RedFame

International Journal of Social Science Studies
Vol. 14, No. 3; September 2026
ISSN 2324-8033 E-ISSN 2324-8041
Published by Redfame Publishing
URL: http://ijsss.redfame.com

Alternative Perspectives on Teaching the Time Value of Money

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Received: March 23, 2026 Accepted: May 19, 2026 Available online: September 3, 2026

doi:10.11114/ijsss.v14i3.9267 URL: https://doi.org/10.11114/ijsss.v14i3.9267

Abstract

The time value of money is a foundational concept generally introduced in the first junior-level finance course at most U.S. business schools. Bond pricing and amortized loans are commonly presented as real-world applications of present value calculations. This paper offers alternative pedagogical approaches to these topics by demonstrating that both future bond prices and remaining loan balances can also be derived using future value methods. By broadening the analytical framework, these alternative perspectives should deepen students' conceptual understanding of the time value of money and better prepare them to make informed financial decisions in both professional and personal contexts.

Keywords: Time Value of Money, Bond Price, Amortized Loan Balance, Present and Future Value Calculations

1. Introduction

The concept of the time value of money is typically introduced in the very first finance class, usually at the junior level, in U.S. universities. And such finance class is generally a core requirement for all business majors, reflecting the essential role the concept plays across disciplines. This topic is important not just for finance majors but also for other business and even non-business majors. Its significance extends beyond finance majors; students in accounting, marketing, management, and even non-business fields benefit from understanding the time value of money concept. Recently, the *Wall Street Journal* reported that a personal finance course is now mandatory in 39 states.[i] This trend further underscores the importance of this concept. The analytical skills and financial reasoning developed through this topic empower students to make informed decisions in areas such as retirement planning, investment evaluation, and the financing of major purchases like automobiles and homes. In other words, the time value of money represents a foundation of business education, preparing students with essential tools for both professional and personal financial decision-making.

Bond pricing and amortized loans are generally presented as applications of present value calculations in many popular introductory finance textbooks (see, e.g., Berk, DeMarzo, & Harford, 2022; Brigham & Houston, 2022a, 2022b, Ross, Westerfield, & Jordan, 2023). This paper offers an alternative, future-valued based, pedagogical approach to teaching these two topics. By expanding the traditional instructional approach, this alternative perspective will enhance students' understanding of the time value of money concept and thus enrich their business education in general.

The remainder of this paper is organized as follows. We begin by discussing the present value approach to bond pricing. Then we will demonstrate algebraically how it can also be derived using a future value method, and discuss how this alternative technique applies in other areas in finance such as the bond immunization process. This is followed by an examination of how amortized loan balances can be determined by either present value or future calculations which are algebraically identical.

The algebraic equivalence between a present value (backward-looking) method and a future value (forward-looking) approach to solving a financial problem is often examined in textbooks used in mathematical finance and actuarial science programs (see e.g., Chan & Tse, 2018; Kellison, 2008). However, most typical introductory finance textbooks tend to emphasize the present value framework, the use of a financial calculator and Excel while omitting the formal proof. This paper will provide instructors with a readily available pedagogical guideline to incorporate more mathematical rigor into the curriculum, helping students develop critical thinking skills required to evaluate financial problems from different analytical perspectives.

Throughout the paper, numerical examples accompanied by applications using a standard financial calculator, which is commonly required in introductory finance courses, are integrated to reinforce conceptual understanding. As we will demonstrate through an illustration, educators can readily incorporate these insights into their curriculum designs to further enrich students' understanding of the time value of money concepts. The last section will conclude this paper.

2. Bond Valuation

A bond is a fixed-income investment with a defined maturity sold or issued by corporations and governments when they seek to raise funds to finance their activities. The issuing corporations or governments are expected to pay bondholders (investors who bought the investment) fixed coupon payments periodically, usually two times per year (semiannually) and the bond's principal (also called the face, par, or maturity value) on the maturity day. The price an investor needs to pay for a bond is affected by the prevailing interest rate of comparable bond investments, which is heavily influenced by the Federal Reserve monetary policies. This interest rate is also referred to as the market or required interest rate for the bond or the bond's annual nominal yield to maturity.

2.1 Bond Price As a Present Value of Future Cash Flows

Suppose a corporate bond that pays annual coupon payments has the following properties:

N = bond maturity in years;

M = the par value at maturity;

r = annual coupon rate, assuming it is fixed;

CY = annual coupon payment = r(M);

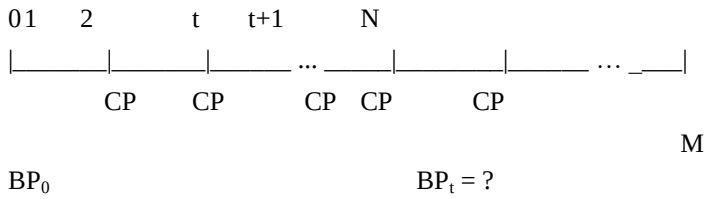
i = the bond's annual yield to maturity or annual required rate of return; and

BP_0 = current (today's) bond price and today is a coupon payment date. Therefore, the next payment will be exactly one year from today.

With a present value method, BP_0 = the present value of N number of remaining coupon payments, which are simply an annuity, plus the present value of M (all discounted at i). Therefore,

$$BP_0 = CP\left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right] + \frac{M}{(1+i)^N}. \text{ [ii]} \quad (1)$$

In addition to many textbooks, this present value method (to compute bond prices) is also discussed and explored in various finance pedagogical articles (see McGowan & Joyner, 2015; Spurrell, 2009; Zhang, 2016). How about the bond price at the end of year t (the coupon payment is just paid and so the next payment will be at t + 1 year)? The following timeline depicts all the bond's coupon payments, M, current price at time 0 (BP_0), and price at year t (BP_t).



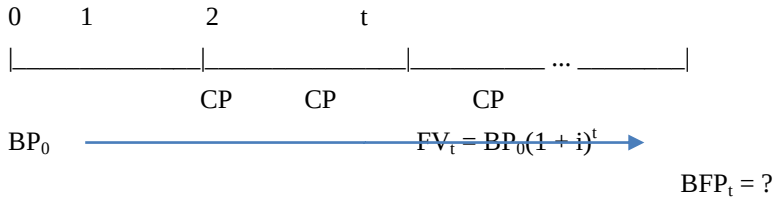
Note that at year t the bond has (N – t) years left till maturity. Therefore, BP_t = the present value of (N – t) number of coupon payments plus the present value of M (all discounted at i). Hence,

$$BP_t = CP\left[\frac{1}{i} - \frac{1}{i(1+i)^{N-t}}\right] + \frac{M}{(1+i)^{N-t}}. \quad (2)$$

2.2 Bond Price as a Future Investment Value

Instead of considering future bond price as a present value of its remaining cash flows, we can also alternatively view it as a *future* value of today's investment, *net* of reinvested coupon payments. Suppose an investor just paid BP_0 for this bond today. This investment value is expected to increase or compound annually over time at the bond's yield to maturity, i. Let the investment future value is FV_t at the end of year t which equals to $BP_0(1 + i)^t$. However, the investor does not receive a lump sum of FV_t. Instead, it consists of the bond's t number of coupon payments, each reinvested at i, and the bond's future price, denoted as BFP_t, at year t. Thus, $BFP_t = FV_t$ *minus* the future value of t number of coupon payments.

The timeline below illustrates the relationship among BP_0, FV_t, and BFP_t:



The future value of the reinvested coupon payments is simply a future value of an annuity which is the t number of coupon payments =

$$\mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right].$$

Therefore,

$$\mathrm{BFP_t} = \mathrm{BP_0}(1+\mathrm{i})^{\mathrm{t}} - \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right].$$

Substituting BP_0 from Eq. (1):

$$\mathrm{BFP_t} = \left\{\mathrm{CP}\left[\frac{1}{\mathrm{i}} - \frac{1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}}\right] + \frac{\mathrm{M}}{(1+\mathrm{i})^{\mathrm{N}}}\right\}(1+\mathrm{i})^{\mathrm{t}} - \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right]$$

$$= \mathrm{CP}\left[\frac{1}{\mathrm{i}} - \frac{1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}}\right](1+\mathrm{i})^{\mathrm{t}} + \frac{\mathrm{M}}{(1+\mathrm{i})^{\mathrm{N}}}(1+\mathrm{i})^{\mathrm{t}} - \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right]$$

$$= \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}} - \frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}}\right] + \frac{\mathrm{M}(1+\mathrm{i})^{\mathrm{t}}}{(1+\mathrm{i})^{\mathrm{N}}} - \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right].$$

Combining the two CP terms:

$$\mathrm{BFP_t} = \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}} - \frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}} - \frac{(1+\mathrm{i})^{\mathrm{t}}-1}{\mathrm{i}}\right] + \frac{\mathrm{M}(1+\mathrm{i})^{\mathrm{t}}}{(1+\mathrm{i})^{\mathrm{N}}}$$

$$= \mathrm{CP}\left[\frac{(1+\mathrm{i})^{\mathrm{t}} - (1+\mathrm{i})^{\mathrm{t}} + 1}{\mathrm{i}} - \frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}}\right] + \frac{\mathrm{M}(1+\mathrm{i})^{\mathrm{t}}}{(1+\mathrm{i})^{\mathrm{N}}}$$

$$= \mathrm{CP}\left[\frac{1}{\mathrm{i}} - \frac{(1+\mathrm{i})^{\mathrm{t}}}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}}\right] + \frac{\mathrm{M}(1+\mathrm{i})^{\mathrm{t}}}{(1+\mathrm{i})^{\mathrm{N}}}.$$

Rewriting $(1+\mathrm{i})^{\mathrm{t}}$ as $\frac{1}{(1+\mathrm{i})^{-\mathrm{t}}}$:

$$\mathrm{BFP_t} = \mathrm{CP}\left[\frac{1}{\mathrm{i}} - \frac{1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}}(1+\mathrm{i})^{-\mathrm{t}}}\right] + \frac{\mathrm{M}}{(1+\mathrm{i})^{\mathrm{N}}(1+\mathrm{i})^{-\mathrm{t}}}$$

$$= \mathrm{CP}\left[\frac{1}{\mathrm{i}} - \frac{1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{N}-\mathrm{t}}}\right] + \frac{\mathrm{M}}{(1+\mathrm{i})^{\mathrm{N}-\mathrm{t}}} = \mathrm{BP_t} \text{ (see Eq. (2) above).} \quad (3)$$

In other words, the future bond price can be calculated using either a present value calculation or a future value

approach. This underscores the principle that certain financial problems can be solved using either method, depending on the framing and available information.

Furthermore, the future value technique of determining a bond future value is critical in understanding the concept of how a pension fund or bond's future value can be protected (or immunized) from changes in the prevailing interest rates. This immunization process is generally discussed in investment textbooks (see Bodie, Kane, & Marcus, 2022). Hence, this alternative perspective on future bond price as future value rather than present value calculation provides students further insight into other finance topics.

2.3 Numerical Example

Assume a corporate bond has the following properties:

N = 10 years;

Par value = $1,000;

Coupon rate = 4%;

Yield to maturity = 6%; and

Annual coupon payment = 4%($1,000) = $40.

2.3.1 Bond Price As Present Value

Current bond price (BP_0) at time zero or today =

$$BP_0 = \$40\left[\frac{1}{0.06} - \frac{1}{0.06(1.06)^{10}}\right] + \frac{\$1{,}000}{(1.06)^{10}} = \$852.80.$$

Bond price *four* years later (BP_4) =

$$BP_4 = \$40\left[\frac{1}{0.06} - \frac{1}{0.06(1.06)^{10-4}}\right] + \frac{\$1{,}000}{(1.06)^{10-4}} = \$901.65.$$

The above calculations can easily be done with the use of a financial calculator such as the *Texas Instruments BAII Plus*. For current bond price (BP_0)[iii]:

TVM Keys:	N	I/Y	**PV**	PMT	FV
Enter	**10**	6		-40	-1,000
Compute (CPT)			**852.80**		

For bond price *four* years later (BP_4), N (maturity) becomes 6:

TVM Keys:	N	I/Y	**PV**	PMT	FV
Enter	**6**	6		-40	-1,000
Compute (CPT)			**901.65**		

2.3.2 Bond Price as a Future Price of Today Value

Bond price four years later:

$$BFP_4 = \$852.80(1.06)^4 - \$40\left[\frac{(1.06)^4 - 1}{0.06}\right] = \$901.65 = BP_4.$$

Using the *Texas Instruments BAII Plus* to compute BFP_4[iv]:

TVM Keys:	N	I/Y	PV	PMT	**FV**
Enter	4	6	852.90	**-40**	
Compute (CPT)					**901.65**

3. Amortized Loans

An amortized loan requires the loan to be paid off over time through a series of equal periodic payments. Each payment covers a portion of the loan's principal and the interest accrued on the outstanding balance until the loan is paid in full. In the U.S., auto loans and home loans (mortgages) are typically amortized loans.

3.1 Loan Balance of An Amortized Loan

Because the payments on an amortized loan are simply an annuity, the present value of the payments is set to equal to the original loan amount. Suppose

L = original loan amount;

i = annual interest rate or annual percentage rate;

N = number of years or the term of the loan; and

PMT = annual payment.

With a present value method,

$$L = PMT\left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right]. \quad (4)$$

Auto loans and mortgages in the U.S. typically call for monthly payments. The above equation and following presentations can be easily adjusted for monthly payments.[v]

3.2 Remaining Loan Balance

Suppose a loanee decides to pay off a loan *immediately* after making t number of payments. Assuming no prepayment penalty, one approach to determine the remaining loan balance right after t number of payments is to discount the remaining (N – t) number of payments to time t or to calculate the present value of the remaining (N – t) payments at time t. Let the remaining loan balance after t payments is LB_t:

$$LB_t = PMT\left[\frac{1}{i} - \frac{1}{i(1+i)^{N-t}}\right]. \quad (5)$$

The present value method of determining amortized loan balance is usually discussed not just in many undergraduate finance textbooks but also in finance education articles (see Gutierrez & Dalsted, 2012; Kruck & Maher, 1999; Vidermanovaa & Melusovaa, 2015; Zhang, 2016).

3.3 Lender's Perspective

An alternative view to conceptualize the remaining loan balance is from the lender's perspective. To the lender, the original loan amount (L) is equivalent to an investment purchased today from which the lender will earn an annual payment of PMT for N years. Suppose:

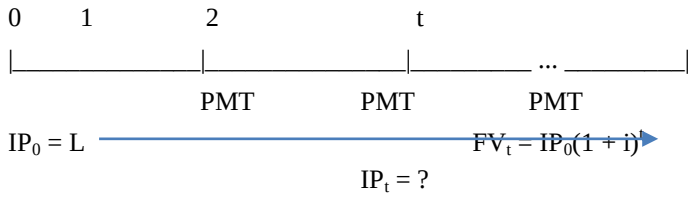
L = the investment value or price at time 0 = IP_0; and

i, N, and PMT are the same as before.

So, what is the investment price (IP_t) after t number of years or payments?

Because the investment earns an annual rate of return of i, the initial investment value IP_0 will therefore compound at i for t years to a future value FV_t, which equals $IP_0(1 + i)^t$. However, the lender does not receive a lump sum of $IP_0(1 + i)^t$ directly. Instead, this future value is the *sum* of the future price of the investment (IP_t) and the future value of the reinvested t number of payments (PMTs).

The timeline below displays the amortized loan:



The future value of reinvested PMTs is just an annuity's future value.

Hence,

$$IP_t = IP_0(1+i)^t - PMT\left[\frac{(1+i)^t - 1}{i}\right].$$

But we know from Eq. (4) that $L = PMT\left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right] = IP_0$.

So,

$$IP_t = PMT\left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right](1+i)^t - PMT\left[\frac{(1+i)^t - 1}{i}\right]$$

$$= PMT\left[\frac{(1+i)^t}{i} - \frac{(1+i)^t}{i(1+i)^N}\right] - PMT\left[\frac{(1+i)^t - 1}{i}\right].$$

Combining the two PMT terms:

$$IP_t = PMT\left[\frac{(1+i)^t}{i} - \frac{(1+i)^t}{i(1+i)^N} - \frac{(1+i)^t - 1}{i}\right]$$

$$= PMT\left[\frac{(1+i)^t}{i} - \frac{(1+i)^t}{i(1+i)^N} - \frac{(1+i)^t}{i} + \frac{1}{i}\right]$$

$$= PMT\left[\frac{1}{i} - \frac{(1+i)^t}{i(1+i)^N}\right].$$

Rewriting $(1+i)^t$ as $\frac{1}{(1+i)^{-t}}$:

$$IP_t = PMT\left[\frac{1}{i} - \frac{1}{i(1+i)^{N-t}}\right] = LB_t \text{ (See Eq. (5) above).}$$

This future value approach to determining an amortized loan balance will strengthen students' grasp of the time value of money concept because it demonstrates ~~how~~ the equivalence of present value and future value methods in solving the same financial problem.

3.4 Numerical Example

Assume an amortized loan has the following terms:

N = 10 years;

Loan amount L = $10,000; and

i = 8%.

Annual payment:

$$\$10{,}000 = PMT\left[\frac{1}{0.08} - \frac{1}{0.08(1+0.08)^{10}}\right] = PMT[6.7101].$$

Or $PMT = \$1{,}490.29$.

Again, the PMT can be computed easily with the *Texas Instruments BAII Plus*[vi]:

TVM Keys:	N	I/Y	PV	**PMT**	FV
Enter	**10**	8	-10,000		0
Compute (CPT)				**1,490.29**	

Suppose the loanee decides to pay off the loan right after the *third* payment. So, the loan balance after three years or right after three payments can be determined as a present value of the remaining seven payments:

$$\mathrm{LB}_3 = \$1{,}490.29\left[\frac{1}{0.08} - \frac{1}{0.08(1+0.08)^{10-3}}\right] = \$7{,}759.00.$$

Using the *Texas Instruments BAII Plus* to compute LB_3[vii]:

TVM Keys:	N	I/Y	**PV**	PMT	FV
Enter	**7**	8		-1,490.29	0
Compute (CPT)			**7,759.00**		

Alternatively, it can also be computed as a future price:

$$\mathrm{IP}_3 = \$10{,}000(1+0.08)^3 - \$1{,}490.29\left[\frac{(1+0.08)^3 - 1}{0.08}\right] = \$7{,}759.04 = \mathrm{LB}_3.$$

With the *Texas Instruments BAII Plus*, IP_3 can be determined as future value of the \$10,000 *minus* the future value of the three reinvested payments (\$1,490.29)[viii]:

TVM Keys:	N	I/Y	PV	PMT	**FV**
Enter	3	8	10,000	**-1,490.29**	
Compute (CPT)					**7,759.04**

4. Pedagogical Recommendations

A salient feature of these alternative perspectives is that they show certain financial problems can be effectively solved by using either a present value (or backward-looking) or future value (or forward-looking) approach. An educator can readily incorporate this insight into both the curriculum design and the learning outcomes. For instance, after showing students how to determine either future bond price or remaining loan balance using both present value and future value methods, the instructor can present this retirement problem. A person who just turned 30 years old and plans to retire at age 60 is given two investment options. Option A is to invest \$20,000 annually (the first payment is made one year from now) in a retirement account that earns 8% annual interest rate, compounded annually. Option B is to wait *five* years and invest an annual amount of \$30,000 (the first investment is at the end of age 35) in the same account (8% annual interest rate, compounded annually). Which option will generate a larger retirement fund at age 60 for the person?

With a future value method, the instructor can ask students to calculate the future values of the two annuities at age 60 and then compare the accumulated amounts to determine which option generates more retirement fund. Next, the instructor can guide students to determine and compare the present values of the two annuities at age 30. The exercise underscores the pedagogical contrast between forward-looking (future value) and backward-looking (present value) approaches.[ix] In conjunction with Poon (2025)'s emphasis on amortized loans, instructors can smoothly incorporate practical and insightful instructional materials into their curriculum that deepens students' comprehension of the concept of the time value of money.

5. Conclusion

The time value of money is a fundamental concept in business education, especially in introductory finance courses. The two commonly taught applications–bond price determination and amortized loan balance–are generally framed as present value calculations in standard finance textbooks. However, this paper demonstrates that, algebraically, both future bond prices and remaining loan balances can also be derived with future value methods. Despite their mathematical equivalence, future value techniques are seldom addressed in typical finance textbooks. Recognizing that either present value or future value techniques can solve the same financial problem will significantly improve students' conceptual understanding and problem-solving skills in time value of money applications.

Disclosure – AI-assisted editing disclosure

AI-based tools were used to polish originally-written text for clarity enhancement and grammatical correctness. However, all content was reviewed and verified by the author, and AI assistance did not displace the scholarly contribution.

Acknowledgments

Not applicable

Author contributions

Not applicable

Funding

Not applicable

Competing interests

Not applicable

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Redfame Publishing.

The journal's policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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[ii] All the equations and calculations regarding bond prices can be easily modified for semiannual coupon bonds. For example, the current bond price of a semiannual coupon bond is

$$\mathrm{BP}_0 = \frac{\mathrm{CP}}{2}\left[\frac{1}{\frac{\mathrm{i}}{2}} - \frac{1}{\frac{\mathrm{i}}{2}\left(1+\frac{\mathrm{i}}{2}\right)^{2\mathrm{N}}}\right] + \frac{\mathrm{M}}{\left(1+\frac{\mathrm{i}}{2}\right)^{2\mathrm{N}}}.$$

[iii] N = bond's maturity, I/Y = bond's yield to maturity, PV = current bond price, PMT = coupon payment, and FV = par value. Visit https://education.ti.com/en/products/calculators/financial/baii-plus for more information.

[iv] N = number (t) of coupon payments the bond investor has received, FV = future bond price or BFP_t at how many years (t) from now. Note that the inputs for PV and PMT have opposite signs (−/+) because we need to *subtract* the future value of coupon payments as part of the future bond price.

[v] For an amortized loan with monthly payments PMT_M,

$$\mathrm{L} = \mathrm{PMT}_\mathrm{M}\left[\frac{1}{\frac{\mathrm{i}}{12}} - \frac{1}{\frac{\mathrm{i}}{12}\left(1+\frac{\mathrm{i}}{12}\right)^{12\mathrm{N}}}\right].$$

[vi] N = term of the loan, I/Y = annual interest rate, PV = loan amount, PMT = annual payment.

[vii] N = number (t) of annual payments the lender has received, FV = future price (IP_t) of investment (L or IP_0) at t. Note that the inputs for PV and PMT have opposite signs (−/+) because the future value of the payments need to be *subtracted* from the loan's future value.

[viii] N = number (t) of annual payments the lender has received, FV = future price (IP_t) of investment (L or IP_0) at t. Note that the inputs for PV and PMT have opposite signs (−/+) because the future value of the payments need to be *subtracted* from the loan's future value.

[ix] The future value of option A (FV_A):

$$\mathrm{FV}_\mathrm{A} = \$20{,}000\left[\frac{(1+0.08)^{(60-30)}-1}{0.08}\right] = \$2{,}265{,}664.$$

For option B, its future value (FV_B):

$$\mathrm{FV}_\mathrm{B} = \$30{,}000\left[\frac{(1+0.08)^{(60-35)}-1}{0.08}\right] = \$2{,}193{,}178.$$

Hence, A is a better choice even though the person invests more annually under B because the amount is unable to compensate for the lost savings from the first 5 years of investment from A.

Now, the present value of option A (PV_A):

$$PV_A = \$20{,}000\left[\frac{1}{0.08} - \frac{1}{0.08(1.08)^{60-30}}\right] = \$225{,}156$$

And for B (PV_B):

$$PV_B = \$30{,}000\left[\frac{1}{0.08} - \frac{1}{0.08(1.08)^{60-35}}\right]\left[\frac{1}{(1.08)^{35-30}}\right] = \$217{,}952$$

Again, A is a better choice.

Note that all the computations can be easily performed using the *Texas Instruments BAII Plus.*