Real Estate Price Cycles and Macroeconomic Variables: A Spectral Analysis with Wavelets for Brazil

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Received: April 11, 2024  Revised: May 28, 2024  Accepted: June 5, 2024
Available online: June 20, 2024  URL: https://doi.org/10.11114/aef.v11i3.6987

Abstract

The aim of this paper is to investigate the relationship between real estate price cycles and two macroeconomic variables, the Gross Domestic Product (GDP) and Interest Rate (Selic) of the Brazilian economy. To achieve this objective, we adopt a data-driven approach by applying the spectral analysis methodology. We adopted wavelet techniques to identify the frequencies of cycles and the coherences (lead-lag situation) between the housing price series and the macroeconomic variables. Our results suggest a situation of a strong spectral relationship between the cycles of housing prices and the investigated macroeconomic variables. In addition to the strong coherence, the cycles of housing prices presented themselves in the leading situation with respect to GDP and a mixed lead-lag behavior for the case of Selic.

Keywords: Real Estate Cycles. Macroeconomics. Spectral Analysis. Wavelet. Coherence

1. Introduction

The end of 2019 went down in history as the beginning of the largest and most recent global health crisis, caused by the new coronavirus Sars-Cov-2 (Wang, Horby, Hayden, & Gao, 2020). All countries were faced with a pandemic, with devastating implications on the way society is organized, hundreds of thousands of deaths and a major economic crisis for the year of 2020. Contradictions and social conflicts were brought to the fore, and, from the economic point of view, the debate about the importance and role of the State as a promoter of economic policies was resumed (Carvalho, 2020).

The return to the debate on the role of the State was, to a certain extent, due to the need to impose physical distancing to reduce the proliferation of the virus and, therefore, mitigate the consequences arising from a pandemic, that is, save lives. In this context, it was necessary to impose isolation, sanitary barriers between countries, quarantines for people who were diagnosed with the COVID19 disease, among many other important and necessary measures to contain the virus were adopted in many countries.

From an economic perspective, these measures inevitably generated an interruption in the production and circulation of products and services, having as a direct consequence, a global economic crisis in 2020. This new crisis, unlike most previous economic crises, where the origin was resulted from problems arising from a shock on the demand side, such as, for example, the sub-prime financial crisis of 2008. The COVID19 pandemic had the economic impact of a negative supply shock, as many countries and companies had to temporarily close or reduce the commercial flow of suppliers, workers and customers (Prabheesh & Padhan, 2021).

In relation to Brazil, in particular, and despite the neoliberal discourse present in the federal government with the election, in 2018, of President Jair Bolsonaro, the role of the State was also placed at the forefront to combat the economic effects of the pandemic. From an institutional point of view, a Constitutional Amendment nº 106 was enacted on the 8th of May of 2020 (Brazil, 1988), which defined an extraordinary fiscal and financial regime to mitigate the problems arising from the pandemic, which became known as the “war budget”.

As for the financial system, given the role of the Central Bank of Brazil (BCB) of maintaining the liquidity and stability of the entire system, several measures were instituted, especially those that sought to provide greater liquidity to banks and financial institutions, such as: a) Working Capital for Business Preservation (CGPE) program, intended for micro, small and medium-sized companies; b) buy private bonds in the secondary market: which authorized the BCB to buy
private bonds in the secondary market in order to maintain the system's liquidity; c) Change in compliance with the requirement for savings deposits; d) property as assurance for more than one loan. Among many other measures adopted, the objective of which was to maintain the level of liquidity or even increase the volume of available resources (BCB, 2020).

In relation to the debate on real estate price cycles, a particularly important measure was the strong reduction in the Brazilian economy's basic interest rate (Selic) to 2%, its historic minimum in October 2020. This movement also reflected in the reduction of average interest rate for real estate financing, presenting, in December 2020, according to data from the Central Bank (BCB, 2021), rates of 6.6% and 7.04%, for individuals and legal entities, respectively, values in levels of their historical minimum.

This movement to reduce interest rates for real estate credit applicants was an essential action, among the monetary variables, for the formation of a new cycle of growth in real estate prices. The other engine to boost a new cycle of growth in real estate supply and demand would be the expansion of gross domestic product (GDP) (Pessoa, Rivero, & Cerejeira, 2021). However, with the new coronavirus crisis in 2020, product growth did not occur, even showing a strong retraction of 4.1%, according to data from the Brazilian Institute of Geography and Statistics (IBGE, 2021).

Given this context, a fact that should be drawn attention to is that, although GDP did not contribute to the formation of a new price cycle, as there was a strong downfall, there was, however, a considerable reduction in the price interest rate generating, what we can characterize as a mini real estate boom. Because, as this market is a segment of the economy that is heavily financially leveraged, that is, it requires long-term financing schemes to acquire the possession, the impact of the interest rate promoted, according to data from the Brazilian Association of Entities of Real Estate Credit and Savings (ABECIP, 2021), a sharp increase in demand for real estate financing of 76% in the first quarter of 2021 and presented an expansion of 112.8%, when compared to the same period in 2020.

As we can see, even in the context of the crisis, there was pressure on housing prices and a possible expansion cycle. Thus, from the point of view of the formation of the price cycle for the housing market, and given its weight in the economy, would this sector be able to lead GDP growth, that is, would it be capable to “drive” the growth of the economy, or would it be necessary for there to first be a solid expansion of the domestic product as a whole, so that the real estate market can then follow? Is this same effect seen in the case of the interest rate on the housing market? In other words, are real estate market cycles consistently aligned with macroeconomic variables or is there a mismatch? And if there is a different frequency between the cycles, which cycle leads the movement?

These questions are difficult to answer using traditional econometric tools, because, usually, they exclusively observe the dominance of time through time series. Therefore, to answer these questions we must go beyond the analysis of time and combine these analyses with frequency dominance (cycles). In this way, the study of the frequency of real estate cycles and macroeconomic variables, namely GDP and interest rate (Selic) can provide information on how these cycles behave and integrate over time, whether in the short or long term, contributing for the economic policy makers to plan necessary measures for countercyclical movements in the economy.

In addition to this introductory section, this article is divided as follows. The next section, 2, presents a discussion of the literature related to price cycles and macroeconomic variables. In the following section, 3, we will present the wavelet methodology that was utilized in our investigation under the frequency domain, apart from presenting the database used. In section 4 we will present the main results of the applied methodology. Section 5 presents a discussion of our results in the light of the literature. Finally, our conclusions will be presented, in section 6.

2. Related Literature

For the real estate cycle, we follow the definition of Natsvaladze and Beraia (2017), which conceptualizes a cyclical period as the movement of change in “market volume”, creating an alteration in prices over a certain period of time. Also, according to the authors, the cyclical period varies according to the conditions of the institutional, economic and social structure of each country and can be very short, up to 5 years, going on to long periods, over 20-30 years.

For Wheaton (1999), in a classic text in the area, he states that the dynamics of the cycle in the real estate market is influenced by other cycles, so it would not be just the internal conditions of supply and demand that would generate the oscillatory behavior of prices. There would also be several other factors, such as demographic, institutional, and various economic variables, such as credit volume and interest rates, for example. For this study, the variables that we will highlight will be the cyclical behavior of the gross domestic product (GDP) and the interest rate (Selic), since they are one of the main macroeconomic indicators and are the most investigated in the literature.

The behavior and formation of the real estate price cycle is still a matter little debated in academia, especially among Brazilian researchers. This low engagement is due to several factors, from an empirical point of view, the lack of availability of data in long and broad series, as is the case in Brazil, which allow for greater depth of exploratory analyses. From a theoretical point of view, studies on macroeconomic cycles and the real estate market traditionally
focus on the theory of real business cycles (RBC) (Davis & Heathcote, 2005), where they consider that the growth phases and economic cycle recessions are responses to shocks on the real economy, therefore, the sense of causality is already given ex-post in modeling (Funke & Paetz, 2013).

As pointed out, studies on macroeconomic cycles and the real estate market have focused on a long tradition on business cycles (Natsvaladze & Beraia, 2017). However, with the 2008 subprime real estate crisis in the United States of America (USA), which generated a financial crisis with global repercussions, a set of studies with different empirical perspectives pointed to the role of real estate markets as directly implicated in the formation of economic cycles from rise to crisis (Igan, Leigh, Simon, & Topalova, 2013; Crowe et al., 2013; Jurgilas & Lansing, 2013).

Thus, there is a very well-established literature that associates the variables of the real estate market, whether in the phase of expansion or depression, with the economic cycle in general. Therefore, there are studies that point to the credit market in general as a good predictor of financial crises or as a generator of instabilities, especially for economically developed markets (Mian, Sufi, & Verner, 2017; Cerutti, Dagher, & Dell’Ariccia, 2017; Schularick & Taylor, 2012).

Other works point to house prices, credit to the families, that is, to the real estate sector they appear as good indicators of crisis leadership (Agnello, Castro, & Sousa, 2019; Jung & Lee, 2017; Alessi, et al., 2015; Borio, 2014; Dufrénot & Malik, 2012; Büyükkarabacak & Valev, 2010). There are also other empirical works that suggest that economic crises, which were preceded by an increase in family debt, especially housing credit, tend to have deeper and longer recessions (Mian, Sufi, & Verner, 2017; Lombardi, Mohanty, & Shim, 2017; Jordà, Schularick, & Taylor, 2016).

The set of these works, with several empirical approaches based on data, highlight the importance and need to delve deeper into the relations between housing cycles, from the perspective of several variables, and macroeconomic cycles, especially in relation to Latin American countries, such as the case of Brazil, as they present few contributions on the topic. Since, as pointed out in related studies, real estate cycles of expansion and decline can arise for several reasons, generating an endogenous propagation effect in the economic system, and even internationally, as was the case with the American sub-prime crisis in 2008.

This irradiation throughout the economic system is due, to a large extent, to the completely interconnected and complex role of relationships in the modern economic system, making the very definition of what we could call the “root” of macroeconomic cycles increasingly difficult to define. And from a logical method point of view, the non-stationary behavior of many macroeconomic series increases the level of challenge imposed on researchers interested in the topic of cyclical fluctuations.

Therefore, even though we have presented studies that point to a positive correlation of the real estate market's leadership over macroeconomic variables, this understanding is not a consensus. As we can see in the study by Kydland, Rupert and Šustek (2016), in which it was identified that there would be a leading role for real estate prices in relation to GDP in the United States and Canada, however, countries such as Australia, Belgium, France, the United Kingdom, which is worth noting that all countries are economically developed, there were a reasonable coincidence of real estate prices with GDP fluctuation, therefore, there was no leadership situation.

The results of the study in the previous paragraph demonstrate a certain ambiguity regarding the meaning of the cycle relation. This result is corroborated by other works, which present different temporal and spatial perspectives, such as the work of Edelstein and Lum (2004), in which it was carried out an empirical research applying a VARX model (Vector Autoregression with Exogenous Inputs) for the Australian market, did not find that price changes, that is, the formation of the real estate cycle, have significant effects on aggregate consumption. As well as the works of Kim (2004), for the South Korean market, and Égert and Mihaljek (2007), for eight transition economies in Central and Eastern Europe and in nineteen member countries of the OECD (Organization for Economic Co-operation and Development), in which they showed that GDP and interest rates play a significant role in the formation of the cycle, in other words, real estate prices would be delayed in the cycle.

There is also a consolidated literature that provides strong evidence that the leadership role of the housing sector depends on the effects of monetary transmission that affect the real estate market (for example: (Umar, Akhtar, & Rao, 2020; Nocera & Roma, 2018; Robstad, 2017; Yang, Wu, & Shen, 2017; Mendonça, 2013; Jarocinski & Smets, 2008; Iacoviello, 2005)). These studies suggest that the effect of monetary transmission on prices is positive, in which case, property prices lag behind the cycle in relation to monetary variables, in particular credit and interest, that is, monetary aggregates influence them. However, they do not change the leading role of the housing sector ahead of GDP.

In short, there is no consensus in the empirical literature regarding the meaning of the causal relation. We have studies that present the causality as being from real estate prices to macroeconomic variables and a large literature that presents the relation inversely. Therefore, to overcome this gap, we must adopt specific strategies to verify the origins of economic fluctuations, and the directions of irradiation in the economy in general. For this purpose, it is essential that
we have a robust methodology to handle different propagation frequencies, as well as being able to handle non-stationary series and transient relations.

Finally, from this overview of the literature, we realize that in addition to there being a relative ambiguity regarding the conclusion about the causalities between real estate prices, the studies mostly use linear models from the family of autoregressive vectors. This research conduct presupposes, in its research design, that the causal relations between the variables must be stationary and do not change over time. And as will be presented in the next section, with the wavelet wave approach it is possible to verify the presence of significant causality, or not, and whether this relationship has changed over time. Therefore, the next section aims to present the fundamental concepts of the wavelet methodology that will be adopted in the empirical part of our research.

3. Methodology

The study of time series applied to economics focuses on investigating the evolution of a variable of interest over a given period. These periods can range from seconds, as is the case with stock exchange prices, to annual periods, such as a country's public debt stock, to decades, such as major growth cycles. In this context, when investigating a variable from the perspective of evolution over time, some challenges arise, for instance, the presence or absence of stationary behavior in the series, change in trend, structural breaks, among other characteristics that alter the structure and the behavior of the series. In many cases, it is precisely in the change in behavior that the series presents, where our interest in the investigative process lies. Especially for researchers dedicated to the processes of formation of economic cycles.

This approach to time series research is called time domain analysis, as the focus is on the temporality of the event. This approach is certainly the most adopted current in the empirical literature, however, there is another that is still little explored in economic sciences which is focused on the analysis of the frequency domain.

The analysis approach under the frequency domain, described as spectral or spectral analysis, aims to investigate how different frequencies in the series can vary over time, as well as verifying whether these frequencies have any spectral relation with other variables of interest (Yang, 2009).

In this regard, although they have different approaches, and are therefore alternative research methods, these two ways of working with a time series are complementary to each other, especially for researchers seeking a greater understanding and refinement of a research problem.

Fourier transform analysis, or more recently wavelet analysis, are methods that reconcile both domains, that is, time and frequency. The wavelet method, especially, presents particularly important characteristics for economic research, such as, for example, no need to work with stationary series or adjustments to break structures (In & Kim, 2013).

In addition to presenting the traditional description of the database, this section aims to make the spectral methodology available to the reader, focusing on wavelet analysis. Thus, as this is a perspective that is still little explored in the empirical literature, we will start with a description of the main concepts, which will facilitate the understanding of how this methodology can be very useful for working with economic series and cycles.

3.1 Signals and Fourier Analysis

When investigating a series from the frequency domain, it is necessary to deal with an even more general term, which is the idea of signals. A signal presents the behavior, state of a variable or a system that can be physical, such as the position of a particle, or completely abstract, such as the situation of hot or cold, that is, an entity that stores information.

From the point of view of mathematical formalism, it is a function with one or more variables which represents information about the nature of a given phenomenon. So, we have \( y = f(x) \), for a one-dimensional signal, or \( y = f(x, z) \), for a two-dimensional signal, and \( y = f(x, z, w) \), when three-dimensional. And regarding the type, the signals can be, in relation to time, defined as continuous or discrete (Yang, 2009).

Signal can be understood as the most general concept of what we can define as a series. Thus, a signal is any entity that represents a behavior or state. A series is the sequence of these representations or signs. Our focus will be, as will be better described in item 3.3, on the series of gross domestic product, interest rates and housing prices to investigate real estate cycles and their various spectral relations.

To work with these series, from another perspective, we will initially present Fourier analysis. This method is well known and already established, especially in other areas of knowledge such as geophysics, medicine and engineering. Thus, although our application is to develop our research using wavelet analysis, however, to some extent, wavelet analysis is a refinement of Fourier analysis. Therefore, we will initially introduce, briefly, the concepts of Fourier analysis and then we will present wavelet analysis, our investigative method.
3.1.1 Fourier Analysis

The French mathematician, Jean Baptiste Joseph Fourier, presented in 1807 the fundamental idea that would later be developed as the transform of a function. According to this author, any periodic function could be written through an infinite sum of sine and cosine functions. This approximation that uses trigonometric functions became known as Fourier transform (Gao & Yan, 2011).

Like the logarithmic transformation technique, which aims to transform an exponential number into a linear relation, the transform is a technique that seeks to simplify the solution of a problem. Thus, as a definition we can understand that the Fourier transform or the wavelets as a convolution of signals in the time domain, that is, given two functions through a linear operator is transformed by the product of these functions into a signal in the frequency domain.

Therefore, as Masset (2008) points out, the Fourier transform is the standard way of working with signals in the frequency domain, where it essentially seeks to project a time series (vector) on an orthonormal basis with trigonometric elements.

The Fourier transform can be expressed as:

\[
f(x) = \frac{1}{2}a_0 + \sum_{j=1}^{\infty} (a_j \cos(jx) + b_j \sin(jx))
\]

In equation 1 the sum goes to infinity, however, it can be adjusted to any function that is in \(L^2\), that is, functions with well-defined integrals in modules, so we have:

\[
H_f(x) = \frac{1}{2}a_0 + \sum_{j=1}^{J} (a_j \cos(jx) + b_j \sin(jx))
\]

According to In and Kim (2013), this equation becomes very useful, as it is capable of transforming any function with a finite sum \((J)\) into basic functions of sines and cosines.

As can be seen in equations 1 and 2, and given by the definition itself, in the Fourier transform the signal can be expressed as the sum of sine and cosine functions. In this sense, this methodology works very well for signals that do not vary with time, that is, a stationary series.

Thus, with the Fourier transform it is not possible to work with signals that vary over time, even though it is possible to check the frequency of the signal, it is not possible to identify at which moment there was a variation in frequency (Rua, 2012). Therefore, Gabor (1946) developed the Fourier transform with windows, where the main idea is to work with the Fourier transform in short times throughout the signal.

The \(f(x)\) signal is divided into smaller sections and each section will be analyzed at its frequency separately. In this way, it is possible to identify when and what frequency there was a cycle change, however, with a limited level of precision, as it is conditioned by the size of the window used by the researcher.

Thus, the major drawback of the Fourier transform with a short time window is that, when choosing the window size, it will be constant throughout the analyzed series. Consequently, in a context with a long series, with the window fixed at a short time there will be a tendency to generate an overrepresentation of high frequency cycles, to the detriment of low frequency cycles. Therefore, it is not an adequate representation of frequency resolutions that vary over time.

This limitation was overcome by the wavelet transform method, which, according to In and Kim (2013), wavelet has three advantages when compared to Fourier transforms. The first advantage is the ability of wavelets to decompose data into many time scales rather than frequency domain. In this way, it is possible to analyze the behavior of signals over several time scales, that is, short or long time cycles. The second advantage is that it allows time windows to vary, unlike the fixed Fourier window. The advantage lies in the possibility of isolating signal discontinuities. And finally, they have the advantage of dealing with series that contain non-stationary data.

For a better understanding of these advantages, graph 1 in Rua (2012) illustrates this discussion and makes it even more evident what a representation in time-frequency space would be. In a time series, which is represented exclusively in space in the time domain, each observation of the variable is present at all frequencies. On the other hand, in the Fourier transform, the points represent the frequencies and at each point all observations in the time domain are present. For short-time Fourier, the time-frequency dimension uses a fixed-size window. In contrast, for the wavelet transform the window size is flexible to the different existing frequencies.
Given these advantages, for researchers who work with economic cycles, wavelet transforms become very useful, since throughout the data the series present trends, structural breaks, among other difficulties typical of this type of investigation.

3.2 Wavelet Analysis

In this section, we will present the properties and advantages of wavelet applications in economics, focusing on continuous wavelets, but we will also present wavelets for discrete time. For a deeper understanding of the fundamentals of wavelet analysis, Daubechies (1993) is the reference book on the subject and presents a broad development of mathematical properties. Seminal works on the application of the wavelet tool in economics include those developed by Ramsey and Lampart (1998b), where they worked on the relation between money and production, and Ramsey and Lampart (1998a) who investigated the link between consumption and income. The works of Aguiar-Conraria and Soares (2011a) and Aguiar-Conraria and Soares (2011b) give us a great contribution to macroeconomic themes and economic cycles. The text by Crowley (2007) presents us with a review of applications in economics and finance, and for a more recent literature review we have the work of He and Nguyen (2015) who presented a review of studies on economics and finance.

3.2.1 Continuous transform with wavelets

The continuous wavelet transform (CWT, from the acronym continuous wavelet transform) is defined as the integral of the signal multiplied by signals that have been dilated (scaled) or moved in position by the function $\psi$. Thus, a CWT for a time series $x(t)$ can be described as:

$$\text{CWT}_{(s,p)} = \int_{-\infty}^{\infty} x(t) \psi_{(s,p,t)} dt$$

Where $s$, $p$ and $t$ are, respectively, scale, position and time. As a result, we will have many CWT coefficients that will be related to each scale and position and can be assumed any values that are compatible with the time series point $x(t)$. Thus, when each coefficient is multiplied by the dilated (scaled) and displaced wavelet, we will have the original signal. This procedure is similar to the Fourier transform, in which it multiplies sines and cosines to obtain the original signal.

The basic function of CWT is $\psi(s,p,t)$. Usually, these basic functions are called mother wavelet and can be defined as:
Where, \( p \) represents the time position and \( s \) the scale parameter. Thus, from a frequency point of view, low scales reveal short-term cycles, that is, high frequency, while high scales show slower cycles, that is, low frequencies.

To be a candidate for a mother wavelet, according to Percival and Walden (2000), \( \psi(t) \) must present the following properties: admissibility, orthogonality and vanishing moment.

The first condition is theoretically useful, since, in the same way as in the Fourier transform, it is necessary to satisfy the condition that the integration interval presents continuity or at least limited discontinuous points. The second condition of orthogonality follows the same definition as the Fourier transform, that is, functions shifted on the same scale are orthogonal, and that functions on other scales are also orthogonal. At last, the vanishing moment, means that the scaling function is more or less smooth. The smoothing effect is defined in the wavelet equation 4.

3.2.2 Morlet Filter of Wavelet Processing

The choice of the mother wavelet \( \psi(t) \) is an important step in the research. The specialized literature on the issue presents a variety of possibilities, such as Haar, Morlet, Daubechies, Mexican Hat, Meyer, Shearlet, among many other possibilities, which will depend on the application and specificity of the research that will be developed (Masset, 2008).

According to Rua (2012), as our objective is to investigate the coherence of real estate prices such as GDP and interest rates, we must look for a wavelet focusing on the phases of the cycles and on the amplitude of the series. And according to the aforementioned author, the most popular and suggested filter for applications in economics for work with continuous wavelet transforms is the Morlet wavelet. In a complementary way, for Aguiar Conraria & Soares (2011a), in addition to recommending the use of the Morlet wavelet, they justify that it is advantageous for presenting user-friendly attributes that simplify the interpretation of the results, as will be seen in section 4, since it will be the filter adopted in our analyses.

To understand the reasons for recommending the Morlet wavelet, it is important to also understand the context, in addition to the characteristics of this filter. Thus, in 1981, Jean Morlet, a French geophysicist from an oil industry, developed pioneering work with earthquakes, presenting a new method of temporal frequency analysis (Morlet, Arens, Fourgeau & Glard, 1982). This method later became known as the Morlet wavelet.

In general terms, geophysicists use sound emission to infer the type of rocky material and thus determine the location of deposits, for example. As sound waves propagate with different frequencies depending on the material in which the seismic shock was generated, signal processing is an important task of this work. However, the passage of sound from one layer to another presents abrupt changes or drops in the frequency of the sound. It was then that Morlet developed a more flexible analysis than that used in the Fourier transform (Russell & Han, 2016).

For a time series \( x(t) \), the Morlet wavelet can be defined as:

\[
\psi(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-\frac{t^2}{2}}
\]  

(5)

As we can see in equation 5, the Morlet wavelet is a complex sine defined within a Gaussian envelope. According to Rua (2012), one of the advantages of working with Morlet is its complex characteristic, allowing amplitude and phase to be time-dependent at each frequency. The parameter \( \omega_0 \) is used to adjust the resolution from the point of view of the frequency of location in time, so, depending on each case, the necessary resolution can be improved or reduced. For a more in-depth look at the characteristics and advantages that made the Morlet wavelet popular in this field of research, see Addison (2002).

3.2.3 Wavelet Analysis

In our analyses, we need an instrument to adequately quantify and detect the relation between two variables, namely, the coherence between real estate prices and GDP in addition to real estate prices and interest rates (Selic).

For this purpose, we will use the wavelet spectrum which is defined as \( |W_x(p,s)|^2 \), which measures the share for series variance at a specific point in time and frequency. For the case of two series, the wavelet cross transform is represented equivalently:

\[
W_{xy}(p, s) = W_x(p, s)W_y(p, s)
\]  

(6)
Given by equation 6, economic shocks, such as crises, for example, will be associated with a greater wavelet power (wavelet power spectrum) and, therefore, variation of the time series.

Another important analysis tool that we will use will be wavelet coherence, defined as between two series x(t) and y(t), as:

$$R^2(p, s) = \frac{\left| S(s^{-1}W_{xy}(p, s)) \right|^2}{S(s^{-1}|W_x(p, s)|^2)S(s^{-1}|W_y(p, s)|^2)}$$  \( (7) \)

Where \( S(.) \) represents smoothing, either in time \((t)\) or scale \((s)\). Smoothing is essential so that the coherence result is not always equal to 1. The objective of wavelet coherence is to verify how related two series are over time and at different frequencies. As results we have a value between 0 and 1, where high values represent high relation and a low result represents weak coherence (Aguiar-Conraria, Azevedo, & Soare, 2008).

As will be presented in section 4 - Results, the wavelet coherence graph is possible to verify in space-time-frequency when we had a stronger relation and perceive both the changes that occurred over time, as well as the frequency level.

A final analysis that will be developed will be to verify the relation (lead-lag), that is, whether the price cycle leads GDP and interest rates, or the leadership relationship occurs inversely. For this analysis, the concept of wavelet phase difference is used, which is formally defined as:

$$\varphi_{x,y}(p, s) = \tan^{-1}\left( \frac{j(W_{xy}(p, s))}{R(W_{xy}(p, s))} \right)$$  \( (8) \)

Where, \( j \) represents the imaginary part and \( R \) the real side. The values of \( \varphi_{x,y}(p, s) \) are between \([-\pi, \pi]\). The interpretation of the result is carried out by the phase difference, therefore, if both series move together and with the same frequency, the phase difference will be equal to zero. If \( \varphi_{x,y}(p, s) \) belongs to the interval \([0, \frac{\pi}{2}]\), the two series are in the same phase, but series \( x \) leads the movement and if \( \varphi_{x,y}(p, s) \) is in the interval \([\frac{\pi}{2}, \pi]\) is the series \( y \) that leads. Now, if \( \varphi_{x,y}(p, s) \) is in the range of \([0, -\frac{\pi}{2}]\), \( y \) leads \( x \) and, finally, if the result is in the range of \([-\frac{\pi}{2}, -\pi]\), \( y \) the one that leads is \( x \). Figure 2 helps us to better visualize and consolidate the information presented about phase differences.

![Figure 2](http://aef.redfame.com)

**Figure 2. Phase differences and their interpretation Roesch and Schmidbauer (2014, p. 7)**

An advantage of this result is that it presents whether the lead-lag relationship for the analyzed frequencies has changed over time. As Klarl (2016) points out, in a general sense, the results can be interpreted as an option for Granger causality analyses. For our study, this result will help us discover whether the Brazilian real estate market leads the GDP or whether it would be the opposite, that is, whether the GDP would be in a position to lead the phases of the cycles. The same analysis will be carried out for the case of Selic and real estate prices.
3.3 Database Description

The time frame of our analyses will be for monthly data from the Brazilian economy, starting in April 2001 until January 2021, totaling 238 observations. The selection of the sample period aims to cover the real estate expansion cycles of the 2000s and its subsequent cooling, in addition to the recent pressure in the sector for the year 2020.

Our objective is to verify, on one hand, the relation between real estate prices and GDP and, on the other hand, the connection of real estate prices with the interest rate. The data for real estate prices that we will use will be the monthly variation of the Index of Guarantee Values of Financed Residential Properties (IVG-R) made available by the time series management system of the Central Bank of Brazil (BCB), thus, we take real estate residential prices as an indicator for the cyclicality of this market. For the data on the gross domestic product (GDP) we will use its monthly variation, which is made available by the Brazilian Institute of Geography and Statistics (IBGE) and the interest rate will be the Selic rate, which is also made available by the BCB.

For the analysis and calculations of continuous wavelets, we will use the WaveletComp package (Roesch & Schmidbauer, 2014) which is available for free access in the official repository of the R programming language.

4. Results

In this section we will present the study of the spectral structure for the Gross Domestic Product (GDP), Interest Rate (Selic) and the Index of Guarantee Values of Financed Residential Properties (IVG-R). For each series we will present the analysis of the wavelet power spectrum, in addition to presenting the wavelet coherences and phase differences between GDP versus IVG-R and Selic versus IVG-R.

4.1 Univariate Wavelet Analysis

The main analysis of this subsection will be to verify the spectral behavior of each variable separately. To do this, we used the wavelet technique for our series of interest in their univariate form. These results will enable us to verify the presence of cyclicalities in the series individually, this presentation is understood as being our first approach to the study of real estate cycles.

In the figures we will present below, the horizontal axis represents time, and the vertical axis represents frequency. To facilitate the interpretation of the results, the frequencies were converted into time units, in our case, years. This way, analyzing the graph, it will be possible to check in which frequency bands, on the vertical axis, and in which time interval, on the horizontal axis, the series showed greater or lesser activity.

The frequency variable presents the number of complete cycles per unit of time, so the higher the frequency means that there was a large cyclical activity in a short time. Otherwise, that is, low frequency represents longer cycles to be completed, denoting the idea of “long tail”. The black lines represent locally significant areas, and the inverted white bell is the area of influence.

4.1.1 Gross Domestic Product

In figure 3 we see the presence of strong frequencies typical of economic cycles, that is, with cyclical fluctuations greater than 2 years and less than 8 years. In particular, throughout the analyzed period there was the presence of high frequencies in the 4-year range, which can be explained, in part, by the theory of political-economic cycles to explain macroeconomic fluctuations (Alesina, Roubini, & Cohen, 1999), that is, being a direct reflection of the political cycles of the federal executive on the economy.
We can still see that in the cycles of 2 there was a great acceleration in the years of the international financial crisis of 2008, moving to a low frequency in the years around 2015, and again presenting a new increase in frequency from 2019 onwards, the latter being effect of the new coronavirus crisis, discussed in the introduction. Therefore, the high frequencies presented can be associated with the shocks caused by the recent crises and the low frequencies with the economic moderation after the great financial crisis of 2008, which Brazil went through until 2016.

Complementarily, figure 3a presents the average wavelet energy spectrum over the entire period. And as we can see, GDP showed high frequencies in short cycles of 2 and 4 years. The long cycle of 16 years, although it presented global statistical significance, was outside the area of local influence, represented by the inverted white bell, plotted in graph 3a.

4.1.2 Interest Rate (Selic)

Figure 4 presents univariate wavelet analysis for the interest rate (Selic). This variable was more stable throughout the period when compared to the wavelet for GDP carried out previously. Thus, in graph 4a we see the existence of 3 large high frequency bands throughout the entire series, namely: 3, 7 and 9 years. In the 2-year range, we see that after 2010 until the start of the new coronavirus crisis, the interest rate entered a low-frequency movement, which began to show a change only at the beginning of 2020, as a response to the shock of the recent crisis, although this information is outside the inverted bell.

As the behavior of the univariate wavelet analysis for local Selic was, to a certain extent, stable throughout the entire series, the global graph is similar, adding that the average frequency for 7 years is the highest for the study, as can be confirmed in figure 4b.

4.1.3 IVG - R

Finally, figure 5 presents univariate wavelet analysis for the Index of Guarantee Values of Financed Residential Properties (IVG-R). Unlike previous wavelet analyses, IVG-R presents, in the sample period, long cycles of high
frequency, figure 5a, while below the 2-year band the cycles were always of low frequency. Graph 5b confirms this analysis of large long-term cycles, where there is a sharp growth in frequency after 9 years.

In summary, the variables demonstrate a cyclicity in the 4-year frequency band, even though the IVG-R has, after 2015, reduced frequencies to very low bands. What suggests that the sub-prime and the coronavirus crises were separated by a moment of low cyclicity. Thus, in general, there seems to be some similarity in the strength spectra of the univariate wavelets, a hypothesis that we will verify more rigorously in the next subsection, in which we will test the coherence relation between these variables.

4.2 Wavelet analysis of coherence and phase difference

Our objective in this subsection is to verify the spectral coherence of the wavelet and the phase differences between GDP and IVG-R, in addition to Selic and IVG-R. The price index is our main variable, as we are seeking to verify the nexus, that is, the relation between real estate price cycles and the two main macroeconomic variables, product and interest. These studies will allow us to identify and understand the frequency relation and synchronization between variables, as well as the lead-lag relation between them. Thus, we seek to answer a question that will finalize our study: what is the relation between real estate price cycles and macroeconomic variables such as GDP and Selic? The results in these analyses are presented in figures 6 and 7, respectively.

As the methodologies for the calculations have already been presented previously, here is a presentation of how to read and interpret the graphs plotted in the following figures.

Graph (a) in the figure is the local coherence wavelet, where the heat map represents the coherence strength; the inverted white cone shows the sample's zone of influence, outside of which it can be affected by spillover effects and the white contours show zones with a significance level of 5% with 100 simulations.

Graph (b) is the global coherence wavelet, where the segments in red represent the frequencies with a significance level of 5% and the peaks of the curve, the points with the highest frequencies.

Finally, graph (c) shows the phase differences at each moment, so if the two series are in the same phase and moving together there will be no difference, therefore, the difference will be 0. If the two series are in the same phase, that is, positively correlated, they will be between 0 and ±\(\frac{\pi}{2}\) and if they are out of phase, or negatively correlated, they will be between the interval, in modulus, greater than \(\frac{\pi}{2}\). The leadership condition for GDP or Selic in relation to real estate prices will occur when the phase difference is between 0 and \(\frac{\pi}{2}\), when in phase, or \(-\frac{\pi}{2}\) and \(-\pi\) in case they are out of phase.

Each black arrow represents the local crossed waves (coherence), as shown in figure 2, and the white lines are the areas of statistical significance between the waves at 5%.

4.2.1 Relation between GDP and Real Estate Prices

Initially, in figure 6, we present information on the strength of local correlation between real estate prices and GDP. Some important results emerge from this analysis: firstly, checking the local coherence between the variables in figure 6a. We can clearly verify local coherence, at low frequency, throughout the entire period analyzed in the 4-year range, that is, over the 3-4 year range there is a strong statistically significant correlation, represented by the area limited by
the lines in white.
Thus, the wavelet wave coherence spectrum suggests that the relation between these variables has been reasonably stable throughout the analyzed period, which can be confirmed with figure 6b, where this low frequency coherence, around four years, presents statistical significance. Even though there is a global peak of 16 years, these frequency bands were not possible to confirm with figure 6a, as they are outside of our local area of influence.

(a) Local Wavelet Coherence

(b) Global Wavelet Coherence

(c) Phase Difference

Figure 6. Bivariate Wavelet Analysis for GDP and IVG-R

Finally, figure 6c presents information about the phase differences between the analyzed variables. And as can be seen throughout the entire series, there was a strong phase coherence, that is, throughout the period of our sample the phases were between $\frac{\pi}{2}$ and $-\frac{\pi}{2}$, that is, they were positively correlated.

In this regard, we have evidence that the real estate cycle, in addition to being in phase coherence like GDP, real estate prices led almost the entire period analyzed. This information can be verified with the dashed line below zero, in the period from 2005 onwards. This denotes that real estate cycles are positively correlated and at the same frequency as GDP, but that real estate prices have led this cycle movement and are, therefore, followed by GDP.
4.2.2 Relation between Interest Rate and Real Estate Prices

Graph 7 presents bivariate wavelet analysis for the interest rate (Selic) and IVG-R. Going in the same direction as the previous analysis carried out for GDP and IVG-R, some results are important, which we must highlight, above all, as they differ in the frequency of the local coherence band and the lead-lag relation.

The first analysis is on the local spectral configuration present in figure 7a, in which we can verify a clear local low-frequency coherence throughout the entire series for the band greater than 4 years and less than 8. The exception is in the period from 2001 to 2008, where there was a local high-frequency coherence for the band below 4 years, that is, short term. We can also verify local high-frequency coherence for the band above 8 years, with the Selic rate playing a local leadership role, which can be verified by the indications of the black arrows in the graph as seen in figure 2.

Figure 7b shows the global coherence of the series, where we can confirm the strong long-term link between interest rates and price cycles, that is, the series present a relation that is statistically significant for bands above 8 years. This result has important long-term implications, which will be further analyzed in the next section, since the housing acquisition depends on long-term financing from the real estate credit system.

Finally, figure 7c shows us, as in the case of GDP, a strong movement in phase between interest and real estate prices, that is, within the range of $\frac{\pi}{2}$ and $-\frac{\pi}{2}$. However, with the interest rate throughout the series, unlike in the case of GDP, there was a change of leadership. Between 2001 and 2010, the Selic rate was leading the movement, that is, the dashed line was between $\frac{\pi}{2}$ and 0, and after 2010, the cycles, still in phase, were reversed, starting to be led by the movement of housing prices, line dashed between 0 and $-\frac{\pi}{2}$. This result is very important and demonstrates that after the 2008 crisis there was a change in the profile of the beginning of the real estate cycle, making the interest rate move slower.

In this subsection on the results, we seek to investigate the possible relation between real estate price cycles and the two main macroeconomic variables: Gross Domestic Product (GDP) and the market reference interest rate (Selic). For this objective, our empirical work carried out a time frame from April 2001 to January 2021, in which comprehends the real estate boom of the Brazilian economy in the 2000s, the great financial crisis of 2008 and the Brazilian economic stagnation of 2015/2016.

In summary, the results show that real estate prices played a leading role in relation to GDP (lead-lag situation) and with local and global coherence in the short term, within 3 years. Regarding the Selic rate, there was long-term
coherence, over 8 years, for the local and global spectrum. However, after the 2008 crisis, the Selic rate stopped being ahead of the housing cycle, starting to lag behind the movement, but always in phase. Last but not least, across the sample spectrum investigated, the wavelet analyses carried out, point to a situation of strong nexus (positive correlation) between the phases of macroeconomic variables and housing prices, that is, there were no countercyclical movements, and in this consistency, real estate prices were in a leading position.

5. Discussion
In this final section, we seek to discuss our results in light of the work developed in specialized literature, where we will place greater emphasis on our similarities and divergences regarding the results found in our research.

In general terms, we can summarize the two wavelet analyses of the real estate cycle and the macroeconomic variables, GDP and Selic, for the fundamental role played by the real estate price cycle in the leadership situation of the series. Thus, for the period analyzed, real estate price cycles were always positively coherent and at the same frequency with the macroeconomic variables investigated, therefore, there would not be a countercyclical situation between them. It is worth mentioning that Selic presented a mixed situation, that is, there was a change of leadership. Therefore, after 2010, Selic stops leading and the real estate cycles start to lead the cycle.

When we cut our analysis only to GDP, we see that throughout the wavelet study there was a stable leadership behavior of the real estate sector in relation to GDP. These results, different from the studies of Klarl (2016), Stock and Watson (2003), in which they present that there would not be, at least for the empirical case of the United States housing market, a stable leadership situation, but rather, a mixed behavior, as occurred in our case for the Selic. Therefore, in situations of economic crises, the housing sector leads, but, in times of housing bubbles, the results suggest that a production shock generates an effect on the real estate market. Thus, unlike these studies mentioned, in our study the Brazilian housing sector leads the production cycle throughout the time frame analyzed, while in the American case only during the recession period, and not in the expansion phase.

Our findings for the Brazilian economy are in line with studies by Fisher (2007), in which it was pointed out, in the case of the United States economy, that family investment leads companies' fixed investment, in this case, the product. In the study by Kydland, Rupert and Šustek (2016), this leadership is observed in countries such as the United States and Canada. Even though, in this study, the cut made by the author was for all investment categories that families have, real estate investment has the greatest weight within the families' portfolio. We also have a study by Green (2002) that points out that real estate prices lead the economic cycle through a wealth effect on the economy.

In this sense, we identified for the Brazilian case that the role of residential investment plays an important role as a signal to the economy for periods of expansion or retraction. Therefore, it makes perfect sense for the Taylor rule to incorporate monitoring of the specific housing market to define the interest rate, as the study by Silva and Besarria (2018), rightly pointed out, where they suggest that the Central Bank is reacting to pressure on housing prices.

With regard to interest rates, it is important to point out the surprising change in leadership. The leadership behavior of the IVG-R cycle over the Selic only appeared after the 2008 crisis. This lead-lag behavior, in favor of housing prices, presents a possibility for agents to anticipate the rise and fall movement of the interest rate.

Another important issue in relation to the interest rate, which to a certain extent was expected, was the strong long-term coherence, since demand in the housing market depends on long-term forms of financing, where interest plays a fundamental role in the final value of financing. In this sense, it also denotes the permanent need to maintain low interest rates for this sector.

In this regard, with the change in causality between interest rates and the housing market over time, literature that adopts only structural econometric models are not able to capture these changes, making it necessary, therefore, to use appropriate methodologies to deal with and treat this behavior with change. To this end, studies like the Markov-Switching (MS) approach by Simo-Kengne et al. (2013) and Chowdhury and Maclennan (2014), are important, however, they still have other limitations such as the need for seasonal series and they also do not use the frequency domain. Aspects that wavelet methodologies are capable of overcoming.

Having said these discussions, some questions that we can delve into are: while there was stability in the leadership of the housing market throughout the formation of cycles for GDP, why was there a change in behavior between Selic and IVG after the crisis of 2008? Why did this happen? What changed in the perception of agents or institutions? The wavelet analysis performed cannot answer these questions and it is a limit of the study. However, with this wavelet analysis evidence in hand, it helps us to develop these and other questions that can (and should) be deepened in future research.

6. Conclusion
This article aimed to investigate the relation between real estate price cycles and two important macroeconomic
variables, the Gross Domestic Product (GDP) and Interest Rate (Selic) of the Brazilian economy. The choice of these macroeconomic variables was anchored in the conclusion in the related empirical literature. This analysis aimed, therefore, to deepen the understanding of the causal relation between the real estate market and other important variables from a macroeconomic point of view, going beyond the simple use of Granger causality or autoregressive vectors, and thus answer our research problem: what is the relation between real estate price cycles and macroeconomic variables (GDP and interest rate)?

To answer this question, we adopted the following methodological route: a review of the literature related to the topic of property price cycles to understand what the main conclusions were about the role of real estate price cycles, with a special focus on housing. In these studies, we looked for whether there would be a leadership situation in the real estate sector, that is, if the housing sector would play a role in “pulling” the different cycles or if it would be behind in the cycle.

This leadership behavior is defined as a lead-lag situation. As a conclusion to this stage of the methodological journey, we understand that there was a certain ambiguity in the results, in addition to a complete scarcity of studies that deal with the empirical situation in Brazil, from the point of view of an approach under the frequency domain, that is, through the nexus of different frequencies for the formation of the cycle.

To contribute to this gap identified in section 2 (Related literature), we investigated the Brazilian residential real estate market, with a time frame ranging from April 2001 to January 2021, totaling 238 observations. We used a wavelet spectral analysis as a methodological approach, which has proven to be very efficient for identifying non-stationary time series or those with structural breaks, in the time domain, and which may present different frequencies (spectrums), throughout the frequency domain. These characteristics of wavelet analysis allowed us to delve deeper into the link between price cycles in the Brazilian residential sector and the cycles of the macroeconomic variables investigated.

With the finalized results, it was possible to better understand the behavior of the different cycles analyzed in this work and respond appropriately to our research problem presented previously. From these results it was possible to identify transitory relations between the series (coherence), as well as identify the relations of these variables that occur at the same time, but with different frequencies.

Given this methodological trajectory, regarding our analysis of GDP and real estate prices, our main results were: 1) Typical frequencies of economic cycles, greater than 2 years and less than 8 years. 2) Presence of positive local coherence throughout the 4-year range, that is, cyclical movements between the series, and 3) Throughout the entire series there was a leadership situation in favor of real estate prices, thus demonstrating the fundamental role of the housing sector, as it was always ahead of the GDP cycle.

For the nexus between Selic and real estate prices, our most important conclusions were: 1) Throughout the sample investigated, local coherence showed a tendency towards increasing participation in a situation of short-term cycles (below 4 years). Thus, between 2001 and 2008, local coherence presented cycles of high frequency (or intermediate in our scale) and then began to have cycles only in the band above 4 to 8 years. 2) Local high-frequency coherence over 8 years, demonstrating a strong long-term relation between the series and, 3) The intriguing exchange of leadership between the real estate market and Selic. Between 2001 and 2010, the Selic rate cycles were leading the movement, in phase, that is, positive cyclical correlation, but after 2010, the cycles began to be led by the movement in housing prices.

In view of the above, the main implication of these results is the fact that the Brazilian real estate market presents itself not only as an important cyclical engine of economic growth, as it has always been in the same cyclical phase, but mainly because it presents itself, most of the time, ahead of the cycle, that is, “pulling” the cycle or at least being an important predictor of the economy's behavior in the short term, in the case of GDP, and in the long term in the case of Selic.

Acknowledgments
Not applicable.

Authors contributions
Dr. Jadson Pessoa and Dr. Sergio L. M. Rivero were responsible for study design and revising. Prof. Jadson Pessoa was responsible for data collection and drafted the manuscript and Prof. Alexsandro S. Brito revised it. All authors read and approved the final manuscript. In this paragraph, also explain any special agreements concerning authorship, such as if authors contributed equally to the study.

Funding
Not applicable.
Competing interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Informed consent
Obtained.

Ethics approval
The Publication Ethics Committee of the Redfame Publishing.

The journal’s policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review
Not commissioned; externally double-blind peer reviewed.

Data availability statement
The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement
No additional data are available.

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