V_{11}^{'(i=0)} = 0 \text{ ; } V_{11}^{(i=0)} = 0 \Rightarrow \lambda_3 = \Gamma_{11} + \Gamma_{12} \cdot \frac{v_{11}}{v_{11}^{'}} \text{ ; } \frac{v_{11}}{v_{11}^{'}} \approx 1 \Rightarrow \lambda_3 = \Gamma_{11} + \Gamma_{12} \quad (32)$$

$$V_{11}^{'(i=0)} = 0 \text{ ; } V_{11}^{(i=0)} = 0 \Rightarrow \lambda_3 = \Gamma_{11} + \Gamma_{12} \cdot \frac{v_{11}}{v_{11}^{'}} \text{ ; } \frac{v_{11}}{v_{11}^{'}} \approx 1 \Rightarrow \lambda_3 = \Gamma_{11} + \Gamma_{12} \text{ ; } \lambda \cdot v_{12}^{'} \cdot e^{\lambda \cdot t} = \Gamma_{33} \cdot [V_{12}^{'(i)} + v_{12}^{'} \cdot e^{\lambda \cdot t}] + \Gamma_{34} \cdot [V_{12}^{(i)} + v_{12} \cdot e^{\lambda \cdot t}] \quad (33)$$

$$V_{12}^{'(i=0)} = 0 \text{ ; } V_{12}^{(i=0)} = 0 \Rightarrow \lambda = \Gamma_{33} + \Gamma_{34} \cdot \frac{v_{12}}{v_{12}^{'}} \text{ ; } \frac{v_{12}}{v_{12}^{'}} \approx 1 \Rightarrow \lambda_4 = \Gamma_{33} + \Gamma_{34} \quad (34)$$

If $\lambda_3>0$ and $\lambda_4>0$ then our fixed point is unstable node. If ($\lambda_3>0$ and $\lambda_4<0$) or ($\lambda_3<0$ and $\lambda_4>0$) or ($\lambda_3<0$ and $\lambda_4<0$) Then our fixed point is saddle point. We define

$$V_{11}^{'}(t-\Delta_1) = V_{11}^{'(i)} + v_{11}^{'} \cdot e^{\lambda \cdot (t-\Delta_1)} \text{ ; } V_{11}(t-\tau_1) = V_{11}^{(i)} + v_{11} \cdot e^{\lambda \cdot (t-\tau_1)} \quad (35)$$

$$V_{12}^{'}(t-\Delta_2) = V_{12}^{'(i)} + v_{12}^{'} \cdot e^{\lambda \cdot (t-\Delta_2)} \text{ ; } V_{12}(t-\tau_2) = V_{12}^{(i)} + v_{12} \cdot e^{\lambda \cdot (t-\tau_2)} \quad (36)$$

Then we get four delayed differential equations respect to coordinates $[V_{11}^{'}\ V_{11}\ V_{12}^{'}\ V_{12}]$ arbitrarily small increments of exponential

$$[v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}] \cdot e^{\lambda \cdot t} . \lambda \cdot e^{\lambda \cdot t} \cdot v_{11}^{'} = \Gamma_{11} \cdot e^{\lambda \cdot (t-\Delta_1)} \cdot v_{11}^{'} + \Gamma_{12} \cdot e^{\lambda \cdot (t-\tau_1)} \cdot v_{11} \quad (37)$$

$$\lambda \cdot e^{\lambda \cdot t} \cdot v_{11} = e^{\lambda \cdot (t-\Delta_1)} \cdot v_{11}^{'} \text{ ; } \lambda \cdot e^{\lambda \cdot t} \cdot v_{12}^{'} = \Gamma_{33} \cdot e^{\lambda \cdot (t-\Delta_2)} \cdot v_{12}^{'} + \Gamma_{34} \cdot e^{\lambda \cdot (t-\tau_2)} \cdot v_{12} \quad (38)$$

$\lambda \cdot e^{\lambda \cdot t} \cdot v_{12} = e^{\lambda \cdot (t-\Delta_2)} \cdot v_{12}^{'}$. In the equilibrium fixed point $V_{11}^{'(i=0)} = 0,\ V_{11}^{(i=0)} = 0$ (39)

$V_{12}^{'(i=0)} = 0,\ V_{12}^{(i=0)} = 0$. The small increments Jacobian of our RFID shifted gate system is as bellow:

$$\Upsilon_{11} = -\lambda + \Gamma_{11} \cdot e^{-\lambda \cdot \Delta_1} \text{ ; } \Upsilon_{12} = \Gamma_{12} \cdot e^{-\lambda \cdot \tau_1} \text{ ; } \Upsilon_{13} = 0 \text{ ; } \Upsilon_{14} = 0 \text{ ; } \Upsilon_{21} = e^{-\lambda \cdot \Delta_1} \text{ ; } \Upsilon_{22} = -\lambda \text{ ; } \Upsilon_{23} = 0 \text{ ; } \Upsilon_{24} = 0 \text{ ; } \Upsilon_{31} = 0 \text{ ; } \Upsilon_{32} = 0 \quad (40)$$

$$\Upsilon_{33} = -\lambda + \Gamma_{33} \cdot e^{-\lambda \cdot \Delta_2} \text{ ; } \Upsilon_{34} = \Gamma_{34} \cdot e^{-\lambda \cdot \tau_2} \text{ ; } \Upsilon_{41} = 0 \text{ ; } \Upsilon_{42} = 0 \text{ ; } \begin{pmatrix} \Upsilon_{11} & \cdots & \Upsilon_{14} \\ \vdots & \ddots & \vdots \\ \Upsilon_{41} & \cdots & \Upsilon_{44} \end{pmatrix} \cdot \begin{pmatrix} v_{11}^{'} \\ v_{11} \\ v_{12}^{'} \\ v_{12} \end{pmatrix} = 0 \text{ ; } \Upsilon_{43} = e^{-\lambda \cdot \Delta_2} \text{ ; } \Upsilon_{44} = -\lambda \quad (41)$$

$$A-\lambda\cdot I=\begin{pmatrix}\Upsilon_{11} & \cdots & \Upsilon_{14}\\ \vdots & \ddots & \vdots\\ \Upsilon_{41} & \cdots & \Upsilon_{44}\end{pmatrix}\ ;\ \det|A-\lambda\cdot I|=0 \qquad (42)$$

$$D(\lambda,\tau_1,\tau_2,\Delta_1,\Delta_2)=\lambda^4+\Gamma_{12}\cdot\Gamma_{34}\cdot e^{-\lambda\cdot[\sum_{i=1}^{2}\tau_i+\sum_{j=1}^{2}\Delta_j]}+\lambda\cdot\{\Gamma_{11}\cdot\Gamma_{34}\cdot e^{-\lambda\cdot[\tau_2+\sum_{j=1}^{2}\Delta_j]}+\Gamma_{33}\cdot\Gamma_{12}\cdot e^{-\lambda\cdot[\tau_1+\sum_{j=1}^{2}\Delta_j]}\}$$
$$+\lambda^2\cdot\{-\Gamma_{34}\cdot e^{-\lambda\cdot(\Delta_2+\tau_2)}-\Gamma_{12}\cdot e^{-\lambda\cdot(\Delta_1+\tau_1)}+\Gamma_{11}\cdot\Gamma_{33}\cdot e^{-\lambda\cdot\sum_{j=1}^{2}\Delta_j}\}-\lambda^3\cdot\{\Gamma_{33}\cdot e^{-\lambda\cdot\Delta_2}+\Gamma_{11}\cdot e^{-\lambda\cdot\Delta_1}\} \qquad (43)$$

We have three stability cases:

$$\tau_1=\tau_2=\tau\ \&\ \Delta_1=\Delta_2=0\ \text{ Or }\ \tau_1=\tau_2=0\ \&\ \Delta_1=\Delta_2=\Delta\ \text{or}\ \tau_1=\tau_2=\Delta_1=\Delta_2=\tau_\Delta \qquad (44)$$

Otherwise $\tau_1\neq\tau_2\ \&\ \Delta_1\neq\Delta_2$ and they are positive parameters. There are other possible simple stability cases:

$$\tau_1=\tau\ ;\ \tau_2=0\ ;\ \Delta_1=\Delta_2=0\ \text{ or }\ \tau_1=0\ ;\ \tau_2=\tau\ ;\ \Delta_1=\Delta_2=0\ ;\tau_1=\tau_2=0\ ;\ \Delta_1=\Delta\ ;\ \Delta_2=0\ \text{ or }\ \tau_1=\tau_2=0\ ;\ \Delta_1=0\ ;\ \Delta_2=\Delta \qquad (45)$$

We need to get characteristics equations for all above stability analysis cases. We study the occurrence of any possible stability switching resulting from the increase of value of the time delays $\tau,\ \Delta,\ \tau_\Delta$ for the general characteristic equation $D(\lambda,\tau/\Delta/\tau_\Delta)$. If we choose τ parameter then $D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)\cdot e^{-\lambda\tau}$. The expression for $P_n(\lambda,\tau)$:

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\cdot\lambda^k=P_0(\tau)+P_1(\tau)\cdot\lambda+P_2(\tau)\cdot\lambda^2+P_3(\tau)\cdot\lambda^3+\ldots\ldots.. \qquad (46)$$

The expression for

$$Q_m(\lambda,\tau):\ Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\cdot\lambda^k=q_0(\tau)+q_1(\tau)\cdot\lambda+q_2(\tau)\cdot\lambda^2+\ldots\ldots.. \qquad (47)$$

3. RFID Shifted Gate System Fourth Order Characteristic Equation $\tau_1=\tau_2=\tau\ \&\ \Delta_1=\Delta_2=0$

The second case we analyze is when there is delay in RFID gate's primary and secondary loop antennas voltages ($\tau_1=\tau_2=\tau$) and no delay in in gate's primary and secondary loop antennas voltages derivatives (Kuang, 1993; Beretta, et al., 2002). The general characteristic equation D(λ, τ) is ad follow:

$$D(\lambda,\tau)=\lambda^4-\lambda^3\cdot(\Gamma_{33}+\Gamma_{11})+\lambda^2\cdot\Gamma_{11}\cdot\Gamma_{33}+\{\Gamma_{12}\cdot\Gamma_{34}\cdot e^{-\lambda\cdot\tau}+\lambda\cdot(\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33})-\lambda^2\cdot(\Gamma_{34}+\Gamma_{12})\}\cdot e^{-\lambda\cdot\tau} \qquad (48)$$

Under Taylor series approximation: $e^{-\lambda\cdot\tau}\approx 1-\lambda\cdot\tau+\frac{1}{2}\cdot\lambda^2\cdot\tau^2$. The Maclaurin series is a Taylor series expansion of a $e^{-\lambda\cdot\tau}$ function about zero (0). We get the following general characteristic equation D(λ, τ) under Taylor series approximation:

$$e^{-\lambda\cdot\tau}\approx 1-\lambda\cdot\tau+\frac{1}{2}\cdot\lambda^2\cdot\tau^2\cdot \qquad (49)$$

$$D(\lambda,\tau)=\lambda^4-\lambda^3\cdot[\Gamma_{33}+\Gamma_{11}]+\lambda^2\cdot\Gamma_{11}\cdot\Gamma_{33}+\{\Gamma_{12}\cdot\Gamma_{34}+\lambda\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]+\lambda^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}\cdot e^{-\lambda\cdot\tau} \qquad (50)$$

$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)\cdot e^{-\lambda\tau}\ ;\ n=4\ ;\ m=2\ ;\ n>m$. The expression for $P_n(\lambda,\tau)$ is

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\cdot\lambda^k=P_0(\tau)+P_1(\tau)\cdot\lambda+P_2(\tau)\cdot\lambda^2+P_3(\tau)\cdot\lambda^3+P_4(\tau)\cdot\lambda^4=\lambda^4-\lambda^3\cdot[\Gamma_{33}+\Gamma_{11}]+\lambda^2\cdot\Gamma_{11}\cdot\Gamma_{33} \qquad (51)$$

$$P_0(\tau)=0\ ;\ P_1(\tau)=0\ ;\ P_2(\tau)=\Gamma_{11}\cdot\Gamma_{33}\ ;\ P_3(\tau)=-[\Gamma_{33}+\Gamma_{11}]\ ;\ P_4(\tau)=1 \quad (52)$$

The expression for

$$Q_m(\lambda,\tau) \text{ is } Q_m(\lambda,\tau)=\sum_{k=0}^{m} q_k(\tau)\cdot\lambda^k = q_0(\tau)+q_1(\tau)\cdot\lambda+q_2(\tau)\cdot\lambda^2 \quad (53)$$

$$Q_m(\lambda,\tau)=\sum_{k=0}^{m} q_k(\tau)\cdot\lambda^k=\Gamma_{12}\cdot\Gamma_{34}+\lambda\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]+\lambda^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\ ;\ q_0(\tau)=\Gamma_{12}\cdot\Gamma_{34}$$
$$q_1(\tau)=\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\ ;\ q_2(\tau)=\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12} \quad (54)$$

The homogeneous system for $V_{11}'\ V_{11}\ V_{12}'\ V_{12}$ leads to a characteristic equation for the eigenvalue λ having the form $P(\lambda,\tau)+Q(\lambda,\tau)\cdot e^{-\lambda\cdot\tau}=0\ ;\ P(\lambda)=\sum_{j=0}^{4}a_j\cdot\lambda^j\ ;\ Q(\lambda)=\sum_{j=0}^{2}c_j\cdot\lambda^j$ and the coefficients $\{a_j(q_i,q_k,\tau),\ c_j(q_i,q_k,\tau)\}\in\mathbb{R}$

Depend on q_i,q_k and delay τ . q_i,q_k are any two shifted gate system's parameters, other parameters keep as a constant

.

$$a_0=0\ ;\ a_1=0\ ;\ a_2=\Gamma_{11}\cdot\Gamma_{33}\ ;\ a_3=-[\Gamma_{33}+\Gamma_{11}]\ ;\ a_4=1 \quad (55)$$

$$c_0=\Gamma_{12}\cdot\Gamma_{34}\ ;\ c_1=\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\ ;\ c_2=\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12} \quad (56)$$

Unless strictly necessary, the designation of the variation arguments (q_i,q_k) will subsequently be omitted from P, Q, a_j, c_j. The coefficients a_j, c_j are continuous, and differentiable functions of their arguments, and direct substitution shows that $a_0+c_0\neq 0$ for $\forall\ q_i,q_k\in\mathbb{R}_+$, i.e. λ=0 is not a of $P(\lambda,\tau)+Q(\lambda,\tau)\cdot e^{-\lambda\cdot\tau}=0$. We assume that $P_n(\lambda,\tau)$ and $Q_m(\lambda,\tau)$ can't have common imaginary roots. That is for any real number ω:

$$p_n(\lambda=i\cdot\omega,\tau)+Q_m(\lambda=i\cdot\omega,\tau)\neq 0\ ;\ p_n(\lambda=i\cdot\omega,\tau)=\omega^4+i\cdot\omega^3\cdot(\Gamma_{33}+\Gamma_{11})-\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33} \quad (57)$$

$$Q_m(\lambda=i\cdot\omega,\tau)=\Gamma_{12}\cdot\Gamma_{34}+i\cdot\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}] \quad (58)$$

$$p_n(\lambda=i\cdot\omega,\tau)+Q_m(\lambda=i\cdot\omega,\tau)=\omega^4-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}+\Gamma_{11}\cdot\Gamma_{33}]+\Gamma_{12}\cdot\Gamma_{34}+i\cdot\omega^3\cdot(\Gamma_{33}+\Gamma_{11})$$
$$+i\cdot\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]\neq 0 \quad (59)$$

$$|P(i\cdot\omega,\tau)|^2=\omega^8+\omega^6\cdot\{(\Gamma_{33}+\Gamma_{11})^2-2\cdot\Gamma_{11}\cdot\Gamma_{33}\}+\omega^4\cdot\Gamma_{11}^2\cdot\Gamma_{33}^2 \quad (60)$$

$$|Q(i\cdot\omega,\tau)|^2=\Gamma_{12}^2\cdot\Gamma_{34}^2+\omega^2\cdot\{[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2-2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}$$
$$+\omega^4\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2 \quad (61)$$

We need to find the expression for

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2 \quad (62)$$

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2=\omega^8+\omega^6\cdot\{(\Gamma_{33}+\Gamma_{11})^2-2\cdot\Gamma_{11}\cdot\Gamma_{33}\}$$
$$+\omega^4\cdot\{\Gamma_{11}^2\cdot\Gamma_{33}^2-[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2\}-\omega^2\cdot\{[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2 \quad (63)$$
$$-2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}-\Gamma_{12}^2\cdot\Gamma_{34}^2$$

We define the following parameters for simplicity:

$$\Xi_0=-\Gamma_{12}^2\cdot\Gamma_{34}^2\ ;\ \Xi_2=-[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2+2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]$$
$$\Xi_4=\Gamma_{11}^2\cdot\Gamma_{33}^2-[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2\ ;\ \Xi_6=(\Gamma_{33}+\Gamma_{11})^2-2\cdot\Gamma_{11}\cdot\Gamma_{33}\ ;\ \Xi_8=1 \quad (64)$$

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2=\Xi_0+\Xi_2\cdot\omega^2+\Xi_4\cdot\omega^4+\Xi_6\cdot\omega^6+\Xi_8\cdot\omega^8=\sum_{k=0}^{4}\Xi_{2\cdot k}\cdot\omega^{2\cdot k} \quad (65)$$

Hence $F(\omega,\tau)=0$ implies $\sum_{k=0}^{4}\Xi_{2\cdot k}\cdot\omega^{2\cdot k}=0$ and its roots are given by solving the above polynomial. Furthermore

$$P_R(i\cdot\omega,\tau)=\omega^4-\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33}\ ;\ P_I(i\cdot\omega,\tau)=\omega^3\cdot(\Gamma_{33}+\Gamma_{11}) \quad (66)$$

$$Q_R(i\cdot\omega,\tau)=\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\ ;\ Q_I(i\cdot\omega,\tau)=\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau] \quad (67)$$

$$\sin\theta(\tau)=\frac{-P_R(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2}\ ;\ \cos\theta(\tau)=-\frac{P_R(i\cdot\omega,\tau)\cdot Q_R(i\cdot\omega,\tau)+P_I(i\cdot\omega,\tau)\cdot Q_I(i\cdot\omega,\tau)}{|Q(i\cdot\omega,\tau)|^2} \quad (68)$$

$$\sin\theta(\tau)=\frac{-\{\omega^4-\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33}\}\cdot\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]+\omega^3\cdot(\Gamma_{33}+\Gamma_{11})\cdot\{\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}}{\Gamma_{12}^2\cdot\Gamma_{34}^2+\omega^2\cdot\{[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2-2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}+\omega^4\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2} \quad (69)$$

$$\cos\theta(\tau)=-\frac{\{\omega^4-\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33}\}\cdot\{\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}+\omega^4\cdot(\Gamma_{33}+\Gamma_{11})\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]}{\Gamma_{12}^2\cdot\Gamma_{34}^2+\omega^2\cdot\{[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2-2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}+\omega^4\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2} \quad (70)$$

That are continuous and differentiable in τ based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2(see Appendix B). This prove the theorem 1.3 (see Appendix C).

4. RFID shifted gate system stability analysis under delayed variables in time

Our RFID shifted gate homogeneous system for $v_{11}^{'}\ v_{11}\ v_{12}^{'}\ v_{12}$ leads to a characteristic equation for the eigenvalue λ having the form $P(\lambda)+Q(\lambda)\cdot e^{-\lambda\cdot\tau}=0$; Second case $\tau_1=\tau_2=\tau$; $\Delta_1=\Delta_2=0$.

$$D(\lambda,\tau_1=\tau_2=\tau,\Delta_1=\Delta_2=0)=\lambda^4-\lambda^3\cdot(\Gamma_{33}+\Gamma_{11})+\lambda^2\cdot\Gamma_{11}\cdot\Gamma_{33}+\{\Gamma_{12}\cdot\Gamma_{34}\cdot e^{-\lambda\cdot\tau}+\lambda\cdot(\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33})-\lambda^2\cdot(\Gamma_{34}+\Gamma_{12})\}\cdot e^{-\lambda\cdot\tau} \quad (71)$$

Under Taylor series approximation: $e^{-\lambda\cdot\tau}\approx 1-\lambda\cdot\tau+\frac{1}{2}\cdot\lambda^2\cdot\tau^2$. The Maclaurin series is a Taylor series expansion of a $e^{-\lambda\cdot\tau}$ function about zero (0). We get the following general characteristic equation D(λ, τ) under Taylor series approximation:

$$e^{-\lambda\cdot\tau}\approx 1-\lambda\cdot\tau+\frac{1}{2}\cdot\lambda^2\cdot\tau^2\cdot \quad (72)$$

$$D(\lambda,\tau)=\lambda^4-\lambda^3\cdot[\Gamma_{33}+\Gamma_{11}]+\lambda^2\cdot\Gamma_{11}\cdot\Gamma_{33}+\{\Gamma_{12}\cdot\Gamma_{34}+\lambda\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]+\lambda^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}\cdot e^{-\lambda\cdot\tau} \quad (73)$$

We use different parameters terminology from our last characteristics parameters definition:

$k \to j$; $p_k(\tau) \to a_j$; $q_k(\tau) \to c_j$; $n = 4$; $m = 2$; $n > m$. Additionally $P_n(\lambda,\tau) \to P(\lambda)$; $Q_m(\lambda,\tau) \to Q(\lambda)$

Then

$$P(\lambda) = \sum_{j=0}^{4} a_j \cdot \lambda^j \; ; \; Q(\lambda) = \sum_{j=0}^{2} c_j \cdot \lambda^j \; ; \; P_\lambda = \lambda^4 - \lambda^3 \cdot [\Gamma_{33} + \Gamma_{11}] + \lambda^2 \cdot \Gamma_{11} \cdot \Gamma_{33} \quad (74)$$

$$Q_\lambda = \Gamma_{12} \cdot \Gamma_{34} + \lambda \cdot [\Gamma_{11} \cdot \Gamma_{34} + \Gamma_{12} \cdot \Gamma_{33} - \Gamma_{12} \cdot \Gamma_{34} \cdot \tau] + \lambda^2 \cdot [\frac{1}{2} \cdot \Gamma_{12} \cdot \Gamma_{34} \cdot \tau^2 - \Gamma_{34} - \Gamma_{12}] \quad (75)$$

$n, m \in \mathbb{N}_0$, $n > m$ and $a_j, c_j : \mathrm{R}_{+0} \to R$ are continuous and differentiable function of τ such that $a_0 + c_0 \neq 0$. In the following "—"denotes complex and conjugate. $P(\lambda), Q(\lambda)$ Are analytic functions in λ and differentiable in τ . The coefficients $\{a_j(C_1, R_1, \text{gate antenna parametrs})$ and $c_j(C_1, R_1, \tau, \text{gate antenna parametrs})\} \in \mathbb{R}$ de pend on RFID shifted gate system's C_1, R_1, τ values and antenna parameters.

$$a_0 = 0 \; ; \; a_1 = 0 \; ; \; a_2 = \Gamma_{11} \cdot \Gamma_{33} \; ; \; a_3 = -[\Gamma_{33} + \Gamma_{11}] \; ; \; a_4 = 1 \quad (76)$$

$$c_0 = \Gamma_{12} \cdot \Gamma_{34} \; ; \; c_1 = \Gamma_{11} \cdot \Gamma_{34} + \Gamma_{12} \cdot \Gamma_{33} - \Gamma_{12} \cdot \Gamma_{34} \cdot \tau \; ; \; c_2 = \frac{1}{2} \cdot \Gamma_{12} \cdot \Gamma_{34} \cdot \tau^2 - \Gamma_{34} - \Gamma_{12} \quad (77)$$

Unless strictly necessary, the designation of the variation arguments $(R_1, C_1, \tau, \text{gate antenna parametrs})$ will subsequently be omitted from P, Q, a_j, c_j. The coefficients a_j, c_j are continuous, and differentiable functions of their arguments, and direct substitution shows that $a_0 + c_0 \neq 0$; $\Gamma_{12} \cdot \Gamma_{34} \neq 0$.

$$\frac{\eta_3 \cdot \xi_3}{\eta_1 \cdot \xi_1} = \frac{\frac{1}{L_{11} \cdot L_{12}} \cdot (1 + \frac{r_{p1}}{R_1})^2}{C_1^2 \cdot (1 + \frac{L_{12}}{L_{11}} + 2 \cdot K \cdot \sqrt{\frac{L_{12}}{L_{11}}}) \cdot (1 + \frac{L_{11}}{L_{12}} + 2 \cdot K \cdot \sqrt{\frac{L_{11}}{L_{12}}})} \neq 0 \quad (78)$$

$\forall\, C_1$, gate antenna parameters $\in \mathbb{R}_+$ i.e $\lambda = 0$ is not a root of characteristic equation. Furthermore $P(\lambda)$, $Q(\lambda)$ are analytic function of λ for which the following requirements of the analysis (see kuang, 1993, section 3.4) can also be verified in the present case (Kuang, 1993; Beretta et al., 2002).

(a) If $\lambda = i \cdot \omega$, $\omega \in \mathbb{R}$ then $P(i \cdot \omega) + Q(i \cdot \omega) \neq 0$, i.e P and Q have no common imaginary roots. This condition was verified numerically in the entire $(R_1, C_1, \text{antenna parametrs})$ domain of interest.

(b) $|Q(\lambda) / P(\lambda)|$ is bounded for $|\lambda| \to \infty$, $\mathrm{Re}\,\lambda \geq 0$. No roots bifurcation from ∞ . Indeed, in the limit

$$|\frac{Q(\lambda)}{P(\lambda)}| = |\frac{\{\Gamma_{12} \cdot \Gamma_{34} + \lambda \cdot [\Gamma_{11} \cdot \Gamma_{34} + \Gamma_{12} \cdot \Gamma_{33} - \Gamma_{12} \cdot \Gamma_{34} \cdot \tau] + \lambda^2 \cdot [\frac{1}{2} \cdot \Gamma_{12} \cdot \Gamma_{34} \cdot \tau^2 - \Gamma_{34} - \Gamma_{12}]\}}{\lambda^4 - \lambda^3 \cdot [\Gamma_{33} + \Gamma_{11}] + \lambda^2 \cdot \Gamma_{11} \cdot \Gamma_{33}}| \quad (79)$$

(c) $F(\omega) = |P(i \cdot \omega)|^2 - |Q(i \cdot \omega)|^2$ (80)

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2=\omega^8+\omega^6\cdot\{(\Gamma_{33}+\Gamma_{11})^2-2\cdot\Gamma_{11}\cdot\Gamma_{33}\}+\omega^4\cdot\{\Gamma_{11}^2\cdot\Gamma_{33}^2-[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2\}$$
$$-\omega^2\cdot\{[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2-2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\}-\Gamma_{12}^2\cdot\Gamma_{34}^2 \quad (81)$$

Has at most a finite number of zeros. Indeed, this is a polynomial in ω (degree in ω^8).

(d) Each positive root $\omega(R_1, C_1, \tau, \text{gate antenna parametrs})$ of $F(\omega)=0$ is continuous and differentiable with respect to $R_1, C_1, \tau, \text{gate antenna parametrs}$. This condition can only be assessed numerically.

In addition, since the coefficients in P and Q are real, we have $\overline{P(-i\cdot\omega)}=P(i\cdot\omega)$, and $\overline{Q(-i\cdot\omega)}=Q(i\cdot\omega)$ thus $\lambda=i\cdot\omega$, $\omega>0$ may be on eigenvalue of characteristic equation. The analysis consists in identifying the roots of characteristic equation situated on the imaginary axis of the complex λ – plane, where by increasing the parameters $R_1, C_1, \tau, \text{gate antenna parametrs}$, Reλ may, at the crossing ,Change its sign from (-) to (+), i.e. from a stable focus $E^{(0)}(V_{11}^{'(0)}, V_{11}^{(0)}, V_{12}^{'(0)}, V_{12}^{(0)})=(0,0,0,0)$ to an unstable one, or vice versa. This feature may be further assessed by examining the sign of the partial derivatives with respect to C_1, R_1, τ and gate antenna parameters.

$$\wedge^{-1}(C_1)=(\frac{\partial\operatorname{Re}\lambda}{\partial C_1})_{\lambda=i\cdot\omega},\ \mathrm{R}_1,\tau,\text{gate antenna parametrs}=const\ ;\ \wedge^{-1}(\mathrm{R}_1)=(\frac{\partial\operatorname{Re}\lambda}{\partial \mathrm{R}_1})_{\lambda=i\cdot\omega},\ C_1,\tau,\text{gate antenna parametrs}=const \quad (82)$$

$$\wedge^{-1}(\mathrm{R}_1)=(\frac{\partial\operatorname{Re}\lambda}{\partial \mathrm{R}_1})_{\lambda=i\cdot\omega},\ C_1,\tau,\text{gate antenna parametrs}=const\ ;\ \wedge^{-1}(L_{11})=(\frac{\partial\operatorname{Re}\lambda}{\partial L_{11}})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\tau=const \quad (83)$$

$$\wedge^{-1}(L_{11})=(\frac{\partial\operatorname{Re}\lambda}{\partial L_{11}})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\tau=const\ ;\ \wedge^{-1}(L_{12})=(\frac{\partial\operatorname{Re}\lambda}{\partial L_{12}})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\tau=const \quad (84)$$

$$\wedge^{-1}(L_{12})=(\frac{\partial\operatorname{Re}\lambda}{\partial L_{12}})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\tau=const\ ;\ \wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial \tau})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\text{gate antenna parametrs}=const,\ \text{where }\omega\in\mathbb{R}_+. \quad (85)$$

$$\wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial \tau})_{\lambda=i\cdot\omega},\ C_1,\mathrm{R}_1,\text{gate antenna parametrs}=const,\ \text{where }\omega\in\mathbb{R}_+. \quad (86)$$

For the case $\tau_1=\tau_2=\tau\ \&\ \Delta_1=\Delta_2=0$ we get the following results:

$$P_R(i\cdot\omega,\tau)=\omega^4-\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33}\ ;\ P_I(i\cdot\omega,\tau)=\omega^3\cdot(\Gamma_{33}+\Gamma_{11}) \quad (87)$$

$$Q_R(i\cdot\omega,\tau)=\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\ ;\ Q_I(i\cdot\omega,\tau)=\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau] \quad (88)$$

$$\Xi_0=-\Gamma_{12}^2\cdot\Gamma_{34}^2\ ;\ \Xi_2=-[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2+2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]$$
$$\Xi_4=\Gamma_{11}^2\cdot\Gamma_{33}^2-[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]^2\ ;\ \Xi_6=(\Gamma_{33}+\Gamma_{11})^2-2\cdot\Gamma_{11}\cdot\Gamma_{33}\ ;\ \Xi_8=1 \quad (89)$$

$$F(\omega,\tau)=|P(i\cdot\omega,\tau)|^2-|Q(i\cdot\omega,\tau)|^2=\Xi_0+\Xi_2\cdot\omega^2+\Xi_4\cdot\omega^4+\Xi_6\cdot\omega^6+\Xi_8\cdot\omega^8=\sum_{k=0}^{4}\Xi_{2\cdot k}\cdot\omega^{2\cdot k} \quad (90)$$

Hence

$$F(\omega,\tau)=0 \text{ implies } \sum_{k=0}^{4}\Xi_{2\cdot k}\cdot\omega^{2\cdot k}=0 \quad (91)$$

When writing $P(\lambda) = P_R(\lambda) + i \cdot P_I(\lambda)$ and $Q(\lambda) = Q_R(\lambda) + i \cdot Q_I(\lambda)$, and inserting $\lambda = i \cdot \omega$

Into RFID Gate system's characteristic equation , ω must satisfy the following :

$$\sin \omega \cdot \tau = g(\omega) = \frac{-P_R(i\cdot\omega)\cdot Q_I(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_R(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \; ; \; \cos \omega \cdot \tau = h(\omega) = -\frac{P_R(i\cdot\omega)\cdot Q_R(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_I(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (92)$$

Where $|Q(i\cdot\omega)|^2 \neq 0$ in view of requirement (a) above, and $(g, h) \in R$. Furthermore, it follows above $\sin \omega \cdot \tau$ and $\cos \omega \cdot \tau$ equations that, by squaring and adding the sides, ω must be a positive root of $F(\omega) = |P(i\cdot\omega)|^2 - |Q(i\cdot\omega)|^2 = 0$. Note that $F(\omega)$ is dependent of τ . Now it is important to notice that if $\tau \notin I$ (assume that $I \subseteq R_{+0}$ is the set where $\omega(\tau)$ is a positive root of $F(\omega)$ and for $\tau \notin I$, $\omega(\tau)$ is not define. Then for all τ in I $\omega(\tau)$ is satisfies that $F(\omega, \tau) = 0$). Then there are no positive $\omega(\tau)$ solutions for $F(\omega, \tau) = 0$, and we cannot have stability switches. For any $\tau \in I$ where $\omega(\tau)$ is a positive solution of $F(\omega, \tau) = 0$, we can define the angle $\theta(\tau) \in [0, 2\cdot\pi]$ as the solution of

$$\sin \theta(\tau) = \frac{-P_R(i\cdot\omega)\cdot Q_I(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_R(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \; ; \; \cos \theta(\tau) = -\frac{P_R(i\cdot\omega)\cdot Q_R(i\cdot\omega) + P_I(i\cdot\omega)\cdot Q_I(i\cdot\omega)}{|Q(i\cdot\omega)|^2} \quad (93)$$

And the relation between the argument $\theta(\tau)$ and $\omega(\tau)\cdot\tau$ fo $\tau \in I$ must be $\omega(\tau)\cdot\tau = \theta(\tau) + n\cdot 2\cdot\pi \;\; \forall \;\; n \in \mathbb{N}_0$

. Hence we can define the maps $\tau_n : I \to R_{+0}$ given by

$\tau_n(\tau) = \frac{\theta(\tau) + n\cdot 2\cdot\pi}{\omega(\tau)}$; $n \in \mathbb{N}_0, \tau \in I$. Let us introduce the functions $I \to R$; $S_n(\tau) = \tau - \tau_n(\tau), \; \tau \in I, \; n \in \mathbb{N}_0$

That are continuous and differentiable in τ . In the following, the subscripts $\lambda, \; \omega, R_1, \; C_1$ and RFID Gate antenna parameters $(L_{a1}, L_{a2}, L_{b1}, L_{b2}, a_1, a_2)$ indicate the corresponding partial derivatives. Let us first concentrate on $\wedge(x)$,remember in $\lambda(L_{a1}, L_{a2}, L_{b1}, L_{b2}, a_1, a_2)$ and $\omega(L_{a1}, L_{a2}, L_{b1}, L_{b2}, a_1, a_2)$, and keeping all parameters except one (x) and τ . The derivation closely follows that in reference [BK]. Differentiating RFID characteristic equation $P(\lambda) + Q(\lambda)\cdot e^{-\lambda\cdot\tau} = 0$ with respect to specific parameter (x), and inverting the derivative, for convenience, one calculates: <u>Remark</u>: $x = R_1, C_1, L_{a1}, L_{a2}, L_{b1}, L_{b2}, a_1, a_2, etc.,$

$$\left(\frac{\partial\lambda}{\partial x}\right)^{-1} = \frac{-P_\lambda(\lambda, x)\cdot Q(\lambda, x) + Q_\lambda(\lambda, x)\cdot P(\lambda, x) - \tau\cdot P(\lambda, x)\cdot Q(\lambda, x)}{P_x(\lambda, x)\cdot Q(\lambda, x) - Q_x(\lambda, x)\cdot P(\lambda, x)} \quad (94)$$

Where $P_\lambda = \frac{\partial P}{\partial \lambda}$,.... etc., Substituting $\lambda = i\cdot\omega$, and bearing i $\overline{P(-i\cdot\omega)} = P(i\cdot\omega)$, $\overline{Q(-i\cdot\omega)} = Q(i\cdot\omega)$

Then $i\cdot P_{\lambda}(i\cdot\omega)=P_{\omega}(i\cdot\omega)$ and $i\cdot Q_{\lambda}(i\cdot\omega)=Q_{\omega}(i\cdot\omega)$ and that on the surface $|P(i\cdot\omega)|^2=|Q(i\cdot\omega)|^2$, one obtains

$$(\frac{\partial\lambda}{\partial x})^{-1}|_{\lambda=i\cdot\omega}=(\frac{i\cdot P_{\omega}(i\cdot\omega,x)\cdot\overline{P(i\cdot\omega,x)}+i\cdot Q_{\lambda}(i\cdot\omega,x)\cdot\overline{Q(\lambda,x)}-\tau\cdot|P(i\cdot\omega,x)|^2}{P_x(i\cdot\omega,x)\cdot\overline{P(i\cdot\omega,x)}-Q_x(i\cdot\omega,x)\cdot\overline{Q(i\cdot\omega,x)}})\quad(95)$$

Upon separating into real and imaginary parts, with

$$P=P_R+i\cdot P_I\ ;\ Q=Q_R+i\cdot Q_I\ ;\ P_{\omega}=P_{R\omega}+i\cdot P_{I\omega}\quad(96)$$

$$Q_{\omega}=Q_{R\omega}+i\cdot Q_{I\omega}\ ;\ P_x=P_{Rx}+i\cdot P_{Ix}\ ;\ Q_x=Q_{Rx}+i\cdot Q_{Ix}\ ;\ P^2=P_R^2+P_I^2\quad(97)$$

When (x) can be any RFID Gate parameters R_1, C_1, And time delay τ etc,.Where for convenience, we have dropped the arguments $(i\cdot\omega,x)$, and where

$$F_{\omega}=2\cdot[(P_{R\omega}\cdot P_R+P_{I\omega}\cdot P_I)-(Q_{R\omega}\cdot Q_R+Q_{I\omega}\cdot Q_I)]\quad(98)$$

$F_x=2\cdot[(P_{Rx}\cdot P_R+P_{Ix}\cdot P_I)-(Q_{Rx}\cdot Q_R+Q_{Ix}\cdot Q_I)]$; $\omega_x=-F_x/F_{\omega}$. We define U and V:

$$U=(P_R\cdot P_{I\omega}-P_I\cdot P_{R\omega})-(Q_R\cdot Q_{I\omega}-Q_I\cdot Q_{R\omega})\ ;\ V=(P_R\cdot P_{Ix}-P_I\cdot P_{Rx})-(Q_R\cdot Q_{Ix}-Q_I\cdot Q_{Rx})\quad(99)$$

We choose our specific parameter as time delay

$$x=\tau.\ P_{R\omega}=2\cdot\omega\cdot[2\cdot\omega^2-\Gamma_{11}\cdot\Gamma_{33}]\ ;\ P_{I\omega}=3\cdot\omega^2\cdot(\Gamma_{33}+\Gamma_{11})\quad(100)$$

$$P_{R\tau}=0\ ;\ P_{I\tau}=0\ ;\ Q_{R\tau}=-\omega^2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\ ;\ Q_{I\tau}=-\omega\cdot\Gamma_{12}\cdot\Gamma_{34}\ ;\ P_{R\omega}\cdot P_R=2\cdot\omega^3\cdot[2\cdot\omega^4-3\cdot\omega^2\cdot\Gamma_{11}\cdot\Gamma_{33}+\Gamma_{11}^2\cdot\Gamma_{33}^2]\quad(101)$$

$$P_{I\omega}\cdot P_I=3\cdot\omega^5\cdot(\Gamma_{33}+\Gamma_{11})^2\ ;\ \omega_{\tau}=-F_{\tau}/F_{\omega}\ ;\ Q_{R\omega}=-2\cdot\omega\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\ ;\ Q_{I\omega}=\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\quad(102)$$

$$Q_{R\omega}=-2\cdot\omega\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\ ;\ Q_{I\omega}=\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\quad(103)$$

$$Q_{R\omega}\cdot Q_R=-2\cdot\omega\cdot[\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12}]\cdot[\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot(\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12})]\quad(104)$$

$$Q_{I\omega}\cdot Q_I=\omega\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]^2\ ;\ F_{\tau}=2\cdot[(P_{R\tau}\cdot P_R+P_{I\tau}\cdot P_I)-(Q_{R\tau}\cdot Q_R+Q_{I\tau}\cdot Q_I)]\quad(105)$$

$$F_{\tau}=2\cdot[(P_{R\tau}\cdot P_R+P_{I\tau}\cdot P_I)-(Q_{R\tau}\cdot Q_R+Q_{I\tau}\cdot Q_I)]\quad(106)$$

$$F_{\tau}=2\cdot\omega^2\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\tau\cdot\omega^2\cdot(\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12})]\quad(107)$$

$$P_R\cdot P_{I\omega}=3\cdot\omega^4\cdot(\omega^2-\Gamma_{11}\cdot\Gamma_{33})\cdot(\Gamma_{33}+\Gamma_{11})\ ;\ P_I\cdot P_{R\omega}=2\cdot\omega^4\cdot(\Gamma_{33}+\Gamma_{11})\cdot(2\cdot\omega^2-\Gamma_{11}\cdot\Gamma_{33})\quad(108)$$

$$Q_R\cdot Q_{I\omega}=[\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot(\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12})]\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]\quad(109)$$

$$Q_I\cdot Q_{R\omega}=-2\cdot\omega^2\cdot(\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau)\cdot(\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12})\quad(110)$$

$$V=(P_R\cdot P_{I\tau}-P_I\cdot P_{R\tau})-(Q_R\cdot Q_{I\tau}-Q_I\cdot Q_{R\tau})\ ;\ P_R\cdot P_{I\tau}=0\ ;\ P_I\cdot P_{R\tau}=0\quad(111)$$

$$Q_R\cdot Q_{I\tau}=-\omega\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot[\Gamma_{12}\cdot\Gamma_{34}-\omega^2\cdot(\frac{1}{2}\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau^2-\Gamma_{34}-\Gamma_{12})]\ ;\ Q_I\cdot Q_{R\tau}=-\omega^3\cdot\Gamma_{12}\cdot\Gamma_{34}\cdot\tau\cdot[\Gamma_{11}\cdot\Gamma_{34}+\Gamma_{12}\cdot\Gamma_{33}-\Gamma_{12}\cdot\Gamma_{34}\cdot\tau]\quad(112)$$

$F(\omega,\tau)=0$. Differentiating with respect to τ and we get

$$F_{\omega}\cdot\frac{\partial\omega}{\partial\tau}+F_{\tau}=0\ ;\ \tau\in I\Rightarrow\frac{\partial\omega}{\partial\tau}=-\frac{F_{\tau}}{F_{\omega}} \tag{113}$$

$$\wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}\ ;\ \wedge^{-1}(\tau)=\operatorname{Re}\{\frac{-2\cdot[U+\tau\cdot|P|^{2}]+i\cdot F_{\omega}}{F_{\tau}+i\cdot 2\cdot[V+\omega\cdot|P|^{2}]}\}\ ;\ \frac{\partial\omega}{\partial\tau}=\omega_{\tau}=-\frac{F_{\tau}}{F_{\omega}} \tag{114}$$

$$\wedge^{-1}(\tau)=\operatorname{Re}\{\frac{-2\cdot[U+\tau\cdot|P|^{2}]+i\cdot F_{\omega}}{F_{\tau}+i\cdot 2\cdot[V+\omega\cdot|P|^{2}]}\}\ ;\ \frac{\partial\omega}{\partial\tau}=\omega_{\tau}=-\frac{F_{\tau}}{F_{\omega}}\ ;sign\{\wedge^{-1}(\tau)\}=sign\{(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}\} \tag{115}$$

$$sign\{\wedge^{-1}(\tau)\}=sign\{F_{\omega}\}\cdot sign\{\tau\cdot\frac{\partial\omega}{\partial\tau}+\omega+\frac{U\cdot\frac{\partial\omega}{\partial\tau}+V}{|P|^{2}}\} \tag{116}$$

We shall presently examine the possibility of stability transitions (bifurcations) in a shifted gate double loop RFID system, about the equilibrium point $E^{(0)}(V_{11}^{'(0)},V_{11}^{(0)},V_{12}^{'(0)},V_{12}^{(0)})$ as a result of a variation of delay parameter τ. The analysis consists in identifying the roots of our system characteristic equation situated on the imaginary axis of the complex λ-plane Where by increasing the delay parameter τ, Re λ may at the crossing, change its sign from – to +, i.e. from a stable focus $E^{(*)}$ to an unstable one, or vice versa. This feature may be further assessed by examining the sign of the partial derivatives with respect to τ,

$$\wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega} \tag{117}$$

$$\wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}\ ,C_{1},\mathrm{R}_{1},\text{gate antenna parametrs}=const\ ;\ \text{where}\ \omega\in\mathbb{R}_{+}. \tag{118}$$

For our stability switching analysis we choose typical RFID shifted gate parameters values: L_{11}=4.5mH, L_{12}=2.5mH, C_1=23pF, R_1=100kOhm=10^5, $r_{p1}=100Ohm$, K=0.6, $2\cdot L_m$=0.004 ($2\cdot L_{m}=2\cdot K\cdot\sqrt{L_{11}\cdot L_{12}}$).

$$\eta_{1}=56.22\cdot10^{-12}\ ;\ \eta_{2}=2.49\cdot10^{-5}\ ;\ \eta_{3}=222.42\ ;\ \xi_{1}=101.2\cdot10^{-12}\ ;\ \xi_{2}=4.492\cdot10^{-5} \tag{119}$$

$$\eta_{2}=2.49\cdot10^{-5}\ ;\ \eta_{3}=222.42\ ;\ \xi_{1}=101.2\cdot10^{-12}\ ;\ \xi_{2}=4.492\cdot10^{-5}\ ;\ \xi_{3}=400.4\ ;\ \Gamma_{11}=-\frac{\eta_{2}}{\eta_{1}}=-4.42\cdot10^{5}\ ;\ \Gamma_{12}=-\frac{\eta_{3}}{\eta_{1}}=-3.95\cdot10^{12} \tag{120}$$

$$\Gamma_{33}=-\frac{\xi_{2}}{\xi_{1}}=-4.43\cdot10^{5}\ ;\Gamma_{34}=-\frac{\xi_{3}}{\xi_{1}}=-3.95\cdot10^{12}\ ;\Gamma_{21}=\Gamma_{43}=1\ ;\Gamma_{13}=\Gamma_{14}=\Gamma_{22}=\Gamma_{23}=\Gamma_{24}=0\ ;\Gamma_{31}=\Gamma_{32}=\Gamma_{41}=\Gamma_{42}=\Gamma_{44}=0 \tag{121}$$

Then we get the expression for $F(\omega,\tau)$ for typical RFID shifted gate parameters values.

$$\begin{aligned}F(\omega,\tau)&=|P(i\cdot\omega,\tau)|^{2}-|Q(i\cdot\omega,\tau)|^{2}=\omega^{8}+\omega^{6}\cdot39.16\cdot10^{10}+\omega^{4}\cdot\{383.17\cdot10^{20}-[7.8\cdot10^{24}\cdot\tau^{2}+7.9\cdot10^{12}]^{2}\}\\&-\omega^{2}\cdot\{[34.94\cdot10^{17}-15.6\cdot10^{24}\cdot\tau]^{2}-31.2\cdot10^{24}\cdot[7.8\cdot10^{24}\cdot\tau^{2}+7.9\cdot10^{12}]\}-243.39\cdot10^{48}\end{aligned} \tag{122}$$

We find those ω, τ values which fulfill $F(\omega,\tau)=0$. We ignore negative, complex, and imaginary values of ω for specific τ values. $\tau\in[0.001..10]$ and we can be express by 3D function $F(\omega,\tau)=0$. Since it is a very complex function, we recommend to solve it numerically rather than analytic. We plot the stability switch diagram based on different delay values of our RFID double gate system. Since it is a very complex function we recommend to solve it numerically rather than analytic.

$$\wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}=\operatorname{Re}\{\frac{-2\cdot[U+\tau\cdot|P|^{2}]+i\cdot F_{\omega}}{F_{\tau}+i\cdot 2\cdot[V+\omega\cdot|P|^{2}]}\}\ ;\ \wedge^{-1}(\tau)=(\frac{\partial\operatorname{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}=\frac{2\cdot\{F_{\omega}\cdot(V+\omega\cdot P^{2})-F_{\tau}\cdot(U+\tau\cdot P^{2})\}}{F_{\tau}^{2}+4\cdot(V+\omega\cdot P^{2})^{2}} \tag{123}$$

The stability switch occurs only on those delay values (τ) which fit the equation: $\tau = \frac{\theta_+(\tau)}{\omega_+(\tau)}$ and $\theta_+(\tau)$ is the solution of

$\sin\theta(\tau) = ...$; $\cos\theta(\tau) = ...$ when $\omega = \omega_+(\tau)$ if only ω_+ is feasible. Additionally when all RFID double gate system's parameters are known and the stability switch due to various time delay values τ is describe in the following expression:

$$sign\{\wedge^{-1}(\tau)\} = sign\{F_\omega(\omega(\tau),\tau)\} \cdot sign\{\tau \cdot \omega_\tau(\omega(\tau)) + \omega(\tau) + \frac{U(\omega(\tau)) \cdot \omega_\tau(\omega(\tau)) + V(\omega(\tau))}{|P(\omega(\tau))|^2}\} \quad (124)$$

Remark: we know $F(\omega,\tau)=0$ implies it roots $\omega_i(\tau)$ and finding those delays values τ which ω_i is feasible. There are τ values which ω_i is complex or imaginary number, then unable to analyse stability (Kuang, 1993; Beretta, et al. 2002).

5. Results of RFID Shifted Gate System Stability Switching under Delayed Variables in Time

We find those ω, τ values which fulfil F(ω, τ)=0. We ignore negative, complex, and imaginary values of ω for specific τ values. $\tau \in [0.001..10]$ and we can express by 3D function F(ω, τ)=0. We define new MATLAB script parameters: τ→Tau, G_i(i=1,..,10): G1=39.16e10; G2=383.17e20; G3=7.8e24; G4=7.9e12; G5=34.94e17; G6=15.6e24; G7=31.2e24;G8=7.8e24;G9=7.9e12;G10=243.39e48, Ξ_j→Phi_j ; j=8, 6, 4, 2, 0, Running MATLAB script for τ values $\tau \in [0.001..10]$ gives the following results:

MATLAB script:

```
Tau=10;G1=39.16e10;G2=383.17e20;G3=7.8e24;G4=7.9e12;G5=34.94e17;G6=15.6e24;G7=31.2e24;G8=7.8e24;G9=
7.9e12;G10=243.39e48;Phi8=1;Phi6=G1;Phi4=G2-(G3*Tau*Tau+G4).^2;
Phi2=-((G5-G6*Tau).^2-G7*(G8*Tau*Tau+G9)); Phi0=-G10;p=[Phi8 0 Phi6 0 Phi4 0 Phi2 0 Phi0];r=roots(p)
```

Results: We plot 3D function F(ω, τ)=0. τ:0→10; ω:0→1e13. We define additional MATLAB script parameters: ω→w, τ→t. We get some possible real values for ω which fulfil F(ω, τ)=0, F(ω=0 or ω=2.1437, ω=2.7928, ω=1.0e+006, ω=1.0e+009, ω=1.0e+010, ω=1.0e+011, ω=1.0e+012, ω=1.0e+013, τ)=0; $\tau \in [0.001..10]$. Next is to find those ω, τ values which fulfil

$$\sin\theta(\tau)=.... \ \sin(\omega \cdot \tau) = \frac{-P_R \cdot Q_I + P_I \cdot Q_R}{|Q|^2} \text{ and } \cos\theta(\tau)=.. \quad (125)$$

$$\cos(\omega \cdot \tau) = -\frac{(P_R \cdot Q_R + P_I \cdot Q_I}{|Q|^2} \ ; \ |Q|^2 = Q_R^2 + Q_I^2 \quad (126)$$

Table 2a. Semi-passive RFID TAGs with double loop antennas, $\omega_i(\tau)$.

ω_i	**τ=0**	**τ=0.001**	**τ=0.01**
ω_1	1.0e+006	1.0e+009	1.0e+010
ω_2	-0.0000+3.105i	-2.7928	-2.7928
ω_3	-0.0000-3.105i	-0.0000+2.792i	-0.0000+2.7928i
ω_4	-2.1437	-0.0000-2.792i	-0.0000-2.7928i
ω_5	-1.528+0.0823i	2.7928	2.7928
ω_6	-1.528-0.0823i	0.0000+0.0000i	-0.0000+0.0000i
ω_7	2.1437	0.0000-0.0000i	-0.0000-0.0000i
ω_8	1.528+0.0823i	-0.0000+0.0000i	0.0000+0.0000i
ω_9	1.528-0.0823i	-0.0000-0.0000i	0.0000-0.0000i

Table 2b. Semi-passive RFID TAGs with double loop antennas, $\omega_i(\tau)$.

ω_i	**τ=0.1**	**τ=1**	**τ=10**
ω_1	1.0e+011	1.0e+012	1.0e+013
ω_2	-2.7928	-2.7928	-2.7928
ω_3	0.0000+2.7928i	0.0000+2.7928i	0.0000+2.7928i
ω_4	0.0000-2.7928i	0.0000-2.7928i	0.0000-2.7928i
ω_5	2.7928	2.7928	2.7928
ω_6	-0.0000+0.0000i	-0.0000+0.0000i	-0.0000+0.0000i
ω_7	-0.0000-0.0000i	-0.0000-0.0000i	-0.0000-0.0000i
ω_8	0.0000+0.0000i	0.0000+0.0000i	0.0000+0.0000i
ω_9	0.0000-0.0000i	0.0000-0.0000i	0.0000-0.0000i

Case I: ω=0 then $P_R = 0; P_I = 0; Q_R = \Gamma_{12} \cdot \Gamma_{34}; Q_I = 0$ typical RFID shifted gate parameters values: L_{11}=4.5mH, L_{12}=2.5mH, C_1=23pF, R_1=100kOhm=10^5, $r_{p1} = 100Ohm$, K=0.6, $2\cdot L_m$=0.004 ($2\cdot L_m = 2\cdot K\cdot\sqrt{L_{11}\cdot L_{12}}$).

$$Q_R \neq 0 \; ; \; Q_R > 0 \; ; \; \Gamma_{12} = -\frac{\eta_3}{\eta_1} = -3.95\cdot 10^{12} \; ; \; \Gamma_{34} = -\frac{\xi_3}{\xi_1} = -3.95\cdot 10^{12} \; ; \; Q_R = \Gamma_{12}\cdot\Gamma_{34} = 15.6\cdot 10^{24} \quad (127)$$

sin(ω·τ)=… fulfil and cos(ω·τ)=… can't fulfil since $\cos(\omega\cdot\tau)|_{\omega=0} \neq 0$.

Case II: ω≠0 ; ω>0 ; ω=2.1437, 2.7928, 1.0e+006, 1.0e+009, 1.0e+010, 1.0e+011, 1.0e+012, 1.0e+013 which can fulfil expressions sinθ(τ)=… and cos(ω·τ)=… Finally we plot the stability switch diagram based on different delay values of our RFID shifted gate system.

$$P_R = \omega^4 - \omega^2\cdot 19.58\cdot 10^{10} \; ; \; P_I = -8.85\cdot\omega^3\cdot 10^5 \; ; \; Q_R = 15.6\cdot 10^{24} - \omega^2\cdot[7.8\cdot 10^{24}\cdot\tau^2 + 7.9\cdot 10^{12}] . \quad (128)$$

$$P_I = -8.85\cdot\omega^3\cdot 10^5 \; ; \; Q_R = 15.6\cdot 10^{24} - \omega^2\cdot[7.8\cdot 10^{24}\cdot\tau^2 + 7.9\cdot 10^{12}] \; ; \; Q_I = \omega\cdot[34.95\cdot 10^{17} - 15.6\cdot 10^{24}\cdot\tau] \; ; \; Q_{I\tau} = -\omega\cdot 15.6\cdot 10^{24} \quad (129)$$

$$Q_{R\tau} = -\omega^2\cdot 15.6\cdot 10^{24}\cdot\tau \; ; \; V = -(Q_R\cdot Q_{I\tau} - Q_I\cdot Q_{R\tau}) \; ; \; Q_R\cdot Q_{I\tau} = -\omega\cdot 15.6\cdot 10^{24}\cdot\{15.6\cdot 10^{24} - \omega^2\cdot[7.8\cdot 10^{24}\cdot\tau^2 + 7.9\cdot 10^{12}]\} \quad (130)$$

$$Q_I\cdot Q_{R\tau} = -\omega^3\cdot 15.6\cdot 10^{24}\cdot\tau\cdot[34.95\cdot 10^{17} - 15.6\cdot 10^{24}\cdot\tau] \; ; \; P_{I\omega} = -26.55\cdot\omega^2\cdot 10^5 \; ; \; P_{R\omega} = 2\cdot\omega\cdot(2\cdot\omega^2 - 19.58\cdot 10^{10}) \quad (131)$$

$$Q_{R\omega} = -2\cdot\omega\cdot(7.8\cdot 10^{12}\cdot\tau^2 + 7.9\cdot 10^{12}) \; ; \; Q_{I\omega} = 34.94\cdot 10^{17} - 15.6\cdot 10^{24}\cdot\tau \; ; \; U = (P_R\cdot P_{I\omega} - P_I\cdot P_{R\omega}) - (Q_R\cdot Q_{I\omega} - Q_I\cdot Q_{R\omega}) \quad (132)$$

We plot the function:

$$g(\tau) = \wedge^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega} \; ; \; g(\tau) = \wedge^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega} = \frac{2\cdot\{F_\omega\cdot(V+\omega\cdot P^2) - F_\tau\cdot(U+\tau\cdot P^2)\}}{F_\tau^2 + 4\cdot(V+\omega\cdot P^2)^2} \quad (133)$$

$$sign[g(\tau)] = sign[\wedge^{-1}(\tau)] = sign[(\frac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\cdot\omega}] = sign[\frac{2\cdot\{F_\omega\cdot(V+\omega\cdot P^2) - F_\tau\cdot(U+\tau\cdot P^2)\}}{F_\tau^2 + 4\cdot(V+\omega\cdot P^2)^2}] \quad (134)$$

$$\text{Since} \quad F_\tau^2 + 4\cdot(V+\omega\cdot P^2)^2 > 0 \Rightarrow sign[\wedge^{-1}(\tau)] = sign\{F_\omega\cdot(V+\omega\cdot P^2) - F_\tau\cdot(U+\tau\cdot P^2)\} \quad (135)$$

$$sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[(V+\omega\cdot P^2) - \frac{F_\tau}{F_\omega}\cdot(U+\tau\cdot P^2)]\} \; ; \; \omega_\tau = -\frac{F_\tau}{F_\omega} \; ; \; \omega_\tau = (\frac{\partial\omega}{\partial\tau})^{-1} = -\frac{\partial F/\partial\omega}{\partial F/\partial\tau} \quad (136)$$

$$sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[V + \omega_\tau\cdot U + \omega\cdot P^2 + \omega_\tau\cdot\tau\cdot P^2]\} \; ; \; sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[\frac{1}{P^2}]\cdot[\frac{V+\omega_\tau\cdot U}{P^2} + \omega + \omega_\tau\cdot\tau]\} \quad (137)$$

$$sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[\frac{1}{P^2}]\cdot[\frac{V+\omega_\tau\cdot U}{P^2} + \omega + \omega_\tau\cdot\tau]\} \; ; sign[\frac{1}{P^2}] > 0 \Rightarrow sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[\frac{V+\omega_\tau\cdot U}{P^2} + \omega + \omega_\tau\cdot\tau]\} \quad (138)$$

$$sign[\frac{1}{P^2}] > 0 \Rightarrow sign[\wedge^{-1}(\tau)] = sign\{[F_\omega]\cdot[\frac{V+\omega_\tau\cdot U}{P^2} + \omega + \omega_\tau\cdot\tau]\} \; ; \; sign[\wedge^{-1}(\tau)] = sign[F_\omega]\cdot sign[\frac{V+\omega_\tau\cdot U}{P^2} + \omega + \omega_\tau\cdot\tau] \quad (139)$$

(Table 3)

$$F_\omega = 2\cdot[(P_{R\omega}\cdot P_R + P_{I\omega}\cdot P_I) - (Q_{R\omega}\cdot Q_R + Q_{I\omega}\cdot Q_I)] \quad (140)$$

We check the sign of $\wedge^{-1}(\tau)$ according the following rule: If sign[$\Lambda^{-1}(\tau)$] > 0 then the crossing proceeds from (-) to (+) respectively (stable to unstable). If sign[$\Lambda^{-1}(\tau)$] < 0 then the crossing proceeds from (+) to (-) respectively (unstable to stable). Anyway the stability switching can occur only for ω≠0; ω>0 ; ω=2.1437, 2.7928, 1.0e+006, 1.0e+009, 1.0e+010, 1.0e+011, 1.0e+012, 1.0e+013 and $\tau \in [0.001..10]$. Since it is a very complex function we recommend solving it numerically rather than analytic. We plot the stability switch diagram based on different delay values of our Semi-passive RFID TAGs with double loop antennas system.

Table 3. Semi-passive RFID TAGs with double loop antennas stability switching criteria.

$sign[F_\omega]$	$sign[\frac{V+\omega_\tau \cdot U}{P^2}+\omega+\omega_\tau \cdot \tau]$	$sign[\wedge^{-1}(\tau)]$
+/-	+/-	+
+/-	-/+	-

6. Discussion

In this paper we consider Semi-passive RFID TAGs with double loop antennas. Due to electromagnetic interferences there are delays in time for voltages and voltages derivatives in the first and second loop. These delays cause to stability switching for our Semi-passive RFID TAGs with double loop antennas. We draw our Semi-passive RFID TAGs with double loop antennas circuit and get system differential equations. Our variables are first and second loop antennas voltages and voltages derivative. Our system dynamic behaviour is dependent on circuit overall parameters and parasitic delays in time. We keep all circuit parameters fix and change parasitic delays over various values $\tau \in [0.001..10]$. Our analysis results extend that of in the way that it deals with stability switching for different delay values. This implies that our system behaviour of the circuit cannot be inspected by short analysis and we must study the full system. Several very important issues and possibilities were left out of the present paper. One possibility is the stability switching by circuit parameters. Every circuit's parameter variation can change our system dynamic and stability behaviour. This case can be solved by the same methods combined with alternative and more technical hypotheses. Moreover, numerical simulations for the Semi-passive RFID TAGs with double loop antennas model studied in references suggest that this result can be extended to enhance models with more general functions.

7. Conclusion

Semi-passive RFID TAGs with double loop antennas environment is characterize by electromagnetic interferences which can influence shifted gate system stability in time. There are four main RFID double loop antenna variables which are affected by electromagnetic interferences, $V_{11}(t),\ V_{11}^{'}(t)\ V_{12}(t), V_{12}^{'}(t)$. Each loop antennas voltages variables under electromagnetic interferences are characterize by time delay respectively $(\Delta_1 \neq \Delta_2\ ;\ \tau_1 \geq 0\ ;\ \tau_2 \geq 0\ ; \Delta_1, \Delta_2 \geq 0)$. The two time delays are not the same but can be categorized to some subcases due to interferences behavior. The first case we analyze is when there is delay in RFID first gate's primary loop antenna voltage and no delay in secondary loop antenna voltage.The second case we analyze is when there is delay in RFID gate's primary and secondary loop antennas voltages ($\tau_1 = \tau_2 = \tau$) and no delay in in gate's primary and secondary loop antennas voltages derivatives (Kuang, 1993; Beretta, et al., 2002).The third case we analyze is when there is delay in RFID gate's primary and secondary loop antennas voltages ($\tau_1 = \tau_2 = \Delta_1 = \Delta_2 = \tau_\Delta$) and delay in in gate's primary and secondary loop antennas voltages derivatives (Steven; Kuang, 1993). For simplicity of our analysis we consider in the third case all delays are the same (there is a difference but it is neglected in our analysis). In each case we derive the related characteristic equation. The characteristic equation is dependent on double loop antennas overall parameters and interferences time delay. Upon mathematics manipulation and [BK] theorems and definitions we derive the expression which gives us clear picture on double loop antennas stability map. The stability map gives all possible options for stability segments, each segment belong to different time delay values segment. Double loop antennas arranged as a shifted gate's stability analysis can be influence either by system overall parameters values. We left this analysis and do not discuss it in the current article.

Appendix A: Lemma 1.1

Assume that $\omega(\tau)$ is a positive and real root of $F(\omega,\tau)=0$

Defined for $\tau \in I$, which is continuous and differentiable. Assume further that if $\lambda = i \cdot \omega$, $\omega \in R$, then $P_n(i \cdot \omega, \tau) + Q_n(i \cdot \omega, \tau) \neq 0,\ \tau \in \mathrm{R}$ hold true. Then the functions $S_n(\tau),\ n \in N_0$, are continuous and differentiable on I.

<u>Appendix B</u>: Theorem 1.2

Assume that $\omega(\tau)$ is a positive real root of $F(\omega, \tau) = 0$ defined for $\tau \in I,\ I \subseteq R_{+0}$, and at some $\tau^* \in I$, $S_n(\tau^*) = 0$. For some $n \in N_0$ then a pair of simple conjugate pure imaginary roots

$$\lambda_+(\tau^*) = i \cdot \omega(\tau^*),\ \lambda_-(\tau^*) = -i \cdot \omega(\tau^*) \text{ of} \quad (141)$$

$D(\lambda, \tau) = 0$ exist at $\tau = \tau^*$ which crosses the imaginary axis from left to right if $\delta(\tau^*) > 0$ and cross the imaginary axis from right to left if $\delta(\tau^*) < 0$ where

$$\delta(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda = i\omega(\tau^*)}\} = sign\{F_\omega(\omega(\tau^*), \tau^*)\} \cdot sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (142)$$

<u>Appendix C</u>: Theorem 1.3

The characteristic equation has a pair of simple and conjugate pure imaginary roots $\lambda = \pm\omega(\tau^*),\ \omega(\tau^*)$ real at $\tau^* \in I$ if $S_n(\tau^*) = \tau^* - \tau_n(\tau^*) = 0$ for some $n \in N_0$. If $\omega(\tau^*) = \omega_+(\tau^*)$, this pair of simple conjugate pure imaginary roots crosses the imaginary axis from left to right if $\delta_+(\tau^*) > 0$ and crosses the imaginary axis from right to left if $\delta_+(\tau^*) < 0$ where

$$\delta_+(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda = i\omega_+(\tau^*)}\} = sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (143)$$

If $\omega(\tau^*) = \omega_-(\tau^*)$, this pair of simple conjugate pure imaginary roots cross the imaginary axis from left to right if $\delta_-(\tau^*) > 0$ and crosses the imaginary axis from right to left If $\delta_-(\tau^*) < 0$ where

$$\delta_-(\tau^*) = sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda = i\omega_-(\tau^*)}\} = -sign\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \quad (144)$$

If $\omega_+(\tau^*) = \omega_-(\tau^*) = \omega(\tau^*)$ then $\Delta(\tau^*) = 0$ and $sign\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}|_{\lambda = i\omega(\tau^*)}\} = 0$, the same is true when $S_n^{'}(\tau^*) = 0$. The following result can be useful in identifying values of τ where stability switches happened.

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