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Revisiting the Discussion on the Effectiveness of Alternative Portfolio Models: Out-of-sample Analysis of Brazil's Equity Market, 2015-23

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Abstract

Around the world, equity markets draw the attention of investors and financial researchers, who share a common interest in searching for relatively more efficient portfolio strategies. Although numerous new allocation techniques have been proposed, the available literature still give emphasis to more traditional analytical systems. Accordingly, in this research, a large sample, with many stocks and long data series, is applied to the comparative analysis of widely used portfolio models trough results based on out-of-sample data. We also carry out a statistical evaluation of the sample distributions of returns in order to assess if the available data are consistent with the commonly accepted hypothesis that the effects of estimation risk are present in solutions based on the mean-variance model. The results obtained, contrary to many critical evaluations, highlight the superiority of portfolios derived from optimal allocation models over strategies based on market index and equal weights.

Keywords: Equity markets, Asset allocation, Portfolio investment strategies, Portfolio performance, Mean-variance analysis

1. Introduction

Not long after the advent of mean-variance portfolio analysis (Markowitz, 1952), and despite the general recognition of its formal-theoretical refinement, it became clear that this model was subject to certain difficulties and inconsistencies – a finding based on evaluations of different types. In numerous articles on alternative portfolio strategies, it has been frequently mentioned that the application of the optimization model is not widespread among investment professionals, since the portfolios obtained by this method are often marked by weights considered "extreme" and "non-intuitive". In particular, a great number of assets are usually included with short positions – when the model is resolved without imposing non-negativity restrictions – or, when such constraints are introduced (long-only portfolios), solutions often include considerably large weights on assets with low liquidity (Michaud, 1989; Black & Litterman, 1992).

Another type of problem perceived with Markowitz's portfolio analysis is that, even with moderate revisions to the data, the weights in the portfolios are subject to exaggerated fluctuations. Moreover, in evaluations based on out-of-sample analyses, these solutions often do not perform favorably – in particular, when compared to straightforward portfolios with equal weights (DeMiguel, Garlappi & Uppal, 2009). Several authors have stated that, among the factors responsible for the shortcomings pointed out above, the problem of errors in the estimation of parameters stands out, especially in the case of expected returns – which has been called "estimation risk". Deng, Dulaney, McCann & Wang (2013) state that, in addition to errors in the estimation of means, risks, and correlations, the available data on returns are subject to high kurtosis and negative skewness.

A development that results from the perception of the importance of estimation errors is the line of research focused on risk, or "risk-based strategies" (De Carvalho, Lu & Moulin., 2012), since the absence of estimates of expected returns reduces the effects of estimation risk. In this context, the most common portfolio strategies can be prioritized according to the potential effects of estimation risk. Portfolios with equal weights should come first, given that this risk is not present. Secondly, should appear portfolios based exclusively on risk, a group that includes the minimum-risk portfolio obtained from the mean-variance model. At the other extreme in this hierarchy should be included strategies that also use mean estimates and, in particular, portfolios that maximize the "ex-ante" Sharpe ratio.

The general objective of this paper is to pursue a comparative assessment of Markowitz's portfolio analysis from the perspective of the plentiful criticism available in the literature. To reach this overall aim, alternative strategies are evaluated using, on a monthly basis, out-of-sample data obtained for a large number of stocks in the Brazilian market. Each month, the portfolios were rebalanced by incorporating the latest data – a procedure that uses a fixed-sized "window" of data, which is periodically shifted. In this way, each month, ex-post returns that result from the solutions obtained in the previous period are available. In this evaluation, we consider both the unrestricted and restricted (long only) versions of the optimization model. We also impose an upper limit on the individual assets' weights in order to keep the results more closely related to the intuitive behavior of a typical investor.

In this study, the portfolio strategies that were used were: equal weights, sample-based mean-variance (global minimum variance), maximum ex-ante Sharpe ratio; global minimum variance with short-sale constraints; and maximum Sharpe ratio with short-sale constraints. Additionally, results for the major stock-market index in Brazil (Ibovespa) are used as a benchmark. A second objective of this study is the statistical evaluation of the sample distributions of monthly returns in order to assess if the available data are consistent with the general hypothesis that the effects of estimation risk are present, especially in the case of solutions that maximize the Sharpe ratio. In this second type of comparison, the pattern observed in the equal weights' portfolios play a central role.

Contrary to many of the critical evaluations available in the literature, but in accordance with the findings in a more recent study (Theron & van Vuuren, 2018), the overall Markowitz's analysis does not present an unfavorable performance in the data sample that was used. In particular, the global minimum-variance portfolios have indeed met this target – they had the lowest risk. In addition, solutions that maximize the ex-ante Sharpe ratio, based on the mean return and risk of the portfolio, effectively obtained the best results from ex-post data.

2. Review of Literature

One early reference that brought attention to the counter-intuitive nature and the problem of excessive variability in portfolios based on mean-variance analysis is Michaud (1989). This author also emphasizes that the data used in the optimized solutions were subjected to an acute problem of estimation error. As examples of problems of this kind, Black & Litterman (1992) point out that "when investors impose no constraints, the models almost always ordain large short positions in many assets" (p. 28). On the other hand, when non-negativity constraints are imposed, the optimizing solutions often contain "unreasonably" large weights in not-so-liquid assets.[1]

Zakamouline & Koekebakke (2009) affirm that, when one cannot conclude that the returns are normally distributed, solutions based on the maximum ex-ante Sharpe ratio can be "misleading" and "unsatisfactory". Additionally, Deng et. al. (2013) emphasized that, since the Sharpe ratio implicitly assumes that the returns are independently distributed normal random variables, this approach suffers from the problem of estimation errors given that this assumption is not valid in financial markets.

Among the articles investigating the efficiency of alternative portfolio allocation methods, one should mention Haugen & Baker (1991). These authors verify that indices based on market capitalization are not efficient in several situations. For example, this loss of efficiency is observed when investors disagree about the risk and expected return, in cases of short selling being restricted, and when investment income is taxed. Also, this problem is perceived when investment alternatives are not included in the benchmark, and when foreign investors are present in the domestic capital market. The authors conclude that, in these situations, there are alternatives to portfolios based on market capitalization that obtain the same expected return, but with less volatility. Alternatively, Grinold (1992), using an approach that was proposed in Gibbons, Ross & Shanken (1989), conducted tests on the possibility of outperforming the benchmark for five equity markets: German, American, Australian, British, and Japanese. The results indicated that for four of the five markets, the benchmarks (respectively: DAX, S&P 500, ALLORDS, FTA and TOPIX) were not efficient in the period analyzed.

Moreover, in terms of comparing alternative portfolio allocation models, a classic reference is DeMiguel et al. (2009), where several different models are examined and contrasted. These authors analyze out-of-sample data to access the potential effects of estimation risk of solutions based on mean-variance optimization. In particular, strategies with equal weights, minimum variances and maximum Sharpe ratios are included in the evaluation that was performed. Additionally, many authors consider that the problem of estimation errors is more pronounced in the case of expected returns than in moments of second order. This realization leads to the use of portfolio strategies that rely only on risk and diversification. De Carvalho et al. (2012) develop a very detailed empirical study to compare the main alternatives based on this approach, which also include minimum-variance solutions. In Braga (2015), a similar methodology is pursued of comparing portfolio strategies that require a smaller number of parameters in their solutions, since they are

[1] Nevertheless, in the case of this latter problem, one way out is to introduce additional restrictions, with maximum values for the portfolio's weights – an approach that was in fact adopted in this research (Section 3).

less exposed to estimation risk.

On the other hand, a recent article that is closer to the present study is Dolinar, Zoričić & Kožul (2017). These authors seek to evaluate the efficiency of market-capitalization-weighted indices (benchmarks) through the comparison with results obtained from traditional models: equal weights (naïve), and the maximization of the Sharpe ratio. However, the authors did not consider the alternative of imposing non-negative solutions, as was contemplated in the present work. Theron & van Vuuren (2018) also present a comparative empirical analysis of portfolio strategies based on the mean-variance model and derive conclusions similar to the ones in this research.

An earlier paper, that uses data for the Brazilian equity market, is Zanini & Figueiredo (2005). Although the approach followed in that text is different from the present research, the general objective is also to apply alternative portfolio models to data for the Brazilian stock market. In a contemporaneous work, Farias, Vieira & dos Santos (2006), using data for Brazilian equities, also present a comparative analysis for some portfolio selection models. More recently, Santos & Tessari (2012) assess out-of-sample performances of three portfolio allocation models, and contrast those with the Ibovespa benchmark. They also apply alternative estimators for the covariance matrix. In a different perspective, Naibert & Caldeira (2015), using data for Brazilian equities, examine minimum-variance models with alternative covariance matrix estimation methods. Also, Caldeira, Moura, Perlin & Santos (2017), investigate the eventual benefits of using high frequency data to construct optimal minimum-variance portfolios.

3. Mathematical Overview

With the exception of the allocation strategy based on equal weights, the portfolio models in this paper are, from a mathematical perspective, examples of restricted optimization problems given that, in all of them, there is an equality constraint that is used to establish the main characteristic of a *portfolio* – represented by the vector $x\,(n \times 1)$. This constraint can be formulated as the linear function $s^T x = 1$, where $s^T = [1 \quad 1 \; \dots \; 1]$. The general problem of optimization (minimum) with constraints can be represented by:

$$\min_{x} \quad f(x) \tag{1}$$

$$\text{subject to} \quad h_i(x) = 0,\; i = 1, ..., m; \tag{2}$$

$$g_j(x) \geq 0,\; j = 1, ..., p. \tag{3}$$

In the case of portfolio optimization models that do not allow for short selling, besides the basic linear restriction of a portfolio, inequality constraints $x_j \geq 0$; $j = 1, \dots, n$; are also present. In fact, in this research, additional restrictions were introduced in these versions of the optimization problem with the objective of imposing a maximum weight a given stock can have in the portfolio (25%) – which, from the perspective of an investor, does seem reasonable. Therefore, in these versions of portfolio optimization, the additional restrictions are: $0{,}25 - x_j \geq 0$; $j = 1, \dots, n$.

The solutions of models that do not include inequality constraints have relatively simple analytic representations. However, models that include inequality constraints can most often only be solved through numerical methods. In all cases, however, the fundaments of these solutions are the same, and the general aspects are presented below (Simon & Blume, 1994; Gárciga-Otero, 2011).

The most common method to solve an optimization problem with constraints is based on the Lagrange function, which includes multipliers λ_i and μ_j:

$$L(x, \lambda, \mu) = f(x) - \langle \lambda, h(x) \rangle - \langle \mu, g(x) \rangle \tag{4}$$

Proposition 1 (Karush-Khun-Tucker). Necessary conditions for a solution (x^*, λ^*, μ^*): Assuming that the partial derivatives of the functions f, h_i, g_j are well defined, then if x^* is a local solution (minimum point) of problem (1) – (3), there are unique values x^*, λ^*, μ^* such that:

$$\nabla_x L(x^*, \lambda^*, \mu^*) = 0_n; \quad h(x^*) = 0_m; \quad g(x^*) \geq 0_p; \quad \mu^* \geq 0_p; \quad \text{and} \langle \mu^*, g(x^*) \rangle = 0 \tag{5}$$

Proposition 2**.** Sufficient conditions for a solution (x^*, λ^*, μ^*): Assuming that the first and second partial derivatives of the functions f, h_i, g_j are well defined, then one can construct the Hessian matrix $B = \nabla^2_{xx} L(x^*, \lambda^*, \mu^*)$ which, being symmetric, represents a quadratic form $Q(x) = x^T B x$. If $Q(x) > 0$, $\forall\, x \mid x \neq 0_n$ (positive definite), then a solution (x^*, λ^*, μ^*) that satisfies Proposition 1 is a local solution (minimum point) of problem (1) – (3).

From the perspective of a solution method and assuming $f(x)$ is quadratic and all restrictions are linear – which is the case of the portfolio optimization problems examined in this study –, Proposition 1 transforms the original (nonlinear) optimization problem into a linear problem. When inequality constraints are present, numerical solution methods must be used. One such approach is the so-called complementarity method (Murty, 1988; Miranda & Fackler, 2002).

It is well known that, in Markowitz's analysis, portfolios are associated with two variables, namely risk (or volatility) and expected return. In the $\mathbb{R}^2$ space for these variables, the efficient frontier can be specified as the locus of portfolios with the highest expected return for a given level of risk (or variance). For the general mean-variance problem, $f(x)$ is a positive definite quadratic function, and the sufficient conditions for a minimum (Proposition 2) apply. This optimization problem is (Vanini & Vignola, 2001):

General mean-variance portfolio optimization.

$$\min_{x} \quad \frac{1}{2} x^T V x \tag{6}$$

$$\text{subject to} \quad x^T s = 1 \tag{7}$$

$$x^T Er = r_p \tag{8}$$

Matrix V in eq. (6) contains variances and covariances. Further, in eq. (8), Er represents a vector with the mean returns of the n assets, and r_p is a given value on the vertical axis. On the other hand, if restriction (8) is not included, then we have a global minimum-variance problem. When there are no inequality constraints – in particular, short sales are allowed –, the portfolio-optimization problem has a straightforward analytical solution.

Theorem 1. The solution of the global minimum-variance portfolio (6) – (7), without inequality restrictions, is $x^* = (1/a)\, V^{-1} s$; $a = s^T V^{-1} s$.

For a proof, see Vanini & Vignola (2001) and Da Fonseca (2003).

By introducing in the above analysis the return of a risk-free asset, specified on the vertical axis, an optimum point on the frontier of efficient portfolios can be determined by a tangent line to the curve that contains the risk-free return. In this problem, restrictions (7) and (8) are altered to include a risk-free asset (r_f, with proportion invested x_0):

Optimum (tangent) portfolio on the efficient boundary.

$$\min_{x} \quad \frac{1}{2} x^T V x \tag{6}$$

$$\text{subject to} \quad x^T s = 1 - x_0 \tag{7a}$$

$$x^T Er = r_p - r_f x_0 \tag{8a}$$

One aspect that deserves mention is that problem (6) – (8*a*) is equivalent to the maximization of the well-known Sharpe ratio.[2]

Theorem 2. The solution for the optimum (tangent) portfolio (6) – (8*a*), without inequality restrictions, is $x^* = V^{-1}(Er - r_f s)\,(b - a\, r_f)^{-1}$. In this solution, a is defined in Theorem 1, and $b = (Er)^T V^{-1} s$.

For a proof, see Vanini & Vignola (2001) and Da Fonseca (2003).

4. Methodological Elements and Sample Description

Generally, the stocks in the sample used in this research are the ones included in the benchmark index for the Brazilian equity market – the Ibovespa. The total number of equities in this benchmark is not fixed, since it usually changes with each revision of the index. In the three revisions that occurred in 2023, the total number of stocks were, respectively, 88, 85 and 86.

Initially 178 stocks were considered for inclusion in the sample. These stocks were available in a broader index for the Brazilian equity market – the IBrA-B3 – at the end of 2023. Then, from this initial set, only the stocks that were traded in the entire research period and, at the same time, were part of the Ibovespa in at least one edition during this period, were included in the sample that was actually used. The total number of equities incorporated in this final version was 72. The Appendix contains statistical indicators for the sample.

As previously stated, the main objective of this paper is to apply five alternative portfolio selection models to data available for Brazil's equity market from 2015 to 2023. In the context of the earlier researches mentioned in Section 2,

[2] Strictly speaking, for a given portfolio, the Sharpe ratio usually includes the (ex-post) return that was observed in a previous period.

the present work analyzes the following models: a) Equal weights (naïve); b) Sample-based mean-variance (global minimum variance); c) Global minimum variance with inequality constraints; d) Maximum Sharpe ratio; and e) Maximum Sharpe ratio with inequality constraints.[3] Additionally, a comparison is also made with a benchmark for Brazil's equity market. In order to achieve this paper's goal, three basic procedures were implemented:

1. For the 72 stocks, estimates of mean returns, covariances and standard deviations were obtained based on data available daily for the previous two years up to the last trading day of the month of reference.

2. Solutions are constructed for the alternative portfolio strategies using the estimates obtained in Stage 1.

3. For each portfolio selection model, out-of-sample returns were computed using data for the following month. That is, effective returns were obtained for the 72 stocks and these data were used to calculate the actual performance of the portfolio – from the perspective of an investor, these are the gains or losses that would occur by applying a given model.

Procedures 1, 2, and 3 were repeated for each month from January 2017 onwards through a "rolling window", and this scheme provided out-of-sample results for 82 months.[4] In the models based on the Sharpe ratio, it was used the reference rate for Brazil's Treasury bonds (Selic) for the riskless interest rate.

All computations were performed in the software environment R, using several R financial functions available in the package fPortfolio that was developed by Rmetrics (Würtz, Setz, Chalabi, Chen & Ellis, 2015).

5. Analysis of the Results

5.1 Out-of-Sample Performances of Alternative Strategies

As mentioned above, the Ibovespa was used as the benchmark for Brazil's equity market. The use of weights based on broad indices like the Ibovespa is perhaps the most commom procedure that investors apply for portfolio allocation in equities – a solution that unfolds from the established CAPM model. Figure 1 gives a general perspective of the changes in this index in the 82 months for which sample data were used in the present study. As can be perceived in the Figure, there were no substantial changes in this benchmark during the research period.

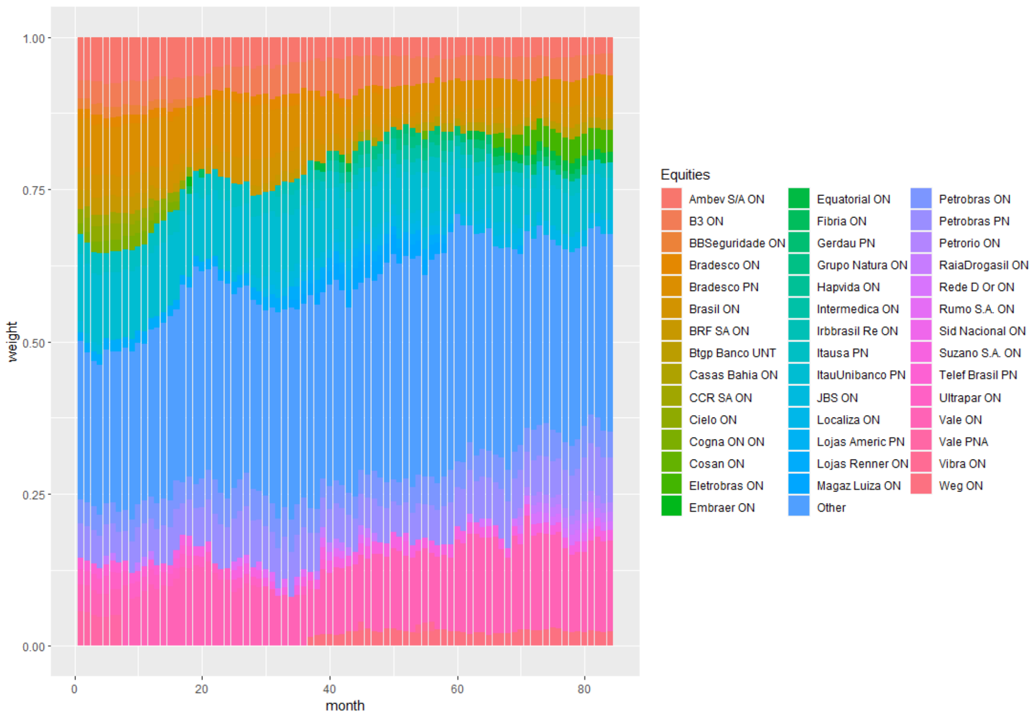

Figure 1. Ibovespa index: Equities with greater weights

[3] The Sharpe ratio is not being considered here as an ex-post indicator of portfolio performance. Instead, the portfolio's ex-ante data are used in the traditional formula.

[4] It is important to emphasize that the procedure in Stage 3 creates a "real world" situation, in the sense that it simulates what would effectively happen to values invested in the portfolios with a one-month maturity.

From the perspective of an investor in equity markets, the portfolios derived from the optimization models with inequality constraints – non-negative restrictions and maximum portfolio weights – certainly seem feasible and not too difficult to implement. This important aspect, from a practical standpoint, is illustrated in Figures 2 (global minimum variance) and 3 (maximum Sharpe ratio). Further, these Figures also reveal that, as intuition would suggest, the maximum Sharpe ratio portfolio is much more diversified and subjected to greater changes in its weights.

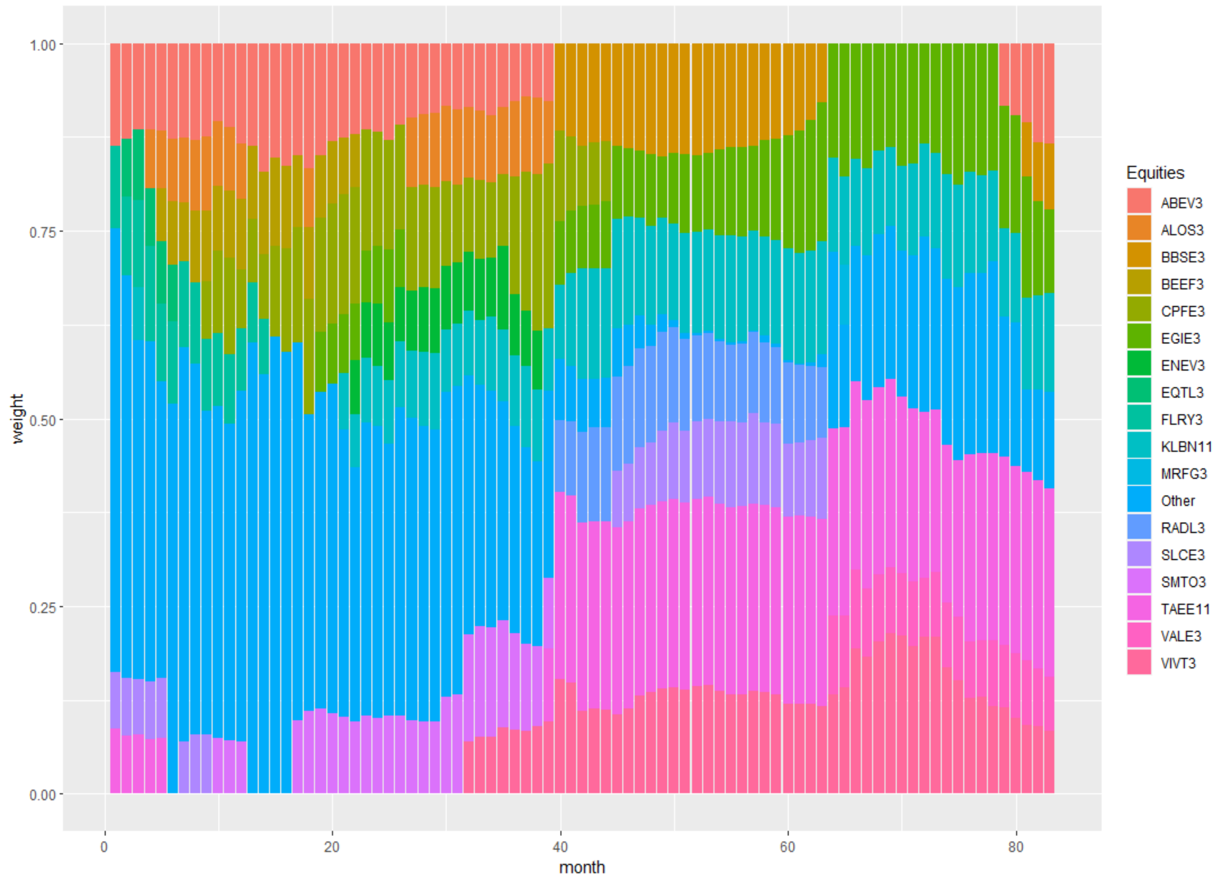

Figure 2. Global minimum variance portfolios with inequality constraints: Equities with greater weights

Note: Optimal solutions with non-negativity and maximum-weight inequalities.

Furthermore, Figure 4 and Table 1 illustrate the results that should be considered the most relevant given that they both are based on out-of-sample data that show the effectiveness of alternative portfolios – that is, the gains and losses that would occur from a “real-world” investment strategy established at the beginning of each month.

In Figure 4, only returns are considered and, in relation to this variable, the huge disparities in the portfolio's performances are evident. It is especially noteworthy that one of the most common allocation strategies – perhaps the most common – based on passive investment in a benchmark portfolio has shown considerably lower results. In particular, it can be seen that the global minimum-variance portfolios were more successful in terms of cumulative return than the Ibovespa. In the case of models that maximize the Sharpe ratio, the more favorable performances in terms of cumulative returns strike out. Also, the very positive performance of the strategy based on equal weights should be highlighted.

Table 1, in turn, includes indicators for both average return and portfolio risk, as well as the ratio that combines them. Based on out-of-sample data, it can be seen that global minimum-variance portfolios have indeed met this target – they have the lowest risk. In addition, models that maximize the ex-ante Sharpe ratio, effectively obtained the best performance from ex-post data based on the mean return and risk of the portfolios. These very favorable results of the solutions from Markowitz’s portfolio analysis are in distinct contrast with a handful of published researches that point out to problems resulting from the presence of estimation risk.

By far the worst result in terms of the Sharpe ratio is that of the Ibovespa, which was outperformed by the equal weights and global minimum-variance portfolios. Additionally, as an example of the comparison between alternative strategies, Figure 5 presents the monthly frequencies for the best and worst portfolios in terms of the Sharpe ratio.

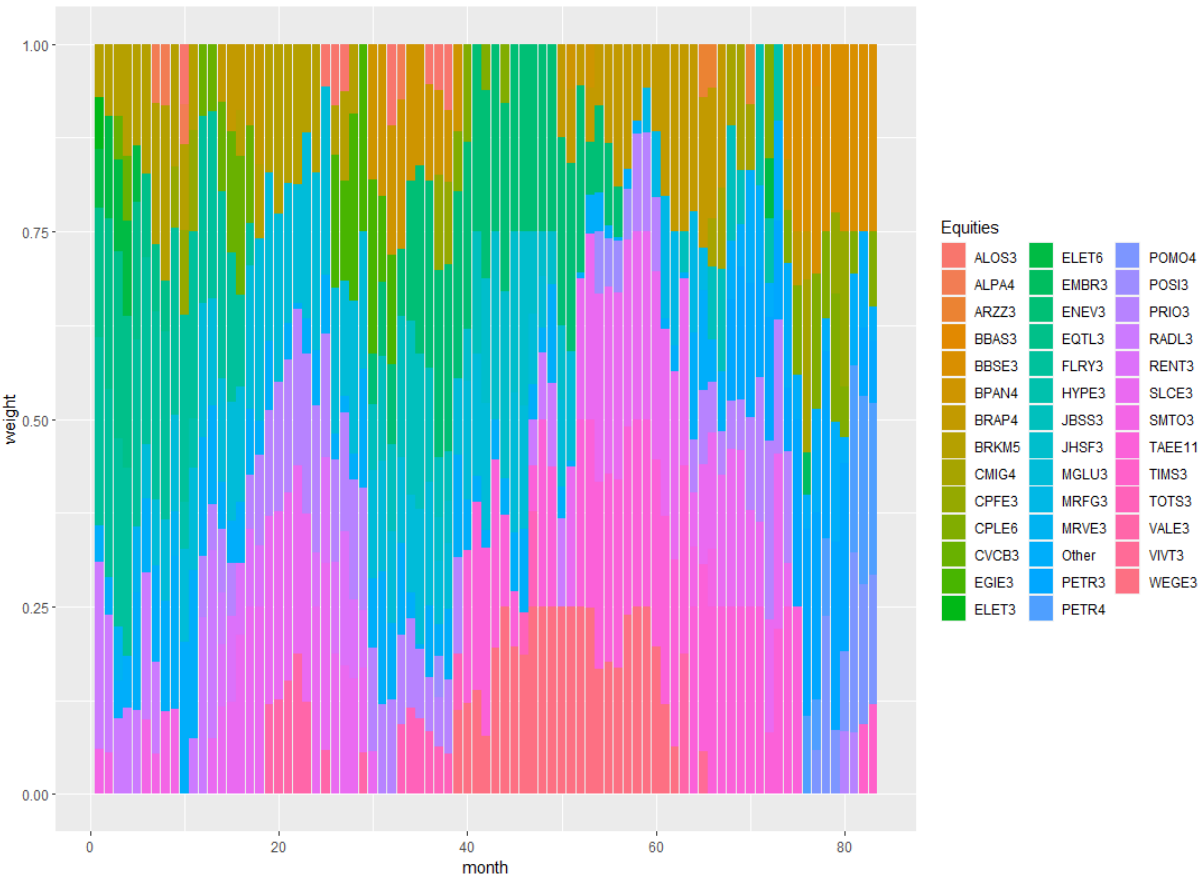

Figure 3. Maximum Sharpe ratio portfolios with inequality constraints: Equities with greater weights

Note: Optimal solutions with non-negativity and maximum-weight inequalities.

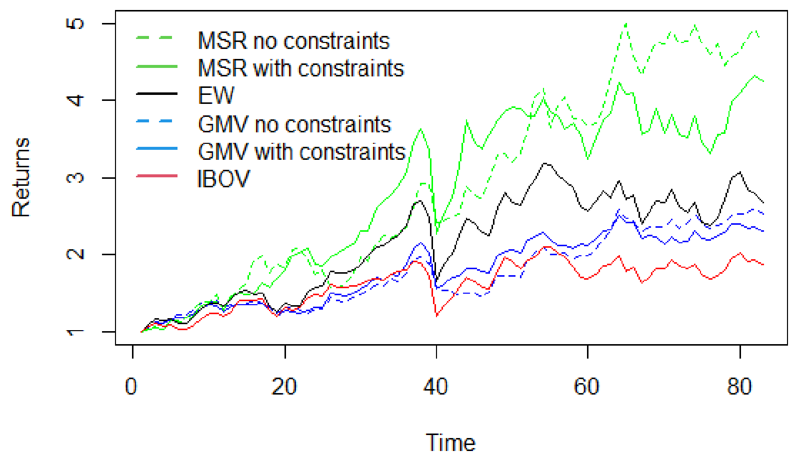

Figure 4. Accumulated returns: Out-of-sample data

Notes: Monthly returns from Jan. 2017 to Oct. 2023. MSR – Max. Sharpe ratio; EW – Equal weights (naïve); GMV – Global min. variance.

Table 1. Out-of-sample portfolio performance: Monthly returns

	Ibovespa	Equal Weights	Global Minimum Variance (GMV)		Maximum Sharpe Ratio (MSR)	
			Unrestricted	Restricted	Unrestricted	Restricted
Average return	0.0099	0.0149	0.0126	0.0116	0.0216	0.0206
Acumulated return	1.8786	2.6706	2.5378	2.3131	4.7507	4.2455
Portfolio risk	0.0647	0.0733	0.0489	0.0500	0.0699	0.0737
Sharpe ratio	0.0415	0.1045	0.1100	0.0866	0.2057	0.1819

Note: In the maximum Sharpe ratio portfolios, the average risk-free monthly rate is 0.0072.

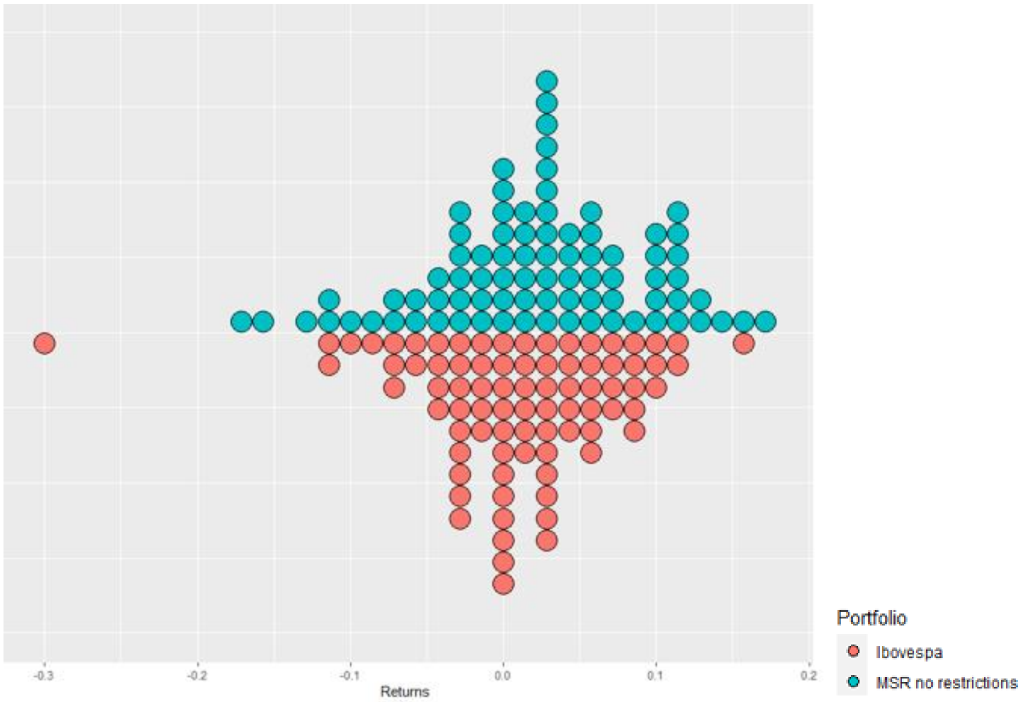

Figure 5. Comparative frequencies for portfolios with highest and lowest ex-post Sharpe ratios

5.2 Analysis of Sample Returns

The purpose of this Section is to evaluate whether the statistical analysis of the monthly returns available in this research is consistent with the common perception of the presence of estimation errors in the solutions based on Markowitz's optimization model. In particular, we sought to access whether the potential effects of this type of problem could be captured in the sample distribution of returns and whether, in comparison with portfolios with equal weights, which are not affected by estimation risk, we could perceive in the other portfolios a more acute presence of asymmetry and kurtosis. In addition, we evaluate whether these distributions can be considered approximately normal. Table 2 contains the key statistics for the portfolios examined.

As it can be perceived from the information in Table 2, and contrary to the commonly accepted view, the returns of portfolios derived from mean-variance analysis do not present a graver problem of asymmetry and kurtosis. In fact, the opposite is true, that is, the unrestricted optimized solutions have the most favorable indicators and, even more importantly, according to the Jarque-Bera statistical test, the unrestricted maximum-Sharpe-ratio portfolio is the only one for which the assumption of normality cannot be rejected.

Table 2. Sample statistics: Monthly returns

	Ibovespa	Equal Weights	Global Min. Variance (GMV)		Max. Sharpe Ratio (MSR)	
			Unrestricted	Restricted	Unrestricted	Restricted
Mean	0.009913	0.014887	0.012602	0.011553	0.021610	0.020625
Median	0.008577	0.018694	0.009705	0.008958	0.026153	0.024715
Minimum	–0.299043	–0,333991	–0.178126	–0.225065	–0.175900	–0.323878
Maximum	0.159027	0.159517	0.146255	0.131861	0.173239	0.184228
Std. Deviation	0.064707	0.073314	0.048877	0.049994	0.069941	0.073667
Std. Error	0.007146	0.008096	0.005398	0.005521	0.007724	0.008135
Skewness	–1.253510	–1.265025	–0.229562	–1.048348	–0.416481	–1.202172
Kurtosis	7.954595	7.713524	5.323047	8.042968	3.169955	7.479945
Jarque-Bera	105.347	97.7797	19.1584	101.911	2.46926	88.3235
Probability	0.000000	0.000000	0.000069	0.000000	0.290942	0.000000

From a different perspective, the information available in Figures 6 and 7 complement the previous results. In these Figures, it is possible to identify the strong impact of the month most affected by the covid epidemic on the sample distributions. As can be seen in the histograms in Figure 6, the portfolio obtained from the solution of the unrestricted maximum Sharpe ratio was the least affected by these adverse effects. On the other hand, the Q-Q plots in Figure 7 indicate that, if this strongly negative period were discarded, the equal-weight portfolio and the solution of the restricted maximum Sharpe ratio would be those closest to the pattern of the normal distribution.

In summary, it seems appropriate to conclude that what this research fundamentally shows is that the data available for the Brazilian stock market point to the superiority of portfolios based on Markowitz's mean-variance analysis, especially in the case of maximum Sharpe-ratio portfolios.

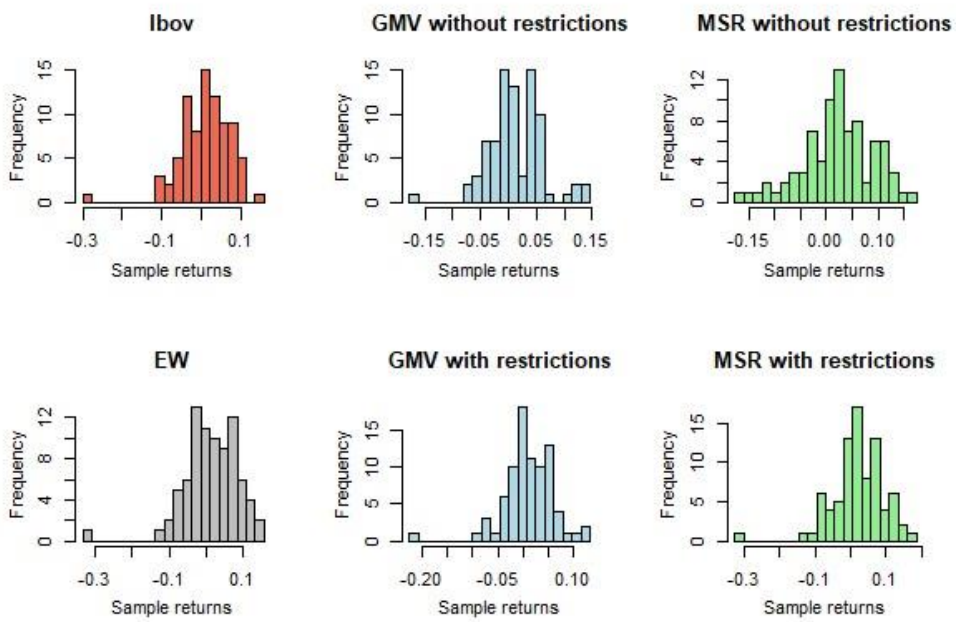

Figure 6. Comparative histograms for portfolio strategies

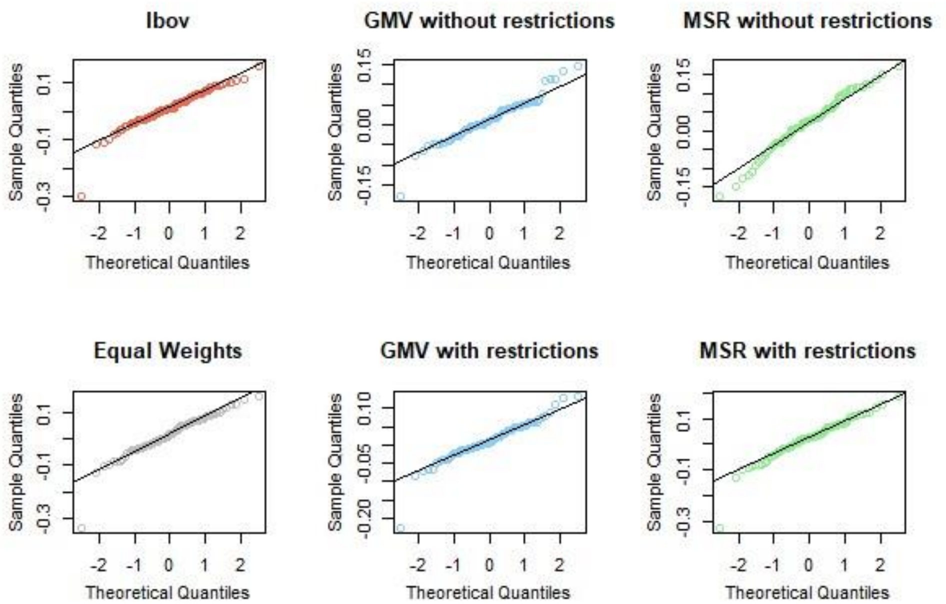

Figure 7. Comparative Q-Q plots for portfolio strategies

6. Conclusions

It is probably appropriate to affirm that, in the available literature on comparative analysis of portfolio models, emphasis continues to be placed on traditional analytical systems and, especially, on Markowitz's mean-variance model. Following this general trend, five models – four of them based on optimized solutions – and one benchmark index (Ibovespa) were contrasted and compared in this research using data from Brazilian stock market over a relatively long period of time. It should also be mentioned that quite a few published studies have pointed out that traditional optimization models are affected by errors in parameter estimation, especially in the case of expected returns. In addition, this perceived deficiency would be one of the factors explaining that, in some applied studies, mean-variance models underperform the commonly used (naïve) strategy of assigning equal weights to a relatively large number of assets.

In this study, the sample includes only the stocks that were traded in the entire research period and, at the same time, were part of the Ibovespa in at least one edition during this interval – the total number of equities was 72. For each portfolio selection model, out-of-sample returns were computed using data for the following month after the period that was used in the estimation and solution. In this way, effective returns were generated for alternative portfolios. When only portfolio returns are considered, it can be seen that the global minimum-variance portfolios were more successful in terms of cumulative return than the Ibovespa. In the case of models that maximize the Sharpe ratio, an even more favorable performance in cumulative returns was obtained. Also, according to both accumulated returns and the Sharpe ratio, the positive performance of the strategy based on equal weights might be highlighted (Table 1).

Considering indicators for portfolio’s return and risk, and especially the Sharpe ratio, it can be seen that, based on out-of-sample data, global minimum-variance portfolios have indeed met this target – they have the lowest risk. In addition, models that maximize the ex-ante Sharpe ratio effectively obtained the best results from ex-post data. By far the worst result in terms of the Sharpe ratio is that of the Ibovespa, which was outperformed by the naïve and global minimum-variance portfolios. Further, another aspect that deserves mention is that, considering the Sharpe-ratio results, the differences between optimization with and without restrictions were not very large. One additional conclusion is that out-of-sample returns derived from mean-variance analysis do not present a graver problem of asymmetry and kurtosis in comparison with equal weights’ portfolios. In fact, the opposite is true, and according to the Jarque-Bera statistical test, the unrestricted maximum-Sharpe-ratio portfolio is the only one for which the assumption of normality cannot be rejected.

Therefore, it seems adequate to conclude that this research essentially shows that, taking into account indicators of both risk and return, available data for the Brazilian stock market point to the superiority of portfolios based on Markowitz's mean-variance analysis, especially in the case of maximum Sharpe-ratio portfolios.

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Authors' contributions

The three authors were responsible and contributed equally for study design, revising and data collection.

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No additional data are available.

References

Black, F., & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*, *48*(5), 28-43. https://doi.org/10.2469/faj.v48.n5.28

Braga, M. D. (2015). Risk parity versus other μ-free strategies: a comparison in a triple view. *Investment Management and Financial Innovations*, *12*(1-2), 277-289.

Caldeira, J. F.; Moura, G. V., Perlin, M. S., & Santos, A. P. (2017). Portfolio management using realized covariances: Evidence from Brazil. *EconomiA*, *18*(3), 328-343. https://doi.org/10.1016/j.econ.2017.04.002

Da Fonseca, M. A. R. (2003). *Álgebra Linear Aplicada a Finanças. Economia e Econometria*. Barueri, SP: Manole.

De Carvalho, R. L., Lu, X., & Moulin, P. (2012). Demystifying Equity Risk–Based Strategies: A Simple Alpha plus Beta Description. *The Journal of Portfolio Management*, Spring issue, 56-70.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? *The Review of Financial Studies*, *22*(5), 1915-53.

Deng, G., Dulaney, T., McCann, C., & Wang, O. (2013). Robust Portfolio Optimization with Value-At-Risk Adjusted Sharpe Ratios. *Journal of Asset Management*, *14*, 293-305. https://doi.org/10.2139/ssrn.2146219

Dolinar, D., Zoričić, D., & Kožul, A. (2017). Towards the Estimation of an Efficient Benchmark Portfolio: The Case of Croatian Emerging Market. *Zagreb International Review of Economics & Business*, *20*(Special Conference Issue), 13-23.

Farias, C. A., Vieira, W. C., & Dos Santos, M. L. (2006). Portfolio Selection Models: Comparative Analysis and Applications to the Brazilian Stock Market. *Revista de Economia e Agronegócio*, *4*(3), 387-408.

Gárciga-Otero, R. (2011). *Matemática: Apostila para MBA em Finanças*. School of Economics, Federal University of Rio de Janeiro.

Gibbons, M. R., Ross, S. A. & Shanken, J. (1989). A Test for the Efficiency of a Given Portfolio. *Econometrica*, *57*(5), 1121-1152. https://doi.org/10.2307/1913625

Grinold, R. C. (1992). Are Benchmark Portfolios Efficient? *The Journal of Portfolio Management*, *19*(1), 34-40. https://doi.org/10.3905/jpm.1992.34

Haugen, R. A., & Baker, N. L. (1991). The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. *The Journal of Portfolio Management*, *17*(3), 35-40. https://doi.org/10.3905/jpm.1991.409335

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1), 77-91.

Michaud, R. O. (1989). The Markowitz Optimization Enigma: Is 'Optimized' Optimal? *Financial Analysts Journal*, Jan.-Feb., 31-42. https://doi.org/10.2469/cp.v1989.n4.6

Miranda, M. J., & Fackler, P. L. (2002). *Applied Computational Economics and Finance*. Cambridge, MA: MIT Press.

Murty, K. G. (1988). *Linear Complementarity, Linear and Nonlinear Programming*. Berlin: Heldermann Verlag. Retrieved from https://public.websites.umich.edu/~murty/books/linear_complementarity_webbook/

Naibert, P. F., & Caldeira, J. (2015). Selection of optimal portfolios under norm constraints in the allocation vectors: an empirical evaluation with data from BM&FBovespa. *Revista Brasileira de Finanças*, *13*(3), 504-43.

Santos, A. A. P., & Tessari, C. (2012). Quantitative Portfolio Optimization Techniques Applied to the Brazilian Stock Market. *Revista Brasileira de Finanças*, *10*(3), 369-94.

Simon, C. P., & Blume, L. (1994). *Mathematics for Economists*. New York, NY: W. W. Norton.

Theron, L., & van Vuuren, G. (2018). The maximum diversification investment strategy: A portfolio performance comparison. *Cogent Economics & Finance*, *6*(1), 1-16. https://doi.org/10.1080/23322039.2018.1427533

Vanini, P., & Vignola, L. (2001). *Markowitz Model*. Available at: https://www.bm.com.tn/ckeditor/files/markowitz.pdf

Würtz, D., Setz, T., Chalabi, Y., Chen, W., & Ellis, A. (2015). *Portfolio Optimization with R/Rmetrics*. Zurich: Rmetrics Association and Finance Online.

Zakamouline, V., & Koekebakke, S. (2009). Portfolio Performance Evaluation with Generalized Sharpe Ratios: Beyond the Mean and Variance. *Journal of Banking and Finance*, *33*(7), 1242-54. https://doi.org/10.2139/ssrn.1028715

Zanini, F. A. M., & Figueiredo, A. C. (2005). As Teorias de Carteira de Markowitz e de Sharpe: Uma Aplicação no Mercado Brasileiro de Ações entre Julho/95 e Junho/2000. *Revista de Administração Mackenzie*, *6*(2), 37-64.

Appendix: Sample statistical indicators

Daily returns (%), Jan. 2015 – Oct. 2023

Equities	Min.	Max.	Mean	St.Dev.	Skew.	Kurt.
ABEV3	-8.649	8.17545	0.02546	1.38147	-0.0935	3.33286
ALOS3	-7.7601	9.69873	0.08828	1.76708	0.57509	2.65997
ALPA4	-11.676	13.815	0.14524	2.18915	0.1183	2.68745
ARZZ3	-11.778	8.29469	0.08133	2.27285	-0.1298	1.67649
B3SA3	-8.8165	9.63918	0.13258	2.07654	0.10257	1.21475
BBAS3	-23.789	13.4291	0.08189	2.80685	-0.477	7.999
BBDC3	-13.926	11.4074	0.0689	2.0192	-0.0395	2.84419
BBDC4	-14.056	12.2462	0.07196	2.06041	-0.0123	3.3952
BBSE3	-10.786	10.3523	0.03981	1.96205	-0.0402	2.57612
BEEF3	-8.2917	8.59033	0.00788	2.25774	0.15861	1.38153
BPAN4	-12.558	20.958	0.11887	3.16932	1.30138	7.08235
BRAP4	-28.085	15.3775	0.09719	3.12091	-0.4495	5.97215
BRFS3	-21.999	11.5806	-0.0442	2.2051	-0.5165	9.57721
BRKM5	-22.042	19.3927	0.05865	2.63072	-0.1403	10.0019
CCRO3	-15.415	10.5884	0.02762	2.31923	-0.2401	2.63373
CIEL3	-10.257	14.264	-0.0561	2.30788	0.35323	2.81565
CMIG4	-23.639	16.3848	0.02896	2.90892	-0.4499	7.77956
COGN3	-16.498	13.5223	-0.0222	2.93037	-0.0546	2.09887
CPFE3	-18.597	8.58726	0.06076	1.68999	-0.9601	14.2281
CPLE6	-13.477	8.98062	0.07762	2.40199	-0.2392	2.20887
CSAN3	-10.477	10.8449	0.07718	2.15076	-0.106	1.40418
CSNA3	-22.951	18.7511	0.05717	3.97169	0.19068	2.70897
CVCB3	-15.502	8.86188	0.09623	2.3182	-0.4173	3.71493
CYRE3	-17.749	11.0435	0.09593	2.27002	-0.2989	4.20235
DXCO3	-14.133	12.2486	0.07242	2.36826	0.1983	2.21913
ECOR3	-16.002	8.79688	0.06181	2.55369	-0.129	1.63166

EGIE3	-7.9922	6.37632	0.07577	1.46281	0.01155	2.12385
ELET3	-23.534	40.0759	0.14703	3.54509	1.25805	15.3772
ELET6	-18.587	27.8243	0.12425	3.1679	0.72171	6.85489
EMBR3	-16.783	20.2928	-0.0098	2.13957	-0.2407	13.0941
ENEV3	-44.183	28.7682	0.005	4.12125	-1.5359	24.8136
EQTL3	-6.4386	7.139	0.12637	1.48138	-0.0867	1.00619
EVEN3	-20.489	11.1005	0.09886	2.61566	-0.2088	3.69234
EZTC3	-13.777	10.2523	0.12543	2.32089	0.00975	1.48801
FLRY3	-7.0261	10.5318	0.12229	1.88372	0.20233	1.10403
GFSA3	-14.64	19.5131	-0.0551	3.18372	0.34078	2.57026
GGBR4	-12.81	14.9188	0.05133	3.03531	0.11576	1.54651
GOAU4	-20.955	16.2707	-0.0255	3.50688	-0.1712	3.34232
GOLL4	-22.444	40.7641	0.08786	4.49217	1.11252	10.3657
HYPE3	-15.383	19.18	0.07304	1.79186	0.4442	15.1094
ITSA4	-10.129	9.77579	0.08176	1.86249	-0.0672	2.15389
ITUB4	-12.836	10.3684	0.07263	1.90775	-0.0678	3.24672
JBSS3	-37.605	20.3245	0.07325	3.23634	-0.7193	18.3953
JHSF3	-19.807	25.4234	0.07358	3.22362	0.78116	5.22312
KLBN11	-6.6559	6.31171	0.04616	1.71487	0.0944	0.70082
LREN3	-8.0651	9.34271	0.12862	1.97652	0.17355	0.83232
MGLU3	-17.751	31.6912	0.30684	3.99687	0.95732	8.30178
MRFG3	-10.583	17.2358	0.04276	2.63692	0.6363	3.53496
MRVE3	-8.6136	12.1949	0.11649	2.17324	0.18969	1.33659
MULT3	-13.293	6.69031	0.0666	1.80422	-0.1001	2.80589
NTCO3	-12.506	13.6711	0.06895	2.46927	0.35952	2.73693
PETR3	-16.154	14.9662	0.0712	3.1187	-0.021	3.01135
PETR4	-17.149	15.0858	0.06752	3.19845	-0.1391	3.33562
POMO4	-10.178	13.6361	0.01981	2.60887	0.29573	1.71455
POSI3	-16.252	31.3483	0.12715	3.33236	0.97549	9.45775
PRIO3	-26.358	60.7989	0.15916	4.5917	2.47063	30.8367
QUAL3	-34.77	31.2153	0.0705	2.87051	-0.763	30.3462
RADL3	-6.636	8.84554	0.1213	1.83815	0.24332	0.96868
RENT3	-7.3285	8.02942	0.12929	2.23208	0.04611	0.58522
SANB11	-11.819	8.45574	0.12059	2.121	-0.1107	1.77555
SBSP3	-12.386	10.4021	0.10088	2.25769	-0.2811	2.41951
SLCE3	-9.323	9.30265	0.11032	2.26761	0.08426	1.39214
SMTO3	-9.6412	9.68997	0.05936	1.79218	0.09813	2.33309
TAEE11	-8.674	8.37174	0.08679	1.5851	-0.1759	1.66165
TIMS3	-8.9029	10.7098	0.02354	2.01964	0.01881	1.54117
TOTS3	-7.4662	8.27476	0.05799	2.05971	0.0702	1.052
UGPA3	-10.717	8.32473	0.00931	1.86238	-0.1859	2.51572
USIM5	-17.598	30.0892	0.04546	3.94261	0.47645	4.91697
VALE3	-28.135	13.7685	0.07905	3.10285	-0.5497	6.75426
VIVT3	-10.831	8.72521	0.04321	1.8545	-0.0148	2.28373
WEGE3	-9.5453	6.85752	0.09657	1.76426	-0.127	1.06274
YDUQ3	-16.434	21.2984	0.06069	3.09667	0.02274	3.37932