Journal of Education and Training Studies
Vol. 4, No. 3; March 2016
ISSN 2324-805X E-ISSN 2324-8068
Published by Redfame Publishing
URL: http://jets.redfame.com

Transition from Model to Proof: Example of Water Treatment Plant

Gürsel Güler

Correspondence: Gürsel Güler, Bozok University, Faculty of Education, Elementary Mathematics Education Department, Yozgat, Turkey.

Received: November 5, 2015 Accepted: December 7, 2015 Online Published: December 12, 2015

doi:10.11114/jets.v4i3.1189 URL: http://dx.doi.org/10.11114/jets.v4i3.1189

Abstract

The aim of this study was to research the prospective mathematics teachers' ability to construct a mathematical model for a real life problem and to prove these models by generalizing them to use in similar situations. The study was conducted with 129 prospective teachers determined on a volunteering basis. The data were obtained with the help of Modelling and Proof Test that were applied to the prospective teachers. In the analysis of the data obtained, the categories which emerged as a result of the classification of the model and proofs formed by the relevant literature and the prospective teachers were used. It was discovered in consequence of the study that the prospective teachers construct seven different models and presented proofs in seven different categories for proving these models. It was observed that the prospective teachers had difficulties in constructing a model for the solution of a real life problem and transition from the model to the proof.

Keywords: mathematical modeling, model, mathematical proof

1. Introduction

For a more efficient mathematical education today, the focus must be placed on the teachers' ability to explain the causes of the concepts they will be teaching and the students' questioning of the reasons behind the mathematical concepts used in problem solving (Hanna & Jahnke, 2004). According to this point-of-view, the teachers must have sufficient knowledge of mathematical concepts in order for them to be able to teach mathematics effectively (Ma, 2010). In terms of the students, it is aimed to raise individuals, who especially query, explore and research. This affected the class environment in teaching mathematics and the explanatory aspects of mathematical proofs used solely as a means of justification (Knuth, 2002) in teaching (Hanna & Jahnke, 2004). For, it is known that each valid proof is not sufficient in explaining why propositions are true. Thus, especially mathematics trainers tried to distinguish the explanatory and non-explanatory proofs from one another (Hanna, 1990; Rav, 1999). In the literature of the relevant field, Hanna (1990) emphasised that distinguishing the role of proof in teaching mathematics by discussing according to different points-of-view may be helpful in this sense and classified the proofs into two groups as formal and acceptable. Formal proofs is a theoretic concepts that occur at a limited number of stages according to certain rules; continues by using axioms or expressions, of which truthfulness is proved, at each step; and carries one through at the last step. Acceptable proofs were defined as proofs based on the rules accepted by qualified mathematicians (Hanna, 1990). The formal proofs defined here show the truthfulness of the prepositions and do not care about whey they are true. While acceptable proofs show the truthfulness of the prepositions and why they are true. Thus, Hanna (1990) classified the proofs as proving proofs and explanatory proofs. The explanatory proofs in this classification can be examined together with proving proof; and proving proofs can be examined together with formal proof (Dede, 2013). Explanatory proofs particularly have attracted the attention of many researchers as they show both the truthfulness of a preposition and why it is true (Hanna & Jahnke, 2004; Hanna, 1990; De Villiers, 2002; Hanna & Sidoli, 2007; Hanna & Jahnke, 2002).

When the literature of the field is examined, it is seen that researchers usually benefit from visualization (Hanna & Sidoli, 2007; Davis, 1993), computer software (De Villiers, 2002; Palais, 1999) and models (Hanna & Jahnke, 2004; Hanna, 1990; Hanna & Jahnke, 2002; Mudaly & De Villiers, 2004) in order to create the explanatory proofs. The results put forth in these researches reveal that visual representations and models help the students understand the proofs (Hanna & Sidoli, 2007). However, it can be said that the materials used by the teachers in the classroom for explanatory proofs are effective in building their capacity and there is no difference between the proofs created using a model (Stylianides, 2007) and other methods of proof (Hawkins, 2007). Nevertheless, from the teachers' point-of-view, mathematical proof and model creation processes are considered totally independent from each other and the applications about these two processes are examined totally independent from each other (Hanna & Jahnke, 2004). There is actually a complementary relationship between both the modelling and proving processes (Hanna & Jahnke, 2004; Mudaly & De Villiers, 2004). However, especially the proofs that could be created with successful representation and models are limited and thus, a portion of the researchers defend that such proofs cannot take the place of a formal proof and others defend that they can further develop the proofs (Hanna & Sidoli, 2007).

It can be said that in our country, we are far from using the potential benefits of the modelling process in mathematical proofs for mathematics and teaching mathematics. In terms of our education programs, we can see that both modelling and reasoning and proving skills are significantly emphasised in the mathematics program of the secondary school (Ministry of National Education [MNE], 2013). However, mathematical modelling and proving can be perceived as totally different from each other as they are examined as different skills. Thus, the gap between these two processes expressed by Hanna and Jahnke (2004) is also seen in our country. In this sense, as distinct from the traditional approach applied in teaching mathematics, modelling activities offer the students rich learning opportunities, through which they can think more deeply and understand previous information and reach generalizable solutions by recreating their knowledge in line with the need (Lesh, Hamilton & Kaput 2007) to reveal the relationships between mathematics and real life (English, 2006). In this approach, it is primarily aimed to develop models that will discover patterns and relationships and use them for solving other problems (Olkun, Şahin, Akkurt, Dikkartın & Gulbağcı, 2009). Furthermore, gaining the students to use the proving skill with the help of appropriate models will ensure that the proof becomes permanent, meaningful and useable for them and that they will perceive the theorems as a structure that exists in life and that is open for development rather than an entirety of systems outside their lives for their further levels of learning (Ünveren, 2010). Thus, the importance of solving real-life problems within the process of proving is as evident as not to be taken lightly (Hodgson & Riley, 2001). However, it can be said that the proofs realized with the help of the models have a positive effect when compared to formal proof in the attitude of prospective teachers (Ünveren, 2010; Doruk, Kıymaz & Horzum, 2012).

In the realization of the proof, it is expected that the hypothesis is taken from a definite place. To this end, if the hypothesis taken is selected from the lives of the students, they can have an education experience as if they are realizing the proof themselves for the first time. This way, they can reconstruct the confirmations and conceptual justifications in the proof itself as if they are mathematicians (Ünveren, 2010). Hence, it can be said that using real life situations taking from the students' lives away from formal structure will provide more effective, permanent and significant learning environments (English, 2003). Besides, the skills of modelling by reasoning and proving are emphasised in secondary school mathematics programs and the students are expected to gain these skills (MNE, 2013). However, there limited researches both in Turkey and abroad on the application of these two skills and especially benefiting from the models in mathematical proofs. Nevertheless, it was seen that there is no research on the skills of the prospective teachers to create a model for a real-life problem and prove the problem by means of this model. Thus, the skills of the prospective teachers to prove the models they will create on a real-life problem and prove the problem with the help of this model are considered important. By this means, important information such as the use of different methods by prospective teachers who will lead the activities in the classroom in the near future in the proofs they will offer to their students and to eliminate the possible problems they may encounter in this sense. Thus, in this study, it was aimed to investigate the skills of prospective mathematics teachers' of secondary school to create models for the water treatment plant problem and prove these models by generalizing them in a way that they can be used in similar situations.

2. Method

This research is a descriptive study carried out with the aim of examining the processes of creating a model by prospective mathematics teachers' in secondary school towards a real-life problem and proving the problem with the help of the model they created, and determining the difficulties that emerge during these processes, if any, and putting forth their reasons. Thus, it was tried to define the modelling and proving processes of prospective teachers as they exist in their specific conditions. For, descriptive research methods are used with the aim of describing a past or present situation in the way that they exist (Mcmillan & Schumacher, 2010).

2.1 Participants

The research was carried out with 129 volunteer prospective teachers taking the class of Special Teaching Methods in fall term among the teachers attending the pedagogical formation training certificate program of a university in Central Anatolia region in 2014-2015 academic year. Prospective teachers carried out personal and group studies within the scope of this lesson on the examination and development of special mathematics teaching methods, activity samples for these methods, secondary school mathematics program and five basic skills included in this program. In this process, modelling and proving skills were especially focused on, and activities were carried out in order to develop these skills and correlate them. These activities were carried out especially in the form of modelling a real-life problem and proving a problem with the help of this model. Furthermore, the prospective teachers participating in the research were informed about the study and told that their names would be kept secret. The candidates participating in the study were given codes as PT1, PT2,…, PT129 symbolizing the prospective teacher.

2.2 Data Collection and Analysis

The research data were collected with the help of the Modelling and Proving Test (MPT) created by the researcher using the Water Treatment Plant (WTP) problem generated by Mudaly and De Villiers (2004). Furthermore, the test was put into its final form by taking the opinions of two academicians, who have studies on modelling and mathematical proof, during the preparation stage of MPT. MPT consists of two stages (Appendix-1). There are two open-ended questions requiring creating an appropriate model for solving the WTP problem in the first stage and proving that the model shows the solution of the problem in the second stage. WTP is a real-life problem and it allows the prospective teachers to create a number of different models in order to achieve the solution of the problem. The prospective teachers endeavoured to create an appropriate model for solving the WTP problem in the first stage, and prove the correctness of the models they created in order to generalize them for all similar situations. Prospective teachers were given a lesson's time (50 minutes) to answer the MPT test. It was determined that prospective teachers created seven different models in accordance with the answers they gave in the modelling stage of the MPT test. The models that prospective teachers created in the solution of the WTP problem were classified as: *Invalid model* [Model1 (M1)], *cone* (M2), *quadrilateral* (M3), *analytical plane* (M4), *rectangle* (M5), *square* (M6) and *circle* (M7). The classifications made by the researcher were created by making amendments in line with the opinions of two academicians of teaching mathematics. These models and their explanations are given in Table 1.

Table 1. Models created by prospective teachers for WTP problem and their explanations

Models	Explanations
Invalid Model (M1)	The models that are left blank, and with which no mathematical correlation could be established with the expressions in the problem statement
Cone (M2)	Models established in the form of cone for the solution of the problem
Quadrilateral (M3)	Models established in the form of quadrilateral for the solution of the problem
Analytical Plane (M4)	Models established in the form of analytical plane for the solution of the problem
Rectangle (M5)	Models established in the form of rectangle for the solution of the problem
Square (M6)	Models established in the form of square for the solution of the problem
Circle (M7)	Models established in the form of circle for the solution of the problem

It was seen that the answers of prospective teachers during the state of proving of the MPT test were classified under seven different categories. The proofs of the prospective teachers were classified as *Unanswered* [Proof1 (P1)], *restatement of the model* (P2), *trial and error* (P3), *sample case* (P4), *symbolic* (P5), *structural* (P6), and *from model to proof* (P7). The classifications made by the researchers were generated by making alterations in line with the studies in the field literature (Ko & Knuth, 2009; Özkaya, Işık & Konyalıoğlu, 2014) and the opinions of two academicians in the field of teaching mathematics. These proof categories and their explanations are given in Table 2.

Table 2. Proofs of the models that prospective teachers created for the solution of MPT problem and their explanation

Category	Explanations
Unanswered (P1)	The answers that are left blanks, irrelevant, given as an estimation or that cannot reach a generalization
Restatement of the Model (P2)	The answers including the repeat of the model rather than proving
Trial and Error (P3)	Answers trying to provide a proof by examples or trial and error over the situations given in the model
Sample Case (P4)	Answers trying to provide a proof only over a single sample case
Symbolic (P5)	Answers that do not create a proof and only include certain features of the model created
Structural (P6)	Answers that use definitions, axioms and features in order to create a valid proof using the model created, but that are logically incorrect or contain errors resulting from the model
From Model to Proof (P7)	Answers including valid proofs created using the generated model and its features

3. Results

3.1 Mathematical Model Creating Abilities of Prospective Teachers

The models created by prospective teachers for the WTP problem in the first stage of the MPT test are given in Table 3.

Table 3. Models created by prospective teachers for the solution of the MPT problem and their explanations

Category	Prospective Teachers (%)
Invalid Model (M1)	PT1-PT35 (%27)
Cone (M2)	PT36-PT38 (%2)
Quadrilateral (M3)	PT39-PT43 (%4)
Analytical Plane (M4)	PT44-PT46 (%2)
Rectangle (M5)	PT47-PT55 (%7)
Square (M6)	PT56-PT87 (%25)
Circle (M7)	PT88-PT129 (%33)

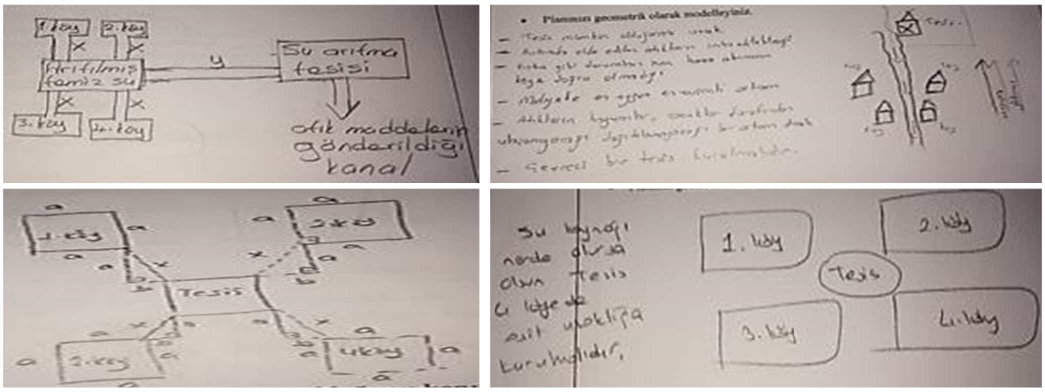

Figure 1. Invalid models created by prospective teachers for the solution of the WTP problem

When the M1 models created by prospective teachers are examined, it is seen that there are answers that are left blank, that no mathematical correlation could be established with the expressions in the problem statement or that solely consist of figures that are irrelevant with the problem statement. It can be said that the reason why the prospective teachers created such models is that they overly stick to the aspects that are included in the problem statement, but that actually have nothing to the with the main problem. For, the prospective teachers in this category only focused on villages and water treatment plants and designed figures for them. Thus, they could not create a geometric shape that these villages and the plant can exist together. Thus, it was seen that they could not turn the WTP problem provided as a real-life problem into a mathematical structure.

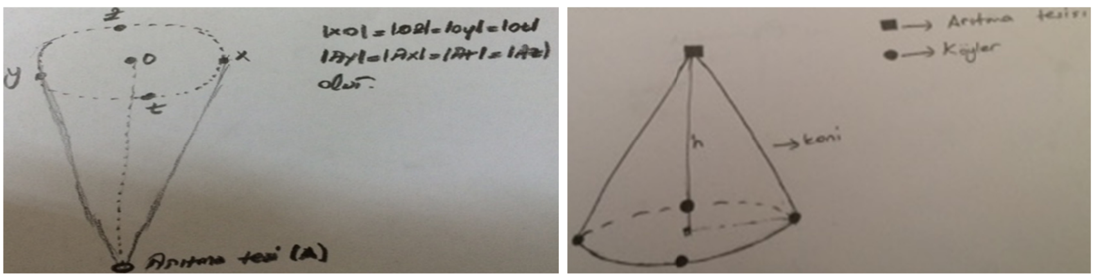

Figure 2. Cone models created by prospective teachers for the solution of the WTP problem

When Figure 2 and Table 3 are examined, it is seen that there are only three prospective teachers in M2 category. It is understood that the prospective teachers in this category can establish a mathematical correlation between the elements required in the WTP problem. In addition, it was seen that they did not take the treatment plant in the geometric model they created to the centre of the geometric shape unlike other prospective teachers and they positioned the plant on the edges of the cone. As the points taken from the circle shape on the ground of the cone-shaped figures created by the prospective teachers are at an equal distance from the end point, the models in M2 category were a valid solution for the WTP problem.

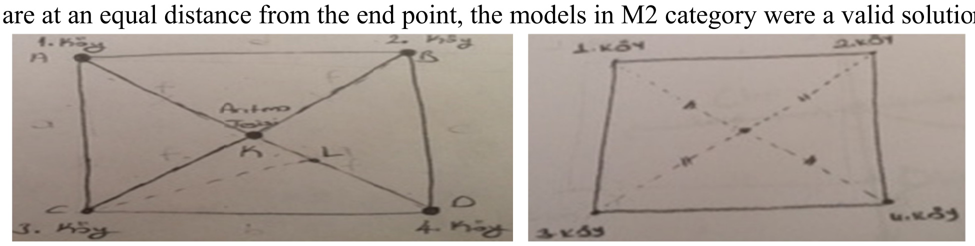

Figure 3. Quadrilateral models created by prospective teachers for the solution of the WTP model

When the models of prospective teachers in M3 category are examined, it is seen that they generally placed the villages at the edges of a quadrilateral and the treatment plant in the centre. The figures created by the prospective teachers in this category were not named as a special quadrilateral. Thus, the equal distance expression in the problem statement could not be established clearly in the models. Hence, it was seen that the prospective teachers could not establish the correlation between the elements required in the problem statement and geometric shapes although they created a model for the solution of the WTP plant.

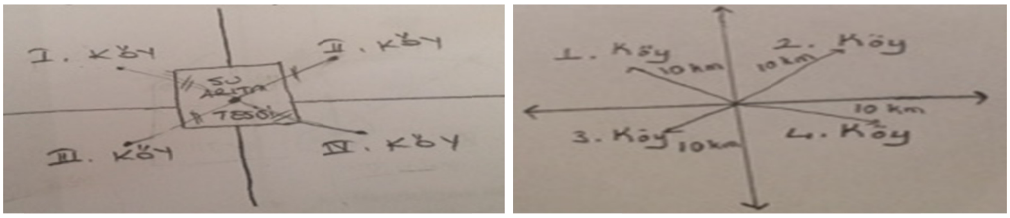

Figure 4. Analytic plane models created by prospective teachers for the solution of the WTP problem

The prospective teachers in M4 category placed the villages at equal distances in different places on the coordinate plane and the treatment plant in (0, 0) point of origin. While the models created by the prospective teachers are correct according to their own drawings, they do not constitute a solution for the problem when the positions of the villages change. Thus, the models of the prospective teachers in M4 category are only valid for sample cases and did not become generalizable and valid for the solution of the problem.

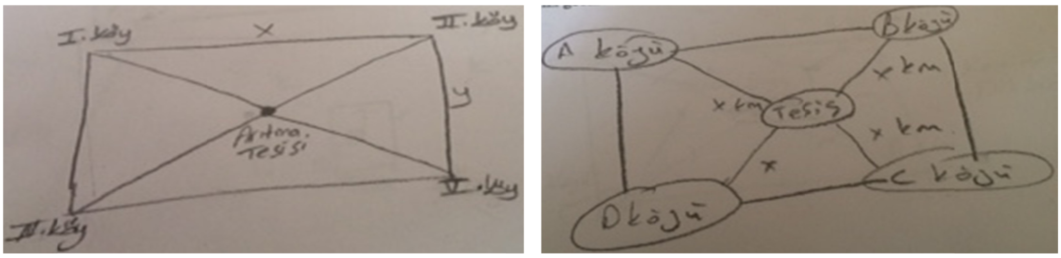

Figure 5. Rectangular models created by prospective teachers for the solution of the WTP problem

It is seen that prospective teachers used the rectangle shape in the models in the M5 category. The prospective teachers placed the villages on the edges of the rectangle and the treatment plant at the cut-off point of the diagonals. In the models in the M5 category, the place of the treatment plant is correct as the diagonal lengths are equal and center one another according to the features of the rectangular. However, these models are not sufficient for the solution of the problem when the villages are not at the edges of the rectangle. It was seen that the prospective teachers created valid models only for a sample case.

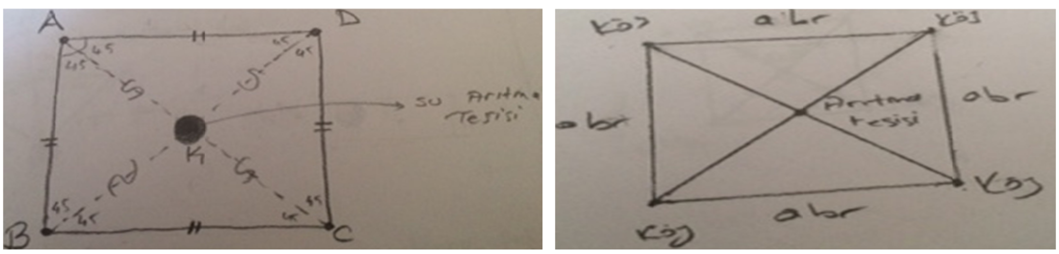

Figure 6. Square models created by prospective teachers for the solution of the WTP problem

It is seen that prospective teachers in the M6 category use the figure square for the solution of the problem. It can be said that the prospective teachers in this category have a similar idea to the candidates using the quadrilateral and rectangular models. It is seen that in the models in the M6 category, the prospective teachers place the villages at the edges of the square and the treatment plant at the cut-off point of the diagonals. The models of the prospective teachers with this idea are valid for sample cases. For, considering the characteristics of square, the lengths of the diagonals and the distance of the points where the diagonals intersect from the corners are equal. In addition, when the places of the villages on the square figure are changed the distance of the point where the treatment plant is located from the villages will also change. Thus, it can be said that the square model, just as the rectangular model, cannot be a general solution as the solutions remain limited to sample cases.

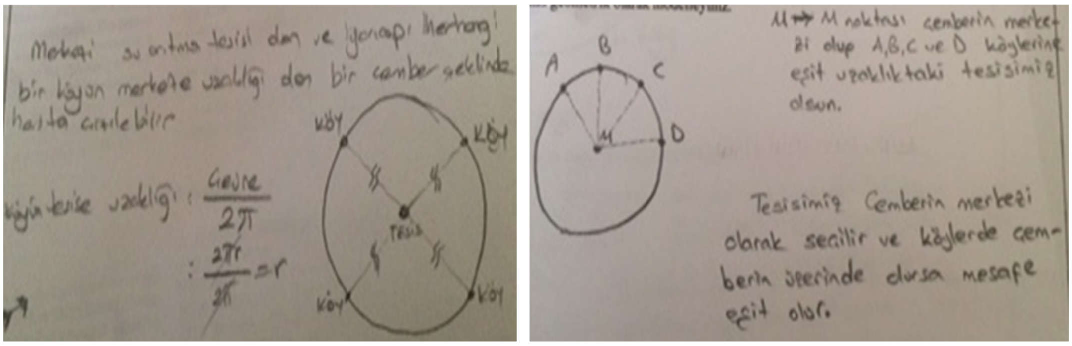

Figure 7. Circular models created by prospective teachers for the solution of the WTP problem

It is observed that prospective teachers used the geometric shape circle in their models in the M7 category. That the prospective teachers used the circular shape for four villages and a treatment plant that is of an equal distance from these villages ensured the creation of a generalizable and valid model. For, circle is a shape that consists of the cluster of the points at equal distance from a stable point. Thus, it was seen that the prospective teachers placed the treatment plant in the centre of the circle and the villages on the circle in the models they created by using the definition and features of the circle. The equality is not impaired as the distance from the treatment plant does not change when the villages on the circle is transferred to different locations.

3.2 Skills of Prospective Teachers to Transit from Model to Proof

The categories on the proofs generated by prospective teachers with the help of the models they created at the second stage of the MPT test are given in Table 4.

Table 4. Categories on the proofs of the models created by prospective teachers for the solution of the WTP problem

Category	Prospective Teachers (%)
Unanswered (P1)	3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 32, 34, 35, 36, 46, 48, 50, 61, 62, 68, 78, 80, 85, 94, 96, 126 (31%)
Restatement of the Model (P2)	1, 2, 10, 25, 37, 40, 41, 95 (6%)
Trial and Error (P3)	31, 39, 47, 49, 54, 64, 71, 89, 121, 123, 128 (9%)
Sample Case (P4)	18, 30, 45, 51, 52, 53, 55, 56, 57, 58, 60, 63, 65, 66, 67, 69, 70, 72, 73, 74, 75, 79, 81, 82, 83, 84, 87, 90, 91, 112, 113, 119, 120, 122, 124, 129 (28%)
Symbolic (P5)	38, 42, 43, 44, 59, 76, 77, 86, 93, 98 (8%)
Structural (P6)	33, 97, 99, 106, 107, 115, 127 (5%)
From Model to Proof (P7)	88, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 114, 116, 117, 118, 125 (13%)

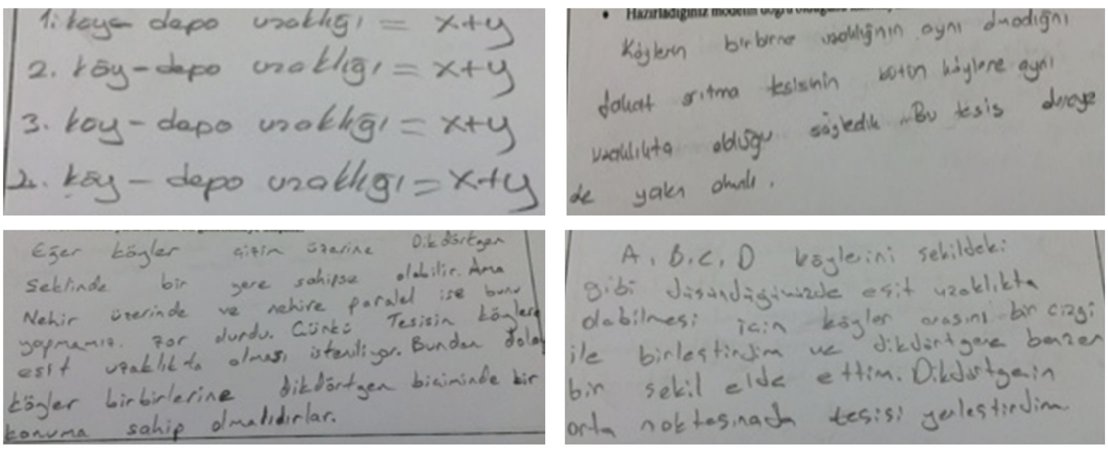

Figure 8. Proof samples of prospective teachers in the unanswered category

It can be said that the proofs of prospective teachers in P1 category are usually the result of the models in the M1 category. The prospective teachers usually gave estimated, irrelevant answers or answers that do not contain a generalization as they cannot correlate the WTP problem with mathematical structures. However, among the answers of the prospective teachers in P1 category, there are also those prospective teachers who can create geometric models for the solution of the WTP problem. It can be said that these prospective teachers could not create a valid proof as they could not use the features of geometric models or generated faulty models. At this point, the answers of PT36 in the M2 category and PT94, PT96 and PT126 in the M7 category stand out. Although these prospective teachers created valid models for the solution of the problem they did not use the characteristics of these structures to create a valid proof.

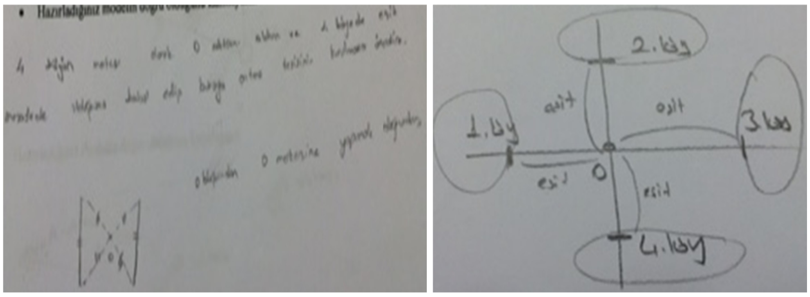

Figure 9. Proof samples of the prospective teachers in the restatement of the model category

When the proofs in the P2 category are examined, it is seen that the prospective teachers repeat the valid or invalid models they create for the solution of the problem. It is observed that the models in M1, M2, M3 and M7 categories are included in this proof category. As the models in M1 and M3 categories are not valid for the solution of the problem, it is obvious that the prospective teachers with this idea will not be able to draw a mathematical conclusion. However, it was seen that (M2 and M7) PT37 and PT95, who created valid models, could not draw any conclusion from the models they created. It can be said that this results from the fact that the prospective teachers who created valid models could not fully internalize the features and definitions of these geometric structures just as in P1 category.

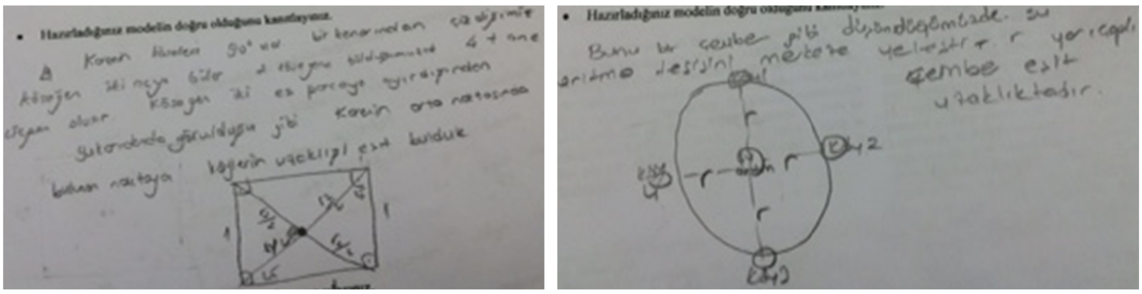

Figure 10. Proof samples of the prospective teachers in the trial and error category

When the answers of prospective teachers in P3 category, it was seen that they endeavoured to provide a proof over the models they created and different shapes. It was determined that the prospective teachers in this category performed trials on more than one shape and could not make valid generalizations. It can be said that this results from the inability of the prospective teachers to create a valid model for the solution of the problem. For, it was observed that problems were encountered during the correctness and generalization of the solution when a valid problem could not be created. However, it is seen that there are valid models (PT89, PT121, PT123, PT128) in M7 category among the answers as well. Thus, it was observed that there are prospective teachers who could not completely interpret the definition of circle among those who created valid models (M7) for the solution.

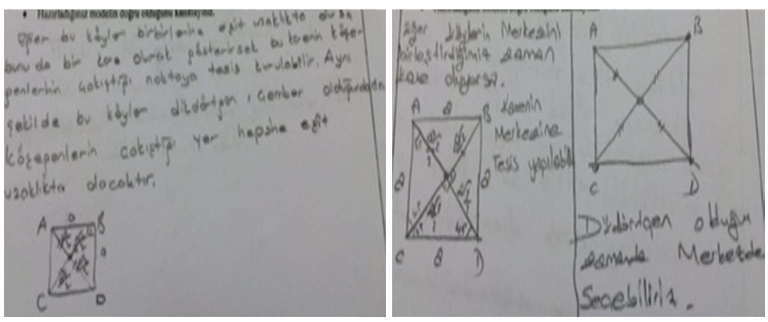

Figure 11. Proof samples of the prospective teachers in the sample case category

When the answers in P4 category are examined, it is seen that the prospective teachers tend to create proof only with one example over the models that the candidates used for the solution of the problem. It can be said that only those prospective teachers who used the M6 models has such an idea. This results from the fact that the prospective teachers, who created a square model, want to prove the correctness of the sample case they generated, in which villages are positioned on the edges of the square and the treatment plant is positioned at the cut-off point of the diagonals. For, this idea in the square model is only valid for similar sample cases. As a general solution could not be found for different positions of the villages on the square, the correctness could naturally not be proved. However, there are prospective teachers (PT90, 91, 112, 113, 119, 120, 122, 124, 129) who created valid (M7) models in P4 category as well. It can be said that the prospective teachers with this idea are only interested in that four points taken from the circle are at an equal distance to the centre rather than using the definition of the circle. Although sufficient arguments were used in terms of the solution of the problem, it was seen that they did not create an argument on the fact that infinite points to be taken on the circle must be at an equal distance to the centre, which is necessary for the proof.

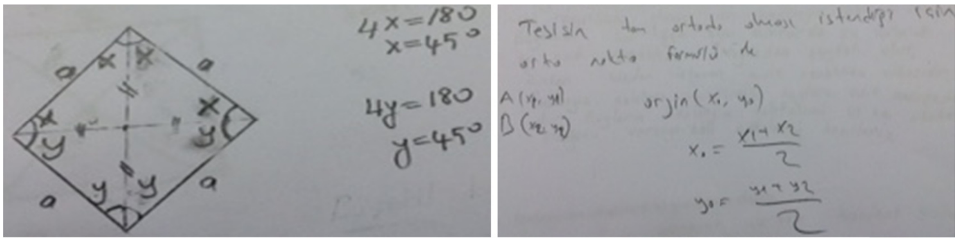

Figure 12. Proof samples of the prospective teachers in the symbolic category

When the answers of prospective teachers in P5 category are examined, it was seen that they tried to express their models symbolically in order to explain and confirm valid (PT93, 98) or invalid (PT38, 42, 43, 44, 59, 76, 77, 86) models they used for the solution of the problem. It was determined that the symbolic expressions used by the prospective teachers who tried to provide a proof were insufficient in terms of producing a general solution. The prospective teachers used symbols solely in order to find the centre of the models they used and they could not achieve a general solution.

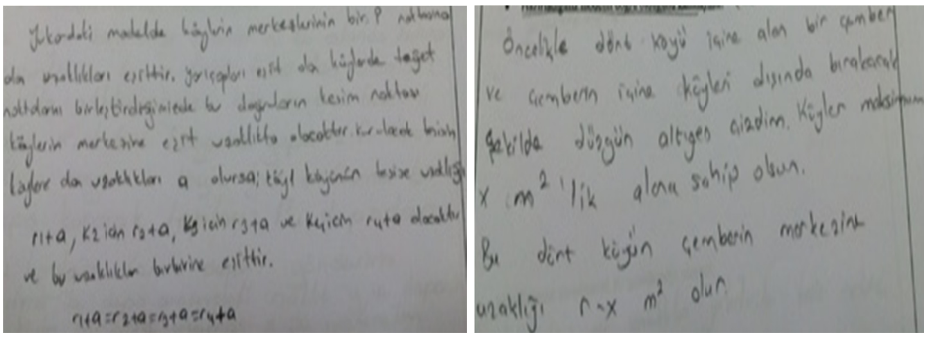

Figure 13. Proof samples of the prospective teachers in the structural category

When the proofs in P6 category are examined, it is seen that 6 prospective teachers apart from PT33 (M1) created M7 models. The prospective teachers in this category could not provide a valid proof with reference to their models although they created valid models for the solution of the problem. It can be said that the most important reason for this is that they used different mathematical models in the proof rather than the definition of circle although they used the circle model in the solution. For, the mathematical expressions used by the prospective teachers in this model are correct, but they cannot provide the confirmation and generalization of the model created for the solution.

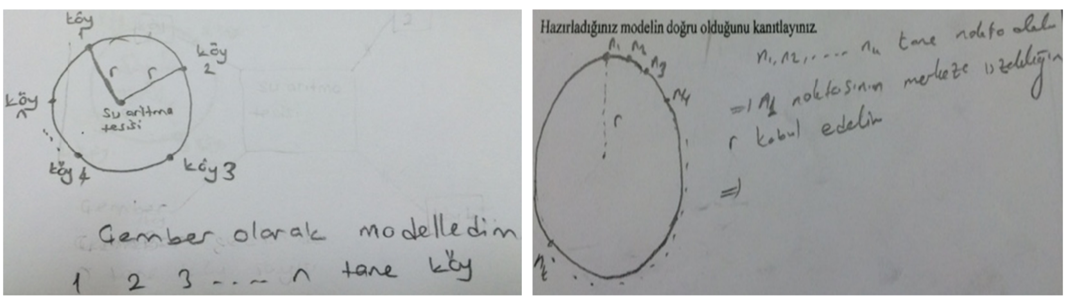

Figure 14. Proof samples of the prospective teachers in from model to proof category

When the answers of prospective teachers in P7 category are examined, it was seen that they created M7 models in the solution of the problem. The prospective teachers made valid confirmation and generalization at the stage of proof using the circle models they created for the solution of the problem. It can be said that the prospective teachers used the models they created and the definition of circle in order to provide a valid proof. It was seen that prospective teachers with this idea achieved a correct generalization that infinite number of points can be chosen on the circle at an equal distance to the centre of the circle. Thus, it can be said that the prospective teachers in P7 category confirmed and generalized the models (M7) they created for the solution of the WTP model and their features (the definition of the circle) for similar situations.

4. Discussion

It was seen that the models used by prospective teachers for solving the WTP program for creating the cluster of the points at an equal distance to a stable point were in the categories of M1 (27%), M2 (2%), M3 (4%), M4 (2%), M5 (7%), M6 (25%) and M7 (33%). It can be said that the models created by the prospective teachers usually focused on the invalid model, square and circle models. In other words, it was determined that only one third of the prospective teachers could create valid models for the solution of the problem. This can be interpreted as prospective teachers could not understand the mathematical information hidden in the WTP problem fully. For, the most important difference distinguishing the problems requiring model creation technique from others is that the available data are not definite and clear as in other problems (Eraslan, 2012). Hence, it can be said that one third of the prospective teachers could not create the models to reveal the mathematical information in the problem. Thus, this result supports the results that prospective teachers had difficulty in determining the hypothesis that will set up their models (Eraslan, 2012; Blum & Leib, 2007). For, all of the prospective teachers other than those in the M2 category usually selected the geometric shape that the villages will located on incorrectly (65%) although the placed the water treatment plant in the centre of the model. While the idea of the prospective teachers to place the water treatment plant in the centre of the geometric shape is a correct approach, attention must be paid in the first step to define the geometric model and its features in a way that they overlap with the mathematical information in the problem. It was seen that this step was neglected especially in the quadrilateral, rectangular and square models where the villages are positioned at the edges of the geometric shapes and the treatment plant at the cut-off point of the diagonals. It can be said that the fact that the problem is only valid for the solution of a single sample case in such type of models created by the prospective teachers and invalid for other cases was neglected. In addition, it was seen that the prospective teachers who created invalid models could not see the mathematical information in the WTP problem and drew schemas for the villages and the treatment plant in the problem rather than a solution. It can be said that the prospective teachers who generated a valid model (M2 and M7) realized that the mathematical information in the WTP problem could be created with the circle geometric model. For, these prospective teachers reflected the information that the distance of four different points on the circle from the centre would be equal in their models with reference to the problem situation. As for the prospective teachers in the M2 category, they generated conic models with a circle base. Thus, only the prospective teachers (PT36, 37, 38) in this category generated valid models with a different idea by placing the water treatment plant at the endpoint of the cone.

In the study, it was seen that the proofs of the models created by prospective teachers for the solution of the WTP at the second stage of the MPT were included in the categories of P1 (31%), P2 (6%), P4 (28%), P5 (8%), P6 (5%) and P7 (13%). It can be said that the proofs provided by the prospective teachers as a result of the M1, M3, M5 and M6 models they used in the solution of the WPT problem are usually included in the categories of P1 and P4. For, it was seen that the models of the prospective teachers in the M1 category led to answers in the category of P1 that are left blank, irrelevant or that do not contain any generalization at the stage of proof with reference to these models as they contain the presentation of the villages and the treatment plant in the WTP problem only with schemas. Similarly, it can be said that the proofs provided with reference to the models of prospective teachers in the categories of M3, M5 and M6 led to the increase of the rates of the answers in categories P3 and P4 as these models are only valid for the special solutions of the WTP problem. Furthermore, it can also be said that the proofs in P5 and P6 categories, in which prospective teachers endeavoured to support the valid or invalid models they created with mathematical expressions, remain incapable. The expressions used in order to generalize the models generated in the answers in the P5 category could not be correlated with the mathematical information in the WTP problem. In the answers in the P6 category, logical mistakes were made although valid mathematical information was used. That M7 models are especially abundant in the P6 category shows that creating a valid model for the solution of the problem is not sufficient in its own for the generalization and confirmation of the models. The answers of the prospective teachers in the P2 category are all about the repeat of the models created and it is far from the correctness and generalization of the models. Thus, it was seen that the proofs of prospective teachers in the categories of P1, P2, P3, P4, P5 and P6 remained incapable for showing the truth and generalizability of the models used in the solution of the problem. This result obtained from the research support the results of “successful visual proofs are scarce” and “their generalizability is limited” (Hanna & Sidoli, 2007; Borwein & Jörgenson, 2001).

Valid results of prospective teachers in the proving stage of the study are in P7 (13%) category. All of the answers of the prospective teachers in this category are a result of the M7 (33%) models. However, while the valid model creating rates of the prospective teachers in the solution of the WTP model is 33%, it was seen that this rate dropped to 13% in the proving stage, which is the stage for showing the correctness of these models and generalizing them for similar situations. This drop in the valid proofs of the prospective teachers can be interpreted that they have difficulty in transiting from the model to the proof. It can be said that the main reason for this is that the definition of the circle hidden in the WTP problem cannot be clearly understood by the teachers. For, it is seen that the prospective teachers cannot achieve a generalization by using the definition of the circle although they create circular models. That the prospective teachers created the model for the real-life problem at the first stage helped them create a deduction chain at the second stage (Hanna & Jahnke, 2004); however it can be said that it is not sufficient. Thus, it can be said that the perception that to use mathematical language and symbol when transiting from model to proof is a must led to the abundance of prospective teachers creating a valid model in P5 and P6 proof categories. For, there is no complementary relationship between proof and modelling and it is necessary to strengthen the relationship between geometric models and their definitions when creating the deduction chain. Thus, as is also specified by Hanna and Sidoli (2007), mathematics and mathematics education is still quite far away from the potential roles of visual representations and especially their roles towards proving. Hence, it is necessary to render similar real-life problems significant in mathematics education and apply them in class environments especially in order for prospective teachers to become aware of the roles between the use of mathematical models in mathematical proofs and the roles of explanation, confirmation, generalization and correlation.

References

Blum, W., & Leib, D. (2007). How do students and teachers deal with modeling problems? In Haines, C. R. Galbraith, P. Blum, W. Khan, S. (Ed.), *Mathematical modeling (ICTMA–12): Education, Engineering and Economics*, 222–231, Horwood Publishing, Chichester.

Borwein, P., & Jörgenson, L. (2001). Visible structures in number theory. *American Mathematical Monthly, 108*, 897-911. http://dx.doi.org/10.2307/2695413

Davis, P. J. (1993). Visual theorems. *Educational Studies in Mathematics, 24*, 333–344. http://dx.doi.org/10.1007/BF01273369

De Villiers, M. (2002). Developing understanding for different roles of proof in dynamic geometry [Online]. Available from: http:// http://mzone.mweb.co.za/residents/profmd/profmat.pdf.

Dede, Y. (2013). Proof in mathematics: Importance, types and historical development. In Zembat İ. Ö., Özmantar, M. F., Bingölbali, E., Şandır, H, & Delice, A. (Ed.), Mathematical concepts with their definitions and historical development, Pegem Academy Publications, Ankara.

Doruk, B. K., Kıymaz, Y., & Horzum, T. (2012). Opinions of prospective classroom teachers on providing proof and using concrete model in proof [Online]. Available from: http:// http://kongre.nigde.edu.tr/xufbmek/dosyalar/tam_metin/pdf/2474-30_05_2012-22_03_11.pdf.

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics*, *63*(3), 303-323. http://dx.doi.org/10.1007/s10649-005-9013-1

English, L. D. (2003). Reconciling theory, research and practice: A models and modeling perspective. *Educational Studies in Mathematics. 54*, 225-248. http://dx.doi.org/10.1023/B:EDUC.0000006167.14146.7b

Eraslan, A. (2012). Prospective elementary mathematics teachers' thought processes on a model eliciting activity. *Educational Sciences: Theory & Practice, 12*(4), 2953-2970.

Hanna, G., & Jahnke, H. N. (2002). Arguments from physics in mathematical proofs: An educational perspective. *For the Learning of Mathematics, 22*(3), 38-45.

Hanna, G., & Jahnke, H. N. (2004). Proving and modeling. In H. W Henn, & W. Blum (Ed.), *Icmi Study 14: Applications and Modelling in Mathematics Education*, (pp. 109-114), Dortmund, Germany.

Hanna, G., & Sidoli, N. (2007). Visualisation and proof: A brief survey of philosophical perspectives. *ZDM-Mathematics Education, 39*, 73-78. http://dx.doi.org/10.1007/s11858-006-0005-0

Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange, 21*(1), 6-13. http://dx.doi.org/10.1007/BF01809605

Hawkins, M. (2007). *Teaching geometric reasoning: Proof by pictures. Unpublished Master Thesis*, North Carolina State University, Raleigh, North Carolina, USA.

Hodgson, T., & Riley, K. (2001). Real-world problems as contexts for proof. *Mathematics Teacher, 94*(9), 724.

Knuth, E. J. (2002). Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teacher Education, 5,* 61-88. http://dx.doi.org/10.1023/A:1013838713648

Ko, Y. Y., & Knuth, E. (2009). Undergraduate mathematics majors' writing performance producing proofs and counterexamples about continuous functions. *Journal of Mathematical Behavior, 28*, 68-77. http://dx.doi.org/10.1016/j.jmathb.2009.04.005

Lesh, R. A., Hamilton, E., & Kaput, J. J. (2007). *Foundations for the future in mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and United States.* Newyork, NJ: Lawrence Erlbaum Associates.

Mcmillan, H. J., & Schumacher, S. (2010). *Research in education*. Boston, USA, Pearson Education.

Ministry of National Education [MNE]. (2013). *Secondary (9-12). Classes Programs Promotion Handbook (In Turkish). Chairman of the Board of Education Ministry of National Education*. Ankara: Department of State Printing House Books.

Mudaly, V., & De Villiers, M. (2004). Mathematical modeling and proof. *Proceedings of the 10th Conference of Association for Mathematics Education of South Africa*, University of the Nort-West, Potchefstroom.

Olkun, S., Şahin, Ö., Akkurt, Z., Dikkartın, F. T., & Gülbağcı, H. (2009). Problem solving and generalization through modeling: A study on elementary school students. *Education and Science, 34*(151), 65-73.

Özkaya, M., Işık, A., & Konyalıoğlu, A. C. (2014). Proving and counter-exampling performances of students in the elementary mathematics education department for continuous functions. *Middle Eastern & African Journal of Educational Research, 11*, 26-42.

Palais, R. S. (1999). The visualization of mathematics: Toward a mathematical Exploratorium. *Notices of the AMS, 46*(6), 647-658.

Rav, Y. (1999). Why do we prove theorems?. *Philosophia Mathematica, 7*(1), 5–41. http://dx.doi.org/10.1093/philmat/7.1.5

Stylianides, A. J. (2007). The notion of proof in the context of elementary school mathematics. *Educational Studies in Mathematics, 65*(1), 1-20. http://dx.doi.org/10.1007/s10649-006-9038-0

Ünveren, E. N. (2010). *An observing on the candidates of the primary mathematics teachers' attitudes towards proof in mathematical modelling process (Msc Thesis)*. Balıkesir University, Institute of Science, Balıkesir.

Appendix 1: Modelling and Proof Test

People who live in many remote villages in under-developed countries such as South Africa do not have access to reliable and clean drinking water. These people depend on nearby creeks and rivers to fulfil their needs of water. Untreated water coming from creeks and rivers in this region has recently become dangerous for the use of people as a result of the cholera epidemic in recent years. Let us assume that you will create a plan of a water treatment plant, which will be at an equal distance to 4 villages in this situation. Where would you suggest the plant to be according to the plan you prepare?

- Model your plan geometrically.
- Prove the correctness of the model you have prepared by generalizing so as it can be used for similar situations as well.