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Improve Student Understanding of Amortized Loans

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Abstract

Amortized loans, such as mortgages and auto loans, are common real-world examples introduced in the first finance college course. Many students mistakenly believe that the monthly payments on an amortized loan should be *larger* than the monthly equivalent of the annual payments (the annual payment divided by 12) of the same loan. This paper mathematically demonstrates that the monthly payments must, in fact, be *smaller* than this monthly equivalent. Also, the paper shows a loanee does pay more interest with monthly payments. Furthermore, this paper presents an alternative approach to treat amortized loan payments as investment or an opportunity cost, thus improving student understanding of amortized loans and financial decision-making.

Keywords: Amortized Loans, Annual Payments, Monthly Equivalent of Annual Payments, Monthly Payments

1. Introduction

The time value of money concepts are essential to college business education because students can gain invaluable finance knowledge and problem-solving skills applicable to both professional and personal financial decision-making. Among the earliest real-world applications discussed in introductory finance courses are probably amortized loans which provide realistic and relatable examples of these principles (see, e.g., Berk, DeMarzo, & Harford, 2022; Brigham and Houston, 2022, 2022; Ross, Westerfield, & Jordan, 2023). Students usually find these examples particularly practical and relevant, given the prevalence of amortized loans, such as mortgages and auto loans, in the U.S. These real-world applications not only encourage engagement but also strengthen students' understanding of core financial principles.

Many students mistakenly assume that the monthly payments on an amortized loan should be *larger* than the monthly equivalent of the annual payments of the same loan because of monthly compounding.[i] This paper refutes this misconception, demonstrating mathematically that the monthly payments must be *smaller* than this monthly equivalent. Also, the paper shows that although the total interest paid with monthly payments appears to be *less* than that with annual payments, a loanee does pay *more* interest with monthly payments. To further enhance student comprehension, the paper introduces an alternative investment-based approach to interpret amortized loan payments, offering a fresh lens through which to evaluate borrowing decisions and opportunity costs.

The remainder of the paper is as follows. First, we will outline how the payments, annual or monthly, on an amortized loan are determined. Next, we will prove mathematically that the monthly payments must be smaller than the monthly equivalent of the annual payments of the same loan and that amortized loan with monthly payments is more costly than with annual payments. Then, we will introduce an alternative investment-based perspective on amortized loans. Specifically, if a loanee were to reinvest the payments at the loan's interest rate, the future value of these payments could be treated as an opportunity cost of obtaining the loan. Numerical examples will be integrated throughout the paper to facilitate a clearer understanding of these concepts. Also, we will discuss how educators can incorporate these insights into their curriculum designs. The last section concludes this paper.

2. Amortized loans

An amortized loan is paid off over time with fixed payments. These periodic payments are applied to both the loan's principal amount and interest accrued until the loan is paid in full. Mortgages and auto loans in the U.S. are typically amortized loans. And part of each payment goes toward the loan principal, and part goes toward interest. Since the scheduled, fixed payments on an amortized loan are simply an *annuity*, the present value of this annuity is set up to equal to the amount borrowed.

2.1 Annual Payments on Amortized Loan

To determine the *annual* payments on an amortized loan, suppose

L = the loan's principal or original amount borrowed = the present value of the annual payments;

i = the loan's nominal interest rate or annual percentage rate;

N = number of years which is also the number of annual payments or the loan term; and

PMT_A = the annual payment.

Therefore,

$$L = PMT_A \left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right] \tag{1}$$

Hence, the annual payments are:

$$PMT_A = L \frac{1}{\frac{1}{i} - \frac{1}{i(1+i)^N}} \tag{2}$$

2.2 Monthly Payments on Amortized Loan

If the loan requires *monthly* instead of annual payments, and let's say PMT_M = the monthly payment while L, i, and N are the same as before.

The monthly payments are:

$$PMT_M = L \frac{1}{\frac{1}{\frac{i}{12}} - \frac{1}{\frac{i}{12}\left(1+\frac{i}{12}\right)^{12N}}} = L \frac{1}{\frac{12}{i} - \frac{12}{i\left(1+\frac{i}{12}\right)^{12N}}} \tag{3}$$

3. Relationship Between Annual and Monthly Payments

3.1 Effective Annual Rate

To fully understand the differences between annual and monthly payments, students must grasp the concept of the effective annual rate (EAR). The nominal interest rate (i) is an annual rate and sometimes referred to as the quoted rate or annual percentage rate which assume payments are made *annually*. However, if the payments are made *monthly*, the actual interest rate a loanee paying is in reality *higher* than i because of monthly compounding. This actual interest rate is called the effective annual rate. If m is the compounding frequency of i or how many payments are made per year, then

$$EAR = (1 + \frac{i}{m})^m - 1 \tag{4}$$

For monthly payments,

$$EAR_M = (1 + \frac{i}{12})^{12} - 1 \tag{5}$$

Note that EAR is *larger* than i when m is more than one time per year and the more frequent i compounds, the higher the EAR.[ii]

3.2 Annual versus Monthly Payment

Many students believe that since monthly payments involve monthly compounding; hence, the amount must be *larger* than the monthly equivalent of the annual payment, or

$$PMT_M > \frac{PMT_A}{12}$$

The following numerical example demonstrates this misunderstanding. Suppose

L = \$100,000;

i = 6%; and

N = 10 years.

Hence, the annual payments and monthly payments can be determined by using Eq. (2) and Eq. (3), respectively:

$$\mathrm{PMT_A} = \$100{,}000\ \frac{1}{\frac{1}{0.06} - \frac{1}{0.06(1+0.06)^{10}}} = \$13{,}586.796$$

$$\mathrm{PMT_M} = \$100{,}000\ \frac{1}{\frac{12}{0.06} - \frac{12}{0.06\left(1+\frac{0.06}{12}\right)^{120}}} = \$1{,}110.205$$

We can also easily compute $\mathrm{PMT_A}$ and $\mathrm{PMT_M}$ using a financial calculator such as the *Texas Instruments BAII Plus*[iii].

Annual payments ($\mathrm{PMT_A}$):

TVM Keys:	N	I/Y	PV	**PMT**	FV
Enter	10	6	-100,000		0
Compute (CPT)				**13,586.796**	

Monthly payments ($\mathrm{PMT_M}$):

TVM Keys:	N	I/Y	PV	**PMT**	FV
Enter	120	0.5	-100,000		0
Compute (CPT)				**1,110.205**	

Note that we enter 120 as N because there are 120 monthly payments and 0.5 as I/Y as the monthly interest rate is 6%/12 or 0.5%.

Therefore, the monthly equivalent of $\mathrm{PMT_A}$ = \$13,586.796/12 = \$1,132.233. So, to many students' surprises, the actual monthly payment is *smaller* than this monthly equivalent:

$$\mathrm{PMT_M} = \$1{,}110.205 < \frac{\mathrm{PMT_A}}{12} = \$1{,}132.233$$

To add to the confusion, some online resources (see Amortization Calculator provided by bankrate.com[iv]) and textbook examples (Ross et al., 2023) discuss two additional features of amortized loans: the total payment (the *sum* of all payments made over the life of the loan) and the total amount of interest paid (the total payment *minus* the original loan principal). So, for our loan example (Section 3.2) the total payments and interest paid for the annual payments and monthly payments are as follows:

Payment Type	Total Payment	Total Interest
Annual	10(\$13,586.796) = \$135,867.96	\$135,867.96 − \$100,000 = \$35,867.96
Monthly	120(\$1,110.205) = \$133,224.60	\$133,224.60 − \$100,000 = \$33,224.60

This type of popular calculation shows a loanee pays *less* total payment and, consequently, *less* total interest with monthly payments appears *contradictory* to the fact that the EAR of the loan is higher than its nominal rate (6%). $\mathrm{EAR_M}$ is $(1 + 0.06/12)^{12} - 1 = 6.1678\%$.

3.3 Monthly Payment Smaller Than Monthly Equivalent of Annual Payment

Helping students understand this seemingly inconsistency will further advance their knowledge of the time value of money concept and sharpen their problem-solving skills. To achieve this, we will first show mathematically that the monthly payment is indeed *smaller* than the monthly equivalent of the annual payment. Next, we will discuss the issues involved in the calculating the total payment and total interest paid on an amortized loan.

From Eq. (2) above, the monthly equivalent of the annual payments is

$$\frac{PMT_A}{12} = \frac{1}{\left[\frac{12}{i} - \frac{12}{i(1+i)^N}\right]}$$

From Eq. (3), we know the monthly payment is

$$PMT_M = L\,\frac{1}{\frac{12}{i} - \frac{12}{i\left(1+\frac{i}{12}\right)^{12N}}}$$

Hence, if

$$\left(1+\frac{i}{12}\right)^{12N} > \ (1+i)^N\text{, then } PMT_M < \frac{PMT_A}{12}$$

Recall from Eq. (5) that for monthly payments,

$$EAR_M = (1+\frac{i}{12})^{12} - 1$$

Hence,

$$(1+\frac{i}{12})^{12} = \ 1 + EAR_M$$

Therefore,

$$(1+\frac{i}{12})^{12N} = (1+EAR_M)^N > \ (1+i)^N \qquad (6)$$

It is because $EAR_M > i$. Hence, mathematically, PMT_M must be *smaller* than $PMT_A/12$.

The reason is the timing difference and thus the compounding effect difference between monthly payments and annual payments. The compounding effect of more frequent payments is obviously more pronounced than that of less frequent payments. After all, time is money and therefore earlier cash flows imply larger present values or future values. That is why the EAR of monthly payments is higher than the nominal rate.

Imagine the monthly payment is *exactly* equal to (not even larger than) the monthly equivalent of annual payment. In that case, the present value of these monthly payments will be *larger* than the loan's principal. To demonstrate using the previous example, suppose the actual monthly payment PMT_M is *really* equal to $PMT_A/12$ = \$13,586.796/12 = \$1,132.233. The implied amount borrowed that requires these monthly payments is simply the present value of 120 of \$1,132.233 which equals \$101,984.13[v], and that is \$1,984.13 more than the actual loan of \$100,000.

However, the calculation of the total payment and therefore total interest paid on amortized loan is simply the sum of all payments minus the loan's principal. Clearly, this calculation does not account for any compounding effect. This, along with the fact that monthly payments are smaller than the monthly equivalent of annual payments, often leads to students' illusion or misunderstanding that an amortized loan with monthly payments is cheaper or incurs less interest than one with annual payments.

3.4 Other Periodic Payments

It should be noted that this relationship also applies to other periodic payments such as quarterly or semiannual payments. For example, if the above amortized loan calls for quarterly payments, the payment (PMT_Q) can be easily determined using the *Texas Instruments BAII Plus*:

TVM Keys:	N	I/Y	PV	**PMT**	FV
Enter	40	1.5	-100,000		0
Compute (CPT)				**3,342.71**	

Note that we enter 40 as N because there are 40 quarterly payments and 1.5 as I/Y as the quarterly interest rate is 6%/4 or 1.5%.

This quarterly payment is *smaller* than the quarterly equivalent of the annual payments which is $13,586.796/4 or $3,396.70. Furthermore, the EAR_Q for quarterly payments is $(1 + 0.06/4)^4 - 1 = 6.14\%$ which is *larger* than the annual interest rate of 6%. Finally, the total interest paid is 40($3,342.71) – $100,000 = $33,708.41 which is *less* than that for annual payments.

4. Alternative Perspective on Amortized Loans

There is an alternative approach to demonstrate that monthly payments result in *more* interest paid (more expensive) than annual payments. From the lender's perspective, the loan's principal (L or $100,000) can be viewed as an investment purchased by an investor. Suppose this investment offers two different payout structures. The first payout is 10 equal annual payments with an annual interest rate of 6%. The second is 120 equal monthly payments (10 years of monthly payments), also with an annual interest rate of 6% but compounds monthly.

In either case, L must equal to the present value of the corresponding annuity─either 10 annual payments or 120 monthly payments (see Equations 2 and 3). We know that the annual payment (PMT_A) is $13,586.796, while the monthly payment (PMT_M) is $1,110.205. So, which payout, annual or monthly payments, should the investor select? If the investor thinks the first payout is better simply because the monthly equivalent of the annual payment ($13,586.796/12 or $1,132.233) is *larger* than the monthly payment, that will be an incorrect choice. It is because it fails to account for the compounding effect of the monthly payments, which can be reinvested at the monthly interest rate, resulting in a greater return over time.

One way to demonstrate that the second payout is a better choice is by comparing the future values of the annual and monthly payments. Students should recognize that these future values can be calculated as the future values of the annuities (10 equal annual payments or 120 equal monthly payments). The future value (FV_A) of the 10 annual payments is:

$$FV_A = PMT_A\left[\frac{(1+i)^N - 1}{I}\right] = \$13{,}856.796\left[\frac{(1+0.06)^{10} - 1}{0.06}\right] = \$179{,}084.77$$

Similarly, the future value (FV_M) of the 120 monthly payments is:

$$FV_M = PMT_M\left[\frac{\left(1+\frac{i}{12}\right)^{12N} - 1}{\frac{i}{12}}\right] = \$1{,}110.205\left[\frac{\left(1+\frac{0.06}{12}\right)^{120} - 1}{\frac{0.06}{12}}\right] = \$181{,}939.67$$

These future values can also be easily determined using the *Texas Instruments BAII Plus*. For the first payout, the future value of the 10 annual payments is:

TVM Keys:	N	I/Y	PV	PMT	**FV**
Enter	10	6	0	-13,586.796	
Compute (CPT)					**179,084.77**

The future value of the 120 monthly payments (second payout) is:

TVM Keys:	N	I/Y	PV	PMT	**FV**
Enter	120	0.5	0	-1,110.205	
Compute (CPT)					**181,939.67**

As expected, the monthly payments generate a *larger* future value.

Another way to verify that the second payout is a better choice is by calculating the future value of the investment (L). For annual payments, L will be compound *annually* for N years, so its future value (FV_A) is

$$FV_A = L(1+i)^N$$

If the interest rate compounds instead *monthly*, the future value (FV_M) is

$$FV_M = L(1+\frac{i}{12})^{12N}$$

But from Eq. (6) we know that

$$(1+\frac{i}{12})^{12N} > (1+i)^{N}$$

Thus, $FV_M > FV_A$. Numerically,

$$FV_A = L(1+i)^N = \$100{,}000(1.06)^{10} = \$179{,}084.77$$

$$FV_M = L(1+\frac{i}{12})^{12N} = \$100{,}000(1+\frac{0.06}{12})^{120} = \$181{,}939.67$$

Again, the second payout is a better choice and the investor should not be deceived by comparing the monthly equivalent of annual payment to the actual monthly payments.

This alternative approach provides students a broader perspective on evaluating the amortized loan payments. If a loanee were investing instead of paying either the annual payments or monthly payments, the monthly payments will generate a larger future value. Put differently, when these payments are reinvested at the nominal interest rate (or monthly rate for monthly payments), the future value of the monthly payments is *larger* than that of the annual payments. In other words, these future values can be interpreted as the opportunity cost of the annual or monthly payments a loanee could have earned. And the more frequent the payments, the larger the opportunity cost, hence the higher the loan's interest cost.

5. Discussion

The critical difference between amortized loans with annual payments and monthly payments is the compounding effect which is one of the most fundamental aspects in finance. Students in the introductory finance class might be inclined to simplify calculations by dividing the annual payment by 12 to approximate the month payment. They then will be confused after realizing that the actual monthly payment is *smaller* rather than *larger* than this approximation (or monthly equivalent).

Inquisitive students may turn to the internet for answers. For instance, the Microsoft Copilot will state that they cannot accurately approximate the monthly payment by simply dividing the annual payment by 12, but often without offering mathematical justification. Thus, it is paramount that students understand *why* the actual monthly payments must be *smaller* than this simplified monthly equivalent of the annual payment─a concept seldom addressed in conventional textbooks. This paper offers educators a practical pedagogical tool to easily facilitate student comprehension of this concept.

For instance, an instructor may incorporate this topic as a learning outcome under the concept of the time value of money. After guiding students through the shortcut of dividing the annual payment on an amortized loan by 12 to approximate the monthly payments, he or she can then elaborate why the actual monthly payments on an amortized loan must be *smaller* than this monthly equivalent. Alternatively, the instructor may ask students to determine the future value of both annual payments and monthly payments. Since the future value of the monthly payments is larger, the instructor can then clarify that this implies more interest paid, making the monthly payments more expensive.

Furthermore, the alternative concept of considering an amortized loan as an investment purchased by an investor and the determining of the future value of such investment can be extended to other finance topics such as the immunization process. This topic is commonly addressed in investment textbooks (see Bodie, Kane, & Marcus, 2022). Immunization involves protecting (or immunizing) the future value of, for example, a pension fund or bond portfolio from fluctuations in the prevailing interest rates. In introductory finance courses, bond pricing is typically taught through present value calculations of a bond's future cash flows, including coupon payments and par value (see, e.g., Berk, DeMarzo, & Harford, 2022; Brigham and Houston, 2022, 2022; Ross, Westerfield, & Jordan, 2023). However, the bond immunization process requires student understanding of the *future* value of a bond portfolio. Hence, this alternative perspective on an amortized loan further develops students understanding of other important finance topics.

6. Concluding Remark

Amortized loans are typically real-world examples discussed in textbooks for the first finance college course. Students will find these illustrations particularly beneficial for understanding the time value of money concept, as mortgages and auto loans in the U.S. are generally structured as amortized loans. There is a confusion or misunderstanding among students that the monthly payments on an amortized loan should be *larger* than the monthly equivalent of the annual payments of the same loan because of monthly compounding. However, this paper demonstrates that, mathematically, the monthly payments must be *smaller* than this monthly equivalent precisely because of the compounding effect.

Also, the paper shows that the total interest paid with monthly payments appears to be less than that with annual payments. However, this total interest paid calculation ignores the monthly compounding effect. Finally, the paper offers an alternative approach to view amortized loan payments. If a loanee were to reinvest the payments at the loan's interest rate, the future value of these payments could be interpreted as an opportunity cost of obtaining the loan. This enhanced understanding of amortized loans will improve students' knowledge and strengthen their problem-solving skills regarding the time value of money concepts and applications.

One potential direction for future pedagogical research is to explore the remaining (or future) balance of an amortized loan, which is generally taught as the present value of the remaining payments. Specifically, one could also demonstrate that the remaining loan balance may also be determined using future value calculations.

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Notes

[i] The annual payments refer to the fixed payments made every year on an amortized loan over the life of the loan, with each payment consists of principal and interest. Instead of annual payments, a loanee will make fixed monthly payments when the loan calls monthly payments. Throughout this paper, the monthly equivalent of the annual payment is defined as the annual payment divided by 12.

[ii] To prove that EAR or $(1 + i/m)^m – 1 > i$, we can rewrite the inequality as $(1 + i/m)^m > 1 + i$. The function $(1 + i/m)^m$ always increases with m. In the extreme case of continuous compounding or m approaches infinity, $(1 + i/m)^m$ approaches e^i, which is always greater than 1 + i (*e* is the Euler's constant approximately equals to 2.71828). Hence, EAR > i. See also Vicknair and Wright (2015) for more discussion of effective annual rate and annual percentage rate.

[iii] The Time Value of Money (TVM) keys are: N = term of the loan (or the number of total payments), I/Y = interest rate per period, PV = initial loan amount, PMT = periodic payment, FV = future value which is zero when computing PMT. Visit https://education.ti.com/en/products/calculators/financial/baii-plus for more information.

[iv] Visit https://www.bankrate.com/mortgages/amortization-calculator/ for more information.

[v] We can easily compute the present value of the payments using the *Texas Instruments BAII Plus*:

TVM Keys:	N	I/Y	**PV**	PMT	FV
Enter	120	0.5		-1,132.233	0
Compute (CPT)			**101,984.13**		