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How to Identify an Arbitrage of Type B on Capital Markets

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Abstract

We propose here a 1-period matrix model of a fraction of the Polish financial market (for our purposes it will suffice to focus on a fraction of the market) built up from the point of view of the Polish biggest listed company KGHM. Using this model we construct an arbitrage portfolio consisting of 5 different assets, namely shares of KGHM, Treasury bills and 3 kinds of stock options. We recall the concept of arbitrage of type A and type B (called also an arbitrage I and arbitrage II, resp.) and illustrate it with examples. To prove that an arbitrage is possible to conduct, we separately distinguish scenarios when options prices are determined by the Black-Scholes formula, and when they deviate from their theoretical values. We prove that in all those cases an arbitrage of type B can be conducted. Since our approach does not rely on the specifics of Poland as a country, it can be equally well implemented in any other country which offers Treasury bills, as well as call and put options on shares of selected companies (KGHM in the studied case). The purpose of this study is to encourage practitioners to conduct an arbitrage in their own country, especially in a case when call and put options are offered on a local OTC market.

Keywords: arbitrage of type A, arbitrage of type B, relative pricing method, complete market

JEL: C02, C18, C54, C60

1. Introduction

In this article we invoke from Cerny (2009) a mathematically rigorous definition of arbitrage of type A and type B (called also an arbitrage I and arbitrage II, resp.) which is also applicable on the markets where the law of one price holds. We show how to apply the notion of arbitrage on the Polish capital market from which we have chosen for analysis the largest listed company, KGHM, one of the leading producers of silver and cooper in the world.

We demonstrate how to identify an arbitrage of type B when a unit price of KGHM's shares equals 119 PLN. We implement an arbitrage by selling 1 million of KGHM's shares and simultaneously purchasing its synthetic replica at 111,562,500 PLN, obtaining a risk-free profit of 7,437,500 PLN. The replica is built up with Treasury bills and 3 kinds of stock options. When these stock options, both in long and short positions, are entered into on an OTC market, the arbitrager does not have to worry about the risk associated with varying daily prices and margins.

The idea behind this methodology is to consider hundreds of scenarios concerning the price of KGHM's shares, together with corresponding prices of specified in this article call an put options, in order to calculate the resulting profit or loss. Therefore, there is no sense to identify factors influencing price movements of KGHM's shares since we took into account all possible scenarios (KGHM's share prices) in our earlier calculations. However, the presented methodology does not guarantee that in a particular period of time, say 2 or 6 nearest trading sessions, the arbitrage will be for sure spotted and conducted.

The readers interested in practical (versus textbook) arbitrage limitations are referred to Shleifer and Vishny (1997). In this article the authors are concerned with so-called professional arbitrage where a small number of highly specialized investors using other people's capital is trying to conduct an arbitrage (p.35). They investigate various implications for security pricing which can be a result of such professional arbitrage, including situations when prices diverge far from their fundamental values. In this paper we offer quite opposite approach by taking asset prices for granted and then demonstrating how to spot and conduct an arbitrage of type B.

Since in this paper we propose a 1-period model of a financial market, we are concerned with *static hedging* on complete and incomplete markets. A financial market is called *complete* if each desired by financial market participants financial instrument can be perfectly replicated by means of liquid financial securities; otherwise, it is called *incomplete*. In reality, all markets are incomplete, unless one considers a fraction of a financial market, as is the case in this paper.

There already exists some literature inclusively devoted to static hedging, however, all known to this author publications are concerned with different topics than the one elaborated in this paper. Many, if not most, are devoted to replication of financial options and other derivative instruments; see, for example, Carr, Ellis and Gupta (1998, p. 1165) where static hedging was developed for several exotic options by means of standard options. See also an article by Glen and Jorion (1993) which investigates benefits resulting from currency hedging in international bond and equity markets. They demonstrate (p.1986) that inclusion of forward contracts leads to statistically significant improvements in the performance of unconditional portfolios containing bonds. This research area is however completely different from ours.

San-Lin Chung and Pai-ta Shih (2009) are concerned with static hedge portfolio (SHP) of an American option. Their results (p. 2140) indicate that the numerical efficiency of their approach is comparable to some recent advanced numerical methods, which is, of course, not relevant to this article.

On the other hand, Michenaud and Solnik (2008) apply an axiomatic behavioral theory, namely the regret theory, to derive closed-form solutions to optimal currency hedging (p.677), which also represents a completely different topic than ours. Summing up, none of the representative articles cited above is concerned with the issue tackled in this paper, namely how to build a synthetic replica of shares of some listed company with the goal to identify an arbitrage opportunity of type B and conduct the arbitrage.

The opposite of static hedging is *dynamic hedging* elaborated for example by Kondor (2009). His approach is also very different from ours. Indeed, in his article (p. 631) the arbitrageurs optimally decide how to allocate their limited capital over time, while in our approach such decision is made once in a time (today only) because we deal with a 1-period model. Besides, Kondor is developing an equilibrium model of convergence trading and its impact on asset prices, while in this paper we are not interested at all how asset prices are being shaped.

Finally, we want to make clear that the concept of arbitrage in finance we are concerned with has nothing to do with the notions of arbitrage functioning in (i) psychology, (ii) law, (iii) sport and (v) political sciences.

2. Some Theory

We present a 1-period model of a fraction of the Polish financial market (see also, Zaremba, 2016, 2017a, 2017c, 2018) in which there are only 2 dates, "today" and "tomorrow", whatever those dates mean. It is assumed that all economic activity (consumption, trading and work) takes place only "today" and "tomorrow". It turns out that such a model quite adequately represents the real financial market; it is specifically adequate for investment funds which do not make frequent trading.

Following Cerny (2009), vectors represent financial instruments, such as the vector **b** below, while matrices represent

financial markets (or their fractions), with columns featuring payouts resulting from all liquid securities tradable on a particular (fraction of) market under consideration. An example of such a matrix, investigated in Zaremba (2017a), is matrix P given below

$$\mathbf{b} = \begin{bmatrix} 65 \\ 80 \\ 95 \\ 110 \\ 125 \\ 140 \end{bmatrix}; \quad P = \begin{bmatrix} 65 & 100 & 0 & 50 \\ 80 & 100 & 0 & 35 \\ 95 & 100 & 5 & 20 \\ 110 & 100 & 20 & 5 \\ 125 & 100 & 35 & 0 \\ 140 & 100 & 50 & 0 \end{bmatrix} \tag{1}$$

It represents a fraction of the Polish financial market interesting from the point of view KGHM, with first column **b** featuring payments resulting "tomorrow" from 1 share of KGHM in six different scenarios. The remaining 3 columns (2, 3, and 4) represent payments generated respectively by a Treasury bill (T-Bill), a call option to buy 1 KGHM's share at strike price of 90 PLN, and a put option to sell 1 KGHM's share at strike price of 115 PLN. The readers interested in prices of KGHM's shares are referred to the website http://kghm.com/pl/inwestorzy/akcje-kghm/wykres-kursu-akcji and www.pl.investing.com/equities/kghm-polska-miedz-sa-historical-data.

In this article we will propose a different model of a fraction of Polish financial market by means of matrix (4) with 4 rows and 5 columns. This is an example of a complete market because each desirable financial instrument (column vector with 4 coordinates) is a linear combination of liquid instruments (columns of matrix (4) whose rank is maximal). We recall from Linear Algebra that the rank of an arbitrary matrix is the dimension of the vector space spanned by its columns.

Each financial market (or its fraction) in our methodology can be viewed as a matrix A with *n* liquid securities (*n* columns), whose payouts "tomorrow" in *m* different states of the financial market (scenarios) are given in *m* rows of matrix

A $= \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}$. In other words, the amount of A_{jk} PLN is to be paid „tomorrow" in scenario "j" by financial instrument "k". "Tomorrow" will always mean in this article 6 months from "today", whatever "today" stands for.

2.1 An Arbitrage of Type A

Let $S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$ be a price vector of (liquid) basis financial instruments (listed as columns of matrix A representing a given financial market). Besides, let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ be a portfolio consisting of x_1 pieces of first basis financial instrument (represented by column 1), x_2 pieces of second basis financial instrument (represented by column 2) and so on. Therefore, according to the method of *relative pricing*, see Cerny (p. 40), the amount $\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & \cdots & s_n \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \sum_{i=1}^{n} s_i x_i = \langle S^T, x \rangle$, represents the price we pay for portfolio **x** on this market.

Definition 1a. We say we performed an arbitrage of type A (Cerny, pp. 38-40) when we bought a portfolio $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ of basis financial instruments for which: (a) $\langle S^T, x \rangle \le 0$; (b) Ax $\geq 0 \in R^m$; (c) Ax $\neq 0 \in R^m$.

Commentary 1. The inequality (a) denotes that a holder of an arbitrage portfolio $\mathbf{x} \in R^n$ will pay nothing to purchase **x** (such case holds when $\langle S^T, x \rangle$=0), or will be even paid a positive amount of -$\langle S^T, x \rangle$ (such case holds when $\langle S^T, x \rangle$ < 0). The inequality (b) tells us that an arbitrage of type A generates no negative cash flow at any state of the financial market, while condition (c) adds that a positive cash flow must appear in at least one state of the market. Let's illustrate this definition with very simple

Example 1. Let a financial market be represented by matrix $D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ and price vector $S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ of its 2 basis financial instruments shown as columns. The first column represents payouts resulting from a share of a certain company XYZ, while the other one features payouts generated by a T-Bill. We will demonstrate that this market admits an arbitrage of type A.

<u>Proof</u>. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ be a portfolio involving a short sale of one T-Bill and the purchase for the received money 1 share of company XYZ. The price of such portfolio is equal to $\langle S^T, x \rangle = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$ PLN, which means that condition (a)

from Definition 1 holds. The remaining 2 conditions are also satisfied because portfolio **x** will generate "tomorrow" the nonnegative payouts $Ax = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Commentary 2. The reason why financial market represented by matrix D with price vector $S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ of 2 basis financial instruments admitted an arbitrage was fact that 1 of them (a share of company XYZ) *stochastically dominated* the other basis instrument (T-Bill) in a sense that the share will generate "tomorrow" higher or the same payouts than the Treasury bill, having the same price as the T-Bill.

Definition 1b. We say we performed an arbitrage of type A in a strong sense when we bought a portfolio $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ of basis financial instruments (i) whose cost was zero or even negative, and (ii) portfolio x will generate "tomorrow" positive cash flow at each state of the financial market.

2.2 Arbitrage of Type B

Definition 2. We say we performed an arbitrage of type B (Cerny, pp. 38-40) when we bought a portfolio **x**= $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ of basis financial instruments such that (i) the purchase of **x** generated an income for us equal to -$\langle S^T, x \rangle > 0$ PLN, while "tomorrow" we will have (ii) A**x = 0**$\in R^m$; **x**$\neq$ **0**, that is, no cash flow will result from portfolio **x** at any state of the financial market.

Commentary 3. The condition A**x = 0** implies that the columns of matrix A are linearly dependent, so there must exist at least 1 (typically it is more than 1) *redundant* basis financial instrument which can be perfectly replicated (as a linear combination of the remaining columns).

Therefore, to achieve a risk-free profit, it is enough to sell this redundant financial instrument (if it is more expensive than its replica) and buy the replica. If, however, the replica is more expensive, we should sell it and buy the redundant financial instrument. In both these cases we earn money today because we sell high and buy low.

In this way we have shortly proved the following

Fact 1 . A financial market admits an arbitrage of type B if and only if there exists at least 1 mispriced basis redundant instrument; it means it is either cheaper or more expensive than its replica built with the remaining basis financial instruments.

Commentary 4. It is easy to see that an arbitrage of type B is stronger than an arbitrage of type A in the strong sense.

Indeed, having bought a portfolio **x** satisfying conditions specified in Definition 2, an investor can easily buy additionally some amount of T-Bills, using the money he or she received when they purchased portfolio x, and next add those T-Bills to portfolio **x**, creating this way a new portfolio $\dot{x}$ whose cost of purchase will still be less than 0 PLN. But this new portfolio $\dot{x}$ will generate "tomorrow" positive payouts at all states of the market; in fact, the T-Bills contained in $\dot{x}$ will alone generate positive payouts at all states of the financial market.

2.3 Two Examples of Arbitrage Portfolios

For better illustration of Commentary 3 and Fact 1, let us analyze the example below.

Example 2. Let a financial market be represented by matrix $E = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and a price vector $S = \begin{bmatrix} 1 \\ 1+p \\ p+\varepsilon \end{bmatrix}$ with $\varepsilon > 0$.

Then that market admits an arbitrage of type B.

Proof. First, let's start with the observation that portfolio $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ replicates the financial instrument $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ because $\begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. But, portfolio **x** is overpriced because its price, 1+p+ε PLN, is higher than the price of $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. In such a situation the portfolio $\mathbf{y} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \neq 0$ has negative price of $-\varepsilon$ PLN, and "tomorrow" **y** will generate no cash since A**x** $= \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, what proves that **y** is indeed an arbitrage portfolio of type B.

Example 3. A financial market represented by matrix $A = \begin{bmatrix} 10 & 100 & 0 & 4 \\ 7 & 100 & 2 & 1 \\ 5 & 100 & 4 & 0 \end{bmatrix}$ and price vector $S = \begin{bmatrix} 6.8 \\ 95 \\ 1.6 \\ 1 \end{bmatrix}$ of its liquid financial instruments (shown in matrix A as 4 columns) admits an arbitrage of type B.

Proof. It follows from the assumptions made that 1 share (column 1) costs 6.80 PLN, 1 T-Bill (column 2) costs 95 PLN, 1 put option of 1 share at strike price of 9 PLN (column 3) costs 1.60 PLN, while 1 call option of 1 share at strike price of 6 PLN (column 4) costs just 1 PLN. Let's note that the call option is a redundant financial instrument because it is a linear combination of columns 1, 2, 3. In fact,

$$\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 10 \\ 7 \\ 5 \end{bmatrix} - 0.16 \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} + 1.5 \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}. \tag{3}$$

Taking into account that the call option costs 1 PLN, while its replica just 0,80 PLN =2(6.8) – 0.16(95) + 1.5(1.6), we see that the call option is overpriced. Therefore one can make an arbitrage by selling the more expensive call for 1 PLN and buy its replica for 0,80 PLN. To end the proof formally, we define portfolio $\mathbf{x} = \begin{bmatrix} 2 \\ -0.16 \\ 1.5 \\ -1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ which satisfies the required 2 conditions from definition 2:

$$\langle S^T, x \rangle = \begin{bmatrix} 6.8 & 95 & 1.6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -0.16 \\ 1.5 \\ -1 \end{bmatrix} = -0{,}20 \text{ PLN}; \quad Ax = \begin{bmatrix} 10 & 100 & 0 & 4 \\ 7 & 100 & 2 & 1 \\ 5 & 100 & 4 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -0.16 \\ 1.5 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

3.1 Arbitrage Involving Replica of KGHM's Shares When Options' Prices Are Close to Their Theoretical Values

Let matrix

$$T = \begin{bmatrix} 60 & 10000 & 0 & 5500 & 0 \\ 85 & 10000 & 0 & 3000 & 0 \\ 110 & 10000 & 2000 & 500 & 1500 \\ 135 & 10000 & 4500 & 0 & 4000 \end{bmatrix} \tag{4}$$

represents a fraction of Polish financial market by featuring payouts resulting from 5 basis instruments 6 months later („tomorrow") in 4 various states of the market. The 1st column shows payouts resulting from 1 KGHM's share, column 2 exhibits risk-free payouts from a T-Bill, columns 3 i 5 show payouts generated by a call option of 100 KGHM's shares at strike prices of respectively 90 PLN (column 3) and 95 PLN (column 5) per each share. Finally, column 4 exhibits payouts in 4 different states of the market generated by 1 put option of 100 KGHM's shares at strike price of 115 PLN per share.

It is well known that each day market prices of all financial instruments move up and down depending on incoming news relevant to their values. For example, in the period from October 1, 2017 through November 16, 2017 (see 2 websites listed in References) KGHM's shares were traded several times at 119 PLN. Therefore, without loss of generality, we assume that buying "today" 1 KGHM's share we pay 119 PLN, while the prices of remaining 4 basis instruments are also shown in vector $\mathbf{p} = \begin{bmatrix} 119 \\ 9875 \\ 3000 \\ 1000 \\ 2500 \end{bmatrix}$. Let's note that the price of a 6-month T-Bill must pretty often fluctuate around 9875 PLN because the risk-free rate in Poland is slightly above 2.5%, so our choice of 9875 PLN was natural. As we will see in the next section, the assumed option prices of 3000 PLN, 1000 PLN and 2500 PLN (see Table 1, Table 2, Table 3) are close to their theoretical values determined by Black-Scholes formulas. The cheapest is the put option of 100 KGHM's shares at 115 PLN (it costs 1000 PLN), while the most expensive is the call option to buy 100 KGHM's shares at 90 PLN (it costs 3000 PLN).

Since the rank of matrix T must be less than 5 (because there are only 4 columns in T), at least 1 of them must be linearly dependent on the remaining ones. For example, column 1 is the following linear combination of the other 4:

$$\begin{bmatrix} 60 \\ 85 \\ 110 \\ 135 \end{bmatrix} = 0.0115 \begin{bmatrix} 10000 \\ 10000 \\ 10000 \\ 10000 \end{bmatrix} - 0.024 \begin{bmatrix} 0 \\ 0 \\ 2000 \\ 4500 \end{bmatrix} - 0.01 \begin{bmatrix} 5500 \\ 3000 \\ 500 \\ 0 \end{bmatrix} + 0.032 \begin{bmatrix} 0 \\ 0 \\ 1500 \\ 4000 \end{bmatrix} \tag{5}$$

It means that 1 KGHM's share is perfectly replicated by the below portfolio of 4 remaining basis instruments:

$$\mathbf{z}_1 = \begin{bmatrix} 0 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix} \tag{6}$$

whose price, according the method of relative pricing (Cerny, p.40), must be equal to

$$0.0115 \cdot 9875 - 0.024 \cdot 3000 - 0.01 \cdot 1000 + 0.032 \cdot 2500 = 111.56 \text{ PLN}, \tag{7}$$

which is clearly smaller than the market price of 119 PLN. Summing up, one sees that $\mathbf{z} = \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix}$ is an arbitrage portfolio of type B. Indeed, $\mathbf{z}$ satisfies the required by Definition 2 conditions: $\langle S^T, \mathbf{z} \rangle = \begin{bmatrix} 119 \\ 9875 \\ 3000 \\ 1000 \\ 2500 \end{bmatrix}^T \cdot \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix} = -7.4375$ PLN; and by virtue of relationship (5), one also has $T\mathbf{z} = \begin{bmatrix} 60 & 10000 & 0 & 5500 & 0 \\ 85 & 10000 & 0 & 3000 & 0 \\ 110 & 10000 & 2000 & 500 & 1500 \\ 135 & 10000 & 4500 & 0 & 4000 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

In this way we have proved

Fact 2. Let **T**=$\begin{bmatrix} 60 & 10000 & 0 & 5500 & 0 \\ 85 & 10000 & 0 & 3000 & 0 \\ 110 & 10000 & 2000 & 500 & 1500 \\ 135 & 10000 & 4500 & 0 & 4000 \end{bmatrix}$ be a mathematical model of a fraction of Polish financial market (from KGHM's point of view) explained in the beginning of this section. Then portfolio $\mathbf{z_1}$ given by formula (6) perfectly replicates 1 KGHM's share, what makes it a synthetic KGHM's share. Its price (111.56 PLN) is well below the market price of 1 KGHM's share (119 PLN).

Corollary 1. The purchase of 1 million portfolios $\mathbf{z_1}$ means (i) purchase of 11500 Treasury bills, (ii) selling 24000 call options of 100 KGHM's shares at strike price of 90 PLN (per share), (iii) writing (selling) 10000 put options of 100 KGHM's shares at strike price of 115 PLN per share, and (iv) purchase of 32000 calls of 100 KGHM's shares at strike price 95 PLN per share. All these financial instruments mature in 6 months from „today". The purchase of 1 million arbitrage portfolios $\mathbf{z} = \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix}$ yields a risk-free profit of 7,437,500 PLN.

3.2 Valuation of Options via Black-Scholes Formula (σ =35%; r_f = 2.5%; Dividend q=0% or q=2% or q=4%)

In the previous subsection we assumed that the prices of 5 basis instruments were given by vector $\begin{bmatrix} 119 \\ 9875 \\ 3000 \\ 1000 \\ 2500 \end{bmatrix}$. We explained that 119 PLN happens to be a market price of KGHM's shares regularly and that the price 9875 PLN of a 6-month T-Bill results from the fact the risk-free rate in Poland fluctuates slightly above 2.5%. In this subsection we want to determine theoretical prices of 3 financial options we selected as our basis instruments (columns 3, 4 and 5 in matrix T). Towards this end we have to make assumptions concerning not only the risk-free rate in Poland, but also volatility of KGHM's share prices (we suppose here that volatility σ =35%), and make also assumptions concerning dividends (q) paid by KGHM as well. According to the Black–Scholes formula, the price of a call option is given by

$$\mathrm{c} = S\exp(-qT)N(d_1) - X\exp(-rT)N(d_2) \tag{8}$$

where T is the time to maturity of the option (in this article we suppose that T = $\frac{1}{2}$) while N(d) stands for the cumulative probability distribution function for the standard normal distribution N(0,1). Furthermore,

$$\mathrm{d_1} = [\ln(\mathrm{S/X}) + (\mathrm{r} - \mathrm{q} + 0{,}5\sigma^2]/\sigma\sqrt{\mathrm{T}}\ ; \quad \mathrm{d_2} = [\ln(\mathrm{S/X}) + (\mathrm{r} - \mathrm{q} - 0{,}5\sigma^2]/\sigma\sqrt{\mathrm{T}}\ . \tag{9}$$

First, let's see how different values of parameter q affect pricing of a call option of 100 KGHM's shares with strike price 90 PLN per share. In the previous subsection the price c_{90} of that call option was supposed to be equal 3000 PLN. We will soon see how much price 3000 PLN is different from its theoretical value. When q = 0% then $\mathrm{d_1}$=1.3028, $\mathrm{d_2}$ = 1.0554 and consequently $\mathrm{N(d_1)}$ = 0.9037, $\mathrm{N(d_2)}$ = 0.8544 so that c_{90} = 3160 PLN.

When dividend yield q is higher, for example q = 2%, then $\mathrm{d_1}$=1.2624, $\mathrm{d_2}$ = 1.0150 and consequently $\mathrm{N(d_1)}$ = 0.8966, $\mathrm{N(d_2)}$ = 0.8449 so that c_{90} = 3053 PLN. Finally, when q = 4% then $\mathrm{d_1}$ = 1.2220, $\mathrm{d_2}$ = 0.9746 and consequently $\mathrm{N(d_1)}$ = 0.8892, $\mathrm{N(d_2)}$ = 0. so that the call option is even more cheaper than 3000 PLN, namely it costs c_{90} = 2949 PLN. We have just proved the following

Fact 3. *The change of parameter q from 0% to 2% and next to 4% implies the corresponding change (decline) of c_{90}:*

Table 1. Theoretical prices of call option with strike 90 PLN in 3 different dividend scenarios

q = 0%	q = 2%	q = 4%
c_{90} = 3160 PLN	c_{90} = 3053 PLN	c_{90} = 2949 PLN

Now, let's price the call option with strike price 95 PLN whose market value c_{95} in the previous subsection was equal to 2500 PLN. When q=0% then $\mathrm{d_1}$=1.0844, $\mathrm{d_2}$ = 0.8369 and consequently $\mathrm{N(d_1)}$ = 0.8609, $\mathrm{N(d_2)}$ = 0.7987 so that c_{95} = 2752 PLN. When dividend is higher, for example, q = 2%, then $\mathrm{d_1}$=1.0440, $\mathrm{d_2}$ = 0.7965 and consequently $\mathrm{N(d_1)}$ = 0.8518, $\mathrm{N(d_2)}$ = 0.7871 so that c_{95} = 2650 PLN. Finally, when q = 4% then $\mathrm{d_1}$ = 1.0036, $\mathrm{d_2}$ = 0.7561 and

$N(d_1)$ = 0.8422, $N(d_2)$ = 0.7752 so that the theoretical value of that call option equals c_{95} = 2551 PLN. In his way we have demonstrated the validity of

Fact 4

The change of parameter q from 0% to 2% and next to 4% implies the following decrease of c_{95}:

Table 2. Theoretical prices of call option with strike 95 PLN in 3 different dividend scenarios

q = 0%	q = 2%	q = 4%
c_{95} = 2752 PLN	c_{95} = 2650 PLN	c_{95} = 2551 PLN

Now, it remains to verify how different dividend yields influence valuation of put option of 100 KGHM's shares at strike price 115 PLN for share, according to the Black–Scholes formula

$$p = S\exp(-qT)N(-d_1) + X\exp(-rT)N(-d_2) \tag{10}$$

When dividend q = 0% then d_1 = 0.3124, d_2 = 0.0649 and consequently $N(d_1)$ = 0.6226, $N(d_2)$ = 0,5259 so that p_{115} = 894 PLN. When dividend raises to q = 2%, then d_1 = 0.2720, d_2 = 0.0245, $N(d_1)$ = 0.6072, and $N(d_2)$ = 0.5098 so that p_{115} = 940 PLN. Now we see that put options cost more when dividend yield is higher. It is understandable, taking into account that they give the right to sell less valuable shares (due to a higher dividend paid in the meantime) for the same price of 115 PLN (in the studied case). Finally, when q = 4% then d_1 = 0.2316, d_2 = -0.0159, what implies that $N(d_1)$ = 0.5916, $N(d_2)$ = 0.4937 and p_{115} = 987 PLN. In this way we have proved

Fact 5

A change of dividend q from 0% to 2%, and next to 4% entails the following changes of theoretical value p_{115} of a put option of 100 KGHM's shares at strike price of 115 PLN for each share:

Table 3. Theoretical prices of put option with strike 115 PLN in 3 different dividend scenarios

q = 0%	q = 2%	q = 4%
p_{115} = 894 PLN	p_{115} = 940 PLN	p_{115} = 987 PLN

3.3 Valuation of Portfolio Replicating KGHM's Shares

We assume as previously that σ =35%; r_f =2.5%; q=0% or q=2% or q=4%. Now, let's valuate portfolio $z = \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix}$

for „today", supposing that the market prices of the 3 financial options we analyze in this article coincide with their theoretical values calculated in Section 3.2. (Tables 1-3). We already know that, depending on dividend yield q, the price vector of 5 basis instruments will be equal to

$$S=\begin{bmatrix} 119 \\ 9875 \\ 3160 \\ 894 \\ 2752 \end{bmatrix} \text{ when q =0%; } S=\begin{bmatrix} 119 \\ 9875 \\ 3053 \\ 940 \\ 2650 \end{bmatrix} \text{ when q = 2%, and } S=\begin{bmatrix} 119 \\ 9875 \\ 2949 \\ 987 \\ 2551 \end{bmatrix} \text{ when q = 4%.} \tag{11}$$

In such situation, the theoretical price of portfolio **z** depends on dividend q, too. When q=0%, portfolio **z** costs „today"

$\begin{bmatrix} 119 \\ 9875 \\ 3160 \\ 894 \\ 2752 \end{bmatrix}^T \begin{bmatrix} -1 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix}$ = - 2.1535 PLN, what translates into a risk-free profit of – 2,153,500 PLN for a holder of 1 million

portfolios **z**. In other words, the replica of 1 million portfolios $\mathbf{z}_1 = \begin{bmatrix} 0 \\ 0.0115 \\ -0.024 \\ -0.010 \\ 0.032 \end{bmatrix}$ costs 116,846.500 PLN. When dividend q paid by KGHM equals 2%, then a single portfolio **z** costs $\begin{bmatrix}119\\9875\\3053\\940\\2650\end{bmatrix}^T\begin{bmatrix}-1\\0.0115\\-0.024\\-0.010\\0.032\end{bmatrix}$ = - 3.3095 PLN, which implies that a risk-free profit for a holder of 1 million portfolios $\mathbf{z}_1$ equals 3,309,500 PLN. Finally, when q=4%, then **z** costs „today" $\begin{bmatrix}119\\9875\\2949\\987\\2551\end{bmatrix}^T\begin{bmatrix}-1\\0.0115\\-0.024\\-0.010\\0.032\end{bmatrix}$ = - 4.4515 PLN, which means that the perfect replica of 1 million KGHM's shares costs just 114,548,500 PLN. In this way we have arrived at

Theorem 1. Let matrix **T**= $\begin{bmatrix}60 & 10000 & 0 & 5500 & 0\\85 & 10000 & 0 & 3000 & 0\\110 & 10000 & 2000 & 500 & 1500\\135 & 10000 & 4500 & 0 & 4000\end{bmatrix}$ be a model of a fraction of the Polish financial market. Then portfolio **z** = $\begin{bmatrix}-1\\0.0115\\-0.024\\-0.010\\0.032\end{bmatrix}$ created „today" (any day when 1 KGHM's share costs 119 PLN) conducts an arbitrage of type B since „tomorrow" (6 months later) (i) the payouts resulting from **z** will be equal to zero PLN in all 4 states of the Polish financial market, and (ii) the purchase of **z** "today" generates an inflow of cash depending on a dividend yield q paid to shareholders by KGHM. The purchase of 1 million of arbitrage portfolios **z** generates an inflow of 2,153,500 PLN when dividends are not paid. When q = 2% then the risk-free profit increases to 3,309,500 PLN. Finally, when q = 4%, then purchase of 1 million portfolios **z** generates a risk-free profit of 4,451,500 PLN.

4. Concluding Remarks

First of all, let us note that the investment horizon length (6 months in this paper) can be arbitrary, depending only on termination dates of call and put options available on a given OTC market and stock market. As we have already mentioned in Introduction, the idea behind the presented above methodology is to first consider hundreds of scenarios concerning the price of KGHM's shares, together with the corresponding prices of specified in this article call an put options, in order to calculate the resulting profit or loss. In this way we will be ready to immediately spot an arbitrage opportunity, when it occurs, independently what factors influence the level of current share prices and call and put option prices.

A good idea, implemented in this article, is to work with such a matrix model of a fraction of given financial market that this model represents a complete market. According to the definition of a complete market, each desired financial instrument has then its perfect replica, which is not the case on incomplete markets; see for example, Zaremba (2018, pp.16-17), Zaremba (2017c, p.101).

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