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Effects of Value Add Tax on Consumption in Developing Countries

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Abstract

The most widespread economic problem today seems to be an abrupt decline in GDP and deep recession. Consumption is one of the most important elements constituting GDP, whose growth leads to GDP growth and thus the economic growth. In this study, the effect of value added tax on consumption has been examined especially on the developing countries. In details, the effects of VAT on the consumption of 19 developing countries for duration of 1995 to 2010 were investigated. At first, VAT was incorporated the consumption function and its impact on the consumption and consequently the saving was explored because consumption and saving are interrelated. For analysing the data, the GMM panel was employed because of the structure of the model. The results revealed that VAT, with a lag, influences the consumption negatively while this finding is consistent with the research literature background as well as Duesenberry theory and the consumers' consumption habits.

Keywords: value added tax (VAT), consumption, economic growth, developing countries, generalized method of moments (GMM).

1. Introducation

As studied by Ghaffarian Kolahi and Zaleha Bt Mohd Noor (2016), VAT is administrated normally by employing the credit mechanism and that VAT is based on the destination principle. The credit mechanism let the sellers to claim credit for any VAT they pay while buying inputs required for producing the goods or services they sell. The sellers are allowed to redeem those VAT credits against any VAT that they are responsible to pay while selling the goods or services. Those VAT credits are claimed by the sellers on condition that they grant invoices for the VAT they had paid on their inputs. In this way, tax evasion is discouraged by VAT as the taxpayers themselves incline to pay and take receipts for the VAT paid on input purchased for allowing them to claim credit against the VAT they themselves need to pay at the point of selling their end product or service. It is possible for the taxpayers themselves to give checks against one another due to the fact that the receipts are required at both ends of the transaction. This method is not commonly used for other types of general sales taxes including turnover tax and retail sales taxes (Ghaffarian Kolahi & Zaleha Bt Mohd Noor, 2016).

It is possible for VAT to boost savings by declining the consumption as savings are exempted from the tax base, implying that the after-tax rate of return savings could be greater (Fritz, et al. 1997).

Furthermore, an intertemporal effect could be gained from the VAT while such an effectcould influencethe relative prices of the current consumption vs. future consumption. This denotes that substitution and income effects will be generated through the tax shock. Nonetheless, the net impact of these two forces on the level of aggregate private consumption is theoretically ambiguous (Ghaffarian Kolahi & Zaleha Bt Mohd Noor, 2016).

It needs to be accentuated that lack of sufficient empirical studies particularly in developing countries has baffled the diagnosis of the way the VAT influences the consumption which per se influences the savings and capital accumulation and growth accordingly.

2. Method

2.1 Theoretical Model

Employing the life cycle hypothesis (LCH), an aggregate consumption function was derived. In fact, a model was developed by Ando and Modigliani (1963) in which the literature was expanded as the consumption tax which itself was an additional factor for measuring the outcome had been incorporated to the model. At the present, it can be highlighted that the model utilised in this research is an application of the mentioned model of Ando and Modigliani (1963). Consumption decisions are envisioned in LCH as integrated in an intertemporal optimization for a representative consumer. To sum up, the income stream of the households allotted over their lifetime is anticipated by LCH. For this reason, a consumption path over their lifetime is determined by the agents in order to maximize their intertemporal lifetime utility function. A subjective discount rate is employed to measure intertemporal lifetime utility function which is dependent on an intertemporal budget constraint controlled by a discounted stream of future income and the wealth they own by birth (Ando and Modigliani 1963).

In the selected model, there are various assumptions that need to be considered:

1) There is no restriction for the individuals to access capital markets.

2) All individual works from i = 1 to i = R and retire from i = (R+1) to i = J.

3) All workers gain the same real wage at any point in time.

4) The government has a balanced budget (cash flow budge constraint).

5) Consumption is a normal good over all the periods.

6) Utility is derived from consumption over all the periods.

7) Individuals' utility function is concave and time-invariant for assuring smoothness of consumption in both periods, that is, U(.)' > 0 and U(.)'' < 0.

8) The tax is imposed on consumption over all the periods.

Following, It is believed, close to the ideas presented by Lewis and Siedman (1999) and (Bakhshi 2000) , that each person would maximize his/her lifetime utility if they select the stream of consumption from $(c_1, c_2, \ldots., c_j)$ subject to his/her lifetime budget constraint as follows:

$$\mathrm{Max}(c_t) \quad U = \sum_{i=0}^{J} \frac{U(c_{t+i})}{(1+\gamma)^i} \tag{1}$$

Where:
c_{t+i}: Consumption in different periods.
J: Certain date of death.
γ: Subjective discount rate.
i: time index.
s.t.

$$\sum_{i=0}^{J} \left(\frac{c_{t+i}}{1-\psi_{t+i}}\right)\left(\frac{1}{1+r}\right)^i = p_t + Z\sum_{i=0}^{R} w_{t+i}\left(\frac{1}{1+r}\right)^i \tag{2}$$

$$Z = \begin{cases} 1 & \text{if } t \le R \\ 0 & \text{if } t \ge (R+1) \end{cases}$$

Where:

ψ_{t+i}: Consumption tax rate in different periods, which equals$\left(T^c_{t+i}/c_{t+i}\right)$,

Where:

T^c_{t+i} : represents tax revenues from the consumption tax in different periods and c_{t+i} represents the tax base which is consumption in different periods (Bakhshi 2000, Lewis and Siedman 1999). Therefore, the tax rate represents the effective rate.

r: real interest rate
p_t: individual's initial holdings of wealth or assets.

w_{t+i}: wage rate in different periods which grows at rate g $\Longrightarrow \mathrm{w_t = w_0(1+g)^t}$

The first-order conditions (F.O.Cs) of the maximization problem are given by:

$$\mathrm{U'(c_{t+i})(1-\psi_{t+i}) = \lambda\left(\frac{1+\gamma}{1+r}\right)^i} \tag{3}$$

Therefore,

$$\mathrm{if\ i = 0 \Longrightarrow U'(c_t)(1-\psi_t) = \lambda} \tag{4}$$

Equation (4) is inserted in (3) for obtaining the Euler equation as follow:

$$\mathrm{\frac{U'(c_{t+i})}{U'(c_t)} = \left(\frac{1-\psi_t}{1-\psi_{t+i}}\right)\left(\frac{1+\gamma}{1+r}\right)^i} \tag{5}$$

At this stage, a particular form is assumed for the utility function in order to enable us to get a closed-form solution. In this research, a CES or iso-elastic utility function will be employed, similar to, to serve our purpose:

$$U(c_t) = \begin{cases} \frac{c_t^{1-\sigma}}{1-\sigma} & if\sigma \neq 1 \\ ln\, c_t & if\ \sigma=1 \end{cases} \tag{6}$$

Where $\sigma = \frac{1}{\rho}$, and ρ is the intertemporal elasticity of substitution.

Moreover, equation (6) is put in (5), and some algebraic manipulation is added; therefore, the following equation will be obtained:

$$\mathrm{\frac{U'(c_{t+i})}{U'(c_t)} = \left(\frac{c_t}{c_{t+i}}\right)^{\sigma} = \left(\frac{1-\psi_t}{1-\psi_{t+i}}\right)\left(\frac{1+\gamma}{1+r}\right)^i} \tag{7}$$

Moreover, equation (7) is inserted into the budget constraint (2) for c_{t+i} to obtain the following (Bakhshi 2000):

$$\mathrm{\sum_{i=0}^{J}\left(\frac{c_t}{1-\psi_{t+i}}\right)\left(\frac{1-\psi_{t+i}}{1-\psi_t}\right)^{\frac{1}{\sigma}}\left(\frac{1+r}{1+\gamma}\right)^{\frac{i}{\sigma}}\left(\frac{1}{1+r}\right)^i = p_t + w_t}$$

Where: w (human wealth) =Z $\mathrm{\sum_{i=0}^{R} w_{t+i}\left[\frac{1}{1+r}\right]}$, and $\mathrm{(p_t + w_t)}$ = total lifetime wealth.

Therefore:

$$c_t = \frac{1}{\beta}(\mathrm{p}_t + w_t) \tag{8}$$

Where β is a proportionality factor which is a function of:

- the interest rate,
- utility function parameters, σ and γ ,
- the length of his/her life,
- consumption tax rates, that is:

$$\mathrm{\beta = \left(\frac{(1-\psi_{t+i})^{1-\sigma}}{(1-\psi_t)}\right)^{\frac{1}{\sigma}}\left(\frac{1+r}{1+\gamma}\right)^{\frac{i}{\sigma}}\left(\frac{1}{1+r}\right)^i}$$

It is asserted hereby that the Equation (8) resembles the equation found by Ando and Modigliani (1963), where $(1\ /\ \beta)$ is equivalent to Ω in their notation.

Equation (8) will be interpreted as representing aggregate consumption function when the population has a particular age and income distribution under the assumption that every consumer behaves similar to a representative one at a particular economic age; hence, our aggregate consumption function will be obtained by:

$$C_t = \frac{1}{\beta}(P_t + W_t) \tag{9}$$

For isolating the impact of a change in consumption tax rate on consumption, at this point, a comparative static analysis will be conducted. We need to concentrate on a tax shock between periods (t) and (t+1) which is generalizable to any number of periods. This is due to the fact that a uniform characterization of the Euler equations for any two periods is needed by optimality conditions (Sqrensen 2005).

Bearing in minding the F.O.Cs, (assuming that tax rates are not equal in the two periods):

$$c_t: U_t - \lambda(1 + \psi_t) = 0 \tag{10}$$

$$c_{t+1}: U_{t+1} - \lambda\left(\frac{(1+\psi_{t+1})}{(1+r)}\right) = 0 \tag{11}$$

$$\lambda: p_t + Z\sum_{i=0}^{R}\left(\frac{w_{t+i}}{(1+r)^i}\right) - \sum_{i=0}^{J}(c_{t+i})\left(\frac{1+\psi_{t+i}}{(1+r)^i}\right) = 0 \tag{12}$$

Where (U_t) refers to the first derivative of the utility function with respect to period's *(t),* consumption (c_t) , or $U_t = \frac{\partial U}{\partial c_t}$, and similarly, $U_{t+1} = \frac{\partial U}{\partial c_{t+1}}$.

A brief mathematical derivation from totally differentiating F.O.Cs is provided in Appendix.

Regarding equation (A.6) if it's supposed $(d\psi_{t+1} = 0)$ it would be obtained:

$$\frac{dc_t}{d\psi_t} = \left(c_t\left[(1 + \psi_t)U_{t+1,t+1} - \frac{(1+\psi_{t+1})}{(1+r)}U_{t,t+1}\right] - \frac{(1+\psi_{t+1})^2\lambda}{(1+r)^2}\right)|D|^{-1} \tag{13}$$

And if it's supposed$(d\psi_t = 0)$:

$$\frac{dc_t}{d\psi_{t+1}} = \left(\frac{(1+\psi_{t+1})}{(1+r)^2}\left[c_{t+1}\left\{\frac{(1+\psi_t)}{(1+r)}U_{t+1,t+1} - U_{t,t+1}\right\} + (1 + \psi_t)\lambda\right]\right)|D|^{-1} \tag{14}$$

The change effect in the current and future tax rates on the current consumption are demonstrated by the Equations (13) and (14), respectively. Nonetheless, the impact of current consumption taxes on current consumption is theoretically unclear because two competing effects are considered:

1) The first one is known as substitution effect given by the term $\left[\left(\frac{(1+\psi_{t+1})^2\lambda}{(1+r)^2}\right)|D|^{-1}\right]$. This effect constantly lessens the current consumption while the current tax rate boosts because by increasing the current tax rate, individuals predict that the future one will have a downward trend. Then, as current tax rates rises, lower levels of current consumption are resulted from the mentioned change in the relative price of consumption Today vs. Tomorrow.

2) Income effect is the second effect which initiates an ambiguous impact on the current consumption because of the fact that the sign of $(U_{t,t+1})$ is undetermined. If it is positive then the term

$\left(c_t\left[(1+\psi_t)U_{t+1,t+1}-\frac{(1+\psi_{t+1})}{(1+r)}U_{t,t+1}\right]|D|^{-1}\right)$ will be attained characterizing that the income effect has a negative sign.

Considering this, it is to be accentuated that both effects work in the same direction in order to enforce each other; hence, the current consumption will be decreased by an increase in the current consumption tax rate. Nonetheless, the sign of the term which represents the income effect is indeterminate when $(U_{t,t+1})$ is negative. Correspondingly, there is no clear indication of any overall impact of an increase in the current tax rate on the current consumption. If we seek to know the impact of future taxes increase on the current consumption, the substitution effect, given by the term$\left(\left[\frac{(1+\psi_{t+1})(1+\psi_t)\lambda}{(1+r)^2}\right]|D|^{-1}\right)$, will be positive concluding that the current consumption increases by the increase of the future tax rate. This assumption seems to be logical as it means that the current consumption is cheaper relative to the future one. Subsequently, it is expected to be positive (it is precisely opposed to the previous case).

As maintained earlier, because of the presence of the term $(U_{t,t+1})$, similar disputes are posed for the ambiguous impact of the income effect. In case of being positive, the income effect will exert a negative sign and the two forces will function in opposite directions. Conversely, the sign of the term representing the income effect is indeterminate when it is negative. Additionally, the total impact of an increase in the future taxes is theoretically ambiguous on the current consumption.

2.2 Econometric Model

Even as a number of factors relevant to the consumption decision were considered by the previously conducted empirical studies, they show a noticeable discrepancy in the data coverage, empirical specification, and econometric procedure.

Through providing a comprehensive characterization of the empirical association between the level of per capita private consumption and the VAT rate, therefore, we would attempt to extend the literature. For the purpose of this study, a reduced form linear equation will be utilised in which inclusion of a wide variety of consumption determinants is allowed. Then, our attention would be directed toward a set of regressors selected based on theoretical connection and analytical relevance; however, the empirical role of a number of less-standard consumption determinants would be also examined.

$$C_{i,t}=\eta C_{i,t-1}+\kappa' X_{i,t}+\mu_{i,t} \qquad |\eta|<1 \tag{15}$$

for $i=1,\dots,N$ and $t=2,\dots,T$

Where:

$C_{i,t}$: Observable dependent variable.

$X_{i,t}$: K x *1* vector of observable independent variables.

κ: K x *1* vector of parameters.

$\mu_{i,t}$: Random disturbance term satisfying the following assumptions:

$E(\mu_{i,t})=0,\ E(\mu_{i,t}^2)=\sigma_\mu^2,\ and\ E(\mu_{i,t},\mu_{j,s})=0\ \ if\ \ i\neq j\ and/or\ t\neq s$

Where:

N: Number of cross-sectional units (countries)

T: Number of time periods (years)

By following Karadag, M., & Westaway, T. (2000) and Bakhshi (2000), estimating regression equation is obtained.

$C_{i,t}=\alpha_0+\alpha_1 C_{i,t-1}+\alpha_2 VATR_t+\alpha_3 lnTT_t+\alpha_4 \ln Y_t+\alpha_6 lndm_T+\alpha_7 UR_t+\alpha_{10} In_t+\mu_{i,t}$

3. Results

The equation of this study has a dynamic nature. Also, one or more of the right hand side variables is/are endogenous. Then in this research, the necessity for an appropriate estimator that deals with these problems is central. The lagged value of the dependent variable in the dynamic model of this research which is a regression is an explanatory variable.

Allowing for dynamics in the underlying process is central to recover the consistent estimates of other parameters although there might be not even a direct tendency in the coefficient on lagged dependent variable (Bond 2002). Several issues must be dealt with through the estimation procedure. Firstly, we need to allow endogeneity of one or more explanatory variables. Moreover, the potential problem of measurement error in measures of the dependent and independent variables needs to be addressed which is believed to be an archetypal problem for macro-level data. Next, the problem of the omitted variable bias must be dealt with. Last of all, the possibility of simultaneity bias ought to be considered by the estimator of this research. The econometric technique in the current research is based on the generalized method of moments (GMM) estimators applied to dynamic models using panel data in order to enable the researcher to address these issues.

Table 1. Exhibits the results of the ultimate estimation using Stata software through xtabond2 method in which the problems have been alleviated.

Table1. Results of Consumption and Value Added Tax

Variables	Coefficient	Std. error	t	p-value
Ln C1	0.5859754	0.1432831	4.09	0.001
lnVAT	0.015567	0.0130502	1.19	0.248
L1	-0.014933	0.0068934	-2.17	0.043
lnY	0.820198	0.1367526	5.99	0
L1	-0.548224	0.1208898	-4.53	0
INTR	-0.000641	0.0003829	-1.68	0.101
L1	-0.000179	0.000416	-0.43	0.672
lnW	0.0096256	0.009475	1.02	0.322
L1	0.0220609	0.0187826	1.17	0.255
UR	-0.001298	0.0019492	-0.67	0.514
L1	-0.000334	0.0022811	-0.15	0.885
Yr-dv13	-0.006599	0.0038944	-1.69	0.106
Yr-dv14	-0.004913	0.0054795	-0.9	0.381
Year	-0.004342	0.0022298	-1.95	0.066

Furthermore, Table 2 reveals the results of the Estat abond test.

Table 2. Arellano-Bond test for zero autocorrelation in first-differenced errors

Ordere	Z	p-value
1	-2.67	0.008
2	1.47	0.142

H0: no autocorrelation

Furthermore, Table 3 demonstrates the results of the Estat sargn test.

Table 3. Sargan test of overidentifying restrictions

Chi2(167)	182.48
Prob > chi2	0.195

H0: overidentifying restrictions are valid

It is found out through the results shown in Tables 2 and 3 that our dynamic GMM estimates are normally supported by the specification tests. Yet, the sargan test of overidentifying restrictions did not reject the null hypothesis which assumed that the instruments were uncorrelated with the error term (p-value = 0.195). Correspondingly, the tests of serial correlation could reject the hypothesis which assumed that the error term is second-order serially correlated (p-value = 0.142). Therefore, further support is provided for using suitable lags of the explanatory variables as instruments for the estimation.

Prior to succinctly introducing the detailed results, their interpretations are to be interpreted. The aim behind our econometric methodology is to isolate the effect of the exogenous component of each of the explanatory variables on aggregate consumption. Accordingly, we would isolate the causal effects of the explanatory variables on aggregate private consumption to the degree that our assumptions would be precise concerning the instruments being used in the GMM procedure.

The specification tests introduced in our results provide a support for the validity of our instruments; therefore, they would let us make inferences on the subject of the connection between the exogenous component and the level of household private consumption. As a result, once the effect of a certain variable on consumption is considered, we practically emphasize the correlation between the exogenous component of that variable and consumption.

Considering the model used in our study and the variables to be examined in this research work, the results of the estimation for the effect of each dependent variable on per capita house hold final consumption is detailed as follows:

As demonstrated by the results, there is a statistically significant relation between the exogenous component of the first lag of lnVAT and the level of private aggregate consumption (at the 5 percent level). As expected, the direction of the relation was found to be negative. Also, the estimated VAT elasticity was 0.014, with a negative significant coefficient. To justify why VAT has an positive effect on the consumption in the level, we can point out to Duesenberry's theory accentuating that the consumer is worried more for his/her consumption in comparison with the consumption of the others rather than the absolute level of his/her own consumption. Moreover, according to the mentioned theory, the current consumption is not only affected by the current levels of absolute income and relative income, but also by the levels of consumption of the previous eras. Therefore, in the first year, VAT failed to have any significant effects on the consumption but in the subsequent year, the consumer reduced his/her consumption under the impact of VAT.

4. Discussion

It was identified that the VAT with a lag had a significant negative effect on consumption and this result fulfilled our expectation indeed. The reason of the lag according to the Duesenberry's theory is that the consumer is worried more for his/her consumption in comparison with the consumption of the others rather than the absolute level of his/her own consumption. Moreover, according to the mentioned theory, the current consumption is not only influenced by the current levels of absolute income and comparative income, but also by the levels of consumption of the previous eras. Therefore, in the first year, VAT failed to have any significant effects on the consumption but in the subsequent year, the consumer reduced his/her consumption under the impact of VAT. It needs to be asserted the average personal income tax rate could exert a significant negative impact on saving; on the other hand, the consumption tax rate exerted an insignificant impact. The empirical results have also substantiated that shifting income taxes to consumption taxes would promote saving (Zheng 2007).

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Appendix

This appendix provides a brief derivation from totally differentiating F.O.Cs (assuming that tax rates are not equal in the two periods and other variables are constant) as follows:

$$U_{t,t}dc_t + U_{t,t+1}\mathrm{dc}_{t+1} - (1+\psi_t)d\lambda - \lambda d\psi_t = 0 \tag{A.1}$$

$$\mathrm{U}_{t+1,t}\mathrm{dc}_t + \mathrm{U}_{t+1,t+1}\mathrm{dc}_{t+1} - \left(\frac{(1+\psi_{t+1})}{(1+r)}\right)\mathrm{d}\lambda - \frac{\lambda}{(1+r)}\mathrm{d}\psi_{t+1} = 0 \tag{A.2}$$

$$-(1+\psi_t)dc_t - \left(\frac{(1+\psi_{t+1})}{(1+r)}\right)dc_{t+1} - c_t d\psi_t - \left(\frac{(c_{t+1})}{(1+r)}\right)d\psi_{t+1} = 0 \tag{A.3}$$

Where: $\mathrm{U}_{t,t} = \frac{\partial^2 \mathrm{U}}{\partial c_t^2}$, $\mathrm{U}_{t,t+1} = \frac{\partial^2 \mathrm{U}}{\partial c_t \partial c_{t+1}}$, (by symmetry), and $\mathrm{U}_{t+1,t+1} = \frac{\partial^2 \mathrm{U}}{\partial c_{t+1}^2}$

The Boardered Hessian is given by:

$$|\mathrm{D}| = \begin{bmatrix} \mathrm{U}_{t,t} & \mathrm{U}_{t,t+1} & -(1+\psi_t) \\ \mathrm{U}_{t+1,t} & \mathrm{U}_{t+1,t+1} & -\frac{(1+\psi_{t+1})}{(1+r)} \\ -(1+\psi_t) & -\frac{(1+\psi_{t+1})}{(1+r)} & 0 \end{bmatrix} \tag{A.4}$$

$|D|$ is implied to be positive in the second-order conditions (S.O.Cs) because by assumption, U_t and U_{t+1} are positive, while $U_{t,t}$ and $\mathrm{U}_{t+1,t+1}$ are negative, and $\mathrm{U}_{t,t+1} = \mathrm{U}_{t+1,t}$ by symmetry.

Therefore, a system of three equations with three unknowns will be obtained:

$$\begin{bmatrix} \mathrm{U}_{t,t} & \mathrm{U}_{t,t+1} & -(1+\psi_t) \\ \mathrm{U}_{t+1,t} & \mathrm{U}_{t+1,t+1} & -\frac{(1+\psi_{t+1})}{(1+r)} \\ -(1+\psi_t) & -\frac{(1+\psi_{t+1})}{(1+r)} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{dc}_t \\ \mathrm{dc}_{t+1} \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} \lambda \mathrm{d}\psi_t \\ \frac{\lambda}{(1+r)}\mathrm{d}\psi_{t+1} \\ c_t \mathrm{d}\psi_t + \frac{(c_{t+1})}{(1+r)}\mathrm{d}\psi_{t+1} \end{bmatrix} \tag{A.5}$$

We can solve $\left(\frac{dc_t}{d\psi_t}\right)$ and $\left(\frac{dc_t}{d\psi_{t+1}}\right)$ by employing Cramer's rule. This is displayed as follows:

$$dc_t = \begin{bmatrix} \lambda d\psi_t & U_{t,t+1} & -(1+\psi_t) \\ \frac{\lambda}{(1+r)}d\psi_{t+1} & U_{t+1,t+1} & -\frac{(1+\psi_{+1})}{(1+r)} \\ c_t d\psi_t + \frac{(c_{t+1})}{(1+r)}d\psi_{t+1} & -\frac{(1+\psi_{t+1})}{(1+r)} & 0 \end{bmatrix} |D|^{-1} \tag{A.6}$$