Studies in Engineering and Technology
Vol. 3, No. 1; August 2016
ISSN 2330-2038 E-ISSN 2330-2046
Published by Redfame Publishing
URL: http://set.redfame.com

A Learning Outcomes Assessment Analysis based on the Mathematical Modeling of RaschGSP Curve, GSM and MSM

Hui-Chung Ho[1], Hsiu-Jye Chiang[2], Duc-Hieu Pham[3], Woody Jann-Der Fann[4], Masatake Nagai[5]

[1]National Taichung University of Education, Taichung, Taiwan (retired)

[2]National United University, Miaoli, Taiwan

[3]Hanoi Pedagogical University 2, Vinh Phuc, Vietnam

[4]Ling-Tung University, Taichung, Taiwan

[5]Teikyo University, Tokyo, Japan (retired)

Correspondence: Hui-Chung Ho, National Taichung University of Education, Taichung, Taiwan

Received: July 6, 2016 Accepted: July 12, 2016 Online Published: July 25, 2016

doi:10.11114/set.v3i1.1766 URL: http://dx.doi.org/10.11114/set.v3i1.1766

Abstract

This paper presents an educational praxis of classroom assessment in curriculum and learning outcomes of "Introduction to Education", and constructs assessment tool and analyzes them based on the mathematical modeling, the former with *Q* Matrix and ISM (Interpretive Structural Modeling), the latter mainly with Nagai's GSP (Grey S-P) Chart, RaschGSP Curve, GSM (Grey Structural Modeling) and MSM (Matrix Based Structure Modeling). It aims: (1) To use and implement numerical value as the code for processing data, (2) To analyze and diagnose based on raw numerical values, (3) To illustrate and explain in visual diagraph analysis. Moreover, it is worth mentioning that applying mathematical logic in educational research, and such tools are not in favor of the quantitative approach, rather claimed unique feature of math logic, in benefit to: (1) Interface qualitative contextual analysis and quantitative numerical characters as the whole, (2) Convert teaching-learning praxis into binary numerical data, (3) Address alternative interdisciplinary educational research.

Keywords: GSP Chart, RaschGSP Curve, GSM, MSM, Educational Research

1. Introduction

Facing web-based information era, academic learning and schooling time is significantly suppressed. "Introduction to Education" is a fundamental curriculum of pedagogy as a profession, the teaching and learning have become complicated and difficulty. "Introduction to Education" is a cross-curricular subject in dealing mainly with educational foundations of philosophy, sociology, psychology and pedagogy. Teaching and learning have been expected to perform as follows: (1) To acknowledge the comprehensive and variety aspects of education, (2) To learn modern educational science and its technique, (3) To trigger students' interest in education itself (Ho, Fann, Chiang, Nguyen, Pham, Nguyen, & Nagai, 2016). No doubt, all lies on effective teaching and assessment.

On account of sophomores majoring in different fields, teaching and learning of "Introduction to Education" vary with difficult and difference. It is necessary to collect data on many tests and exams, which have been not only presented learning outcomes but rather become aware of the knowledge, skills, and beliefs, and even grasp learning for- or backwards to the purpose (Earl, 2003). Today, assessment analysis based on mathematical modeling entails either to design and diagnose, or to remedy and promote, which has highlighted paradigm shift of assessment on the one hand, broaden inform about assessment complex inclusive curricula, teaching and learning, testing and feedback on the other hand.

As described above, applying mathematical modeling to analyze assessment results is trendy. In this paper, it is divided into 2 parts, the one is to select disciplinary concepts and set up a test, the other is testing and analyzing assessment results, which are altogether operated based on mathematical modeling of *Q* Matrix, ISM and Nagai's GSP Chart, RaschGSP Curve, GSM and MSM. It is worth mentioning, tool-based assessment analysis aims to: (1) Discover and implement assessment for, of and by learning scientifically (Ho, Pham, Shih, Sheu, & Nagai, 2014), (2) Define and transform numerical value into education praxis (Ho, Shiu, Pham, Nguyen, & Nagai, 2015), (3) Verify and demo alternative interdisciplinary educational research (Ho, 2014).

2. Mathematical Modeling

Either mathematical modeling or methods are applied to encode items or domain knowledge into numerical value data and make sense of the observed data, and related sets of concepts have been reinforced in the programming language. Furthermore, mathematical modeling as engineered tools have been shown available, performable and effective applied in many fields, such as industry, engineering, commerce, agriculture, medicine and education etc. In this paper, all applied mathematical modeling includes *Q* Matrix, ISM, Nagai's GSP Chart, RaschGSP Curve, GSM and MSM. It is worth mentioning that Nagai's modeling is accessible and reliable, even compatible, which will be furthermore illustrated as follows.

2.1 GSP Chart (Grey S-P Chart)

GSP Chart was proposed by M. Nagai (2010), which is a combination of grey relational analysis (GRA) and Sato's S-P Chart (Sato, 1985). Not only S-P Chart but also GSP Chart are applied as method of non-parametric statistics, nevertheless GSP Chart characters the analysis more concrete and accurate by the localized grey relational grades (See Table 1). GSP Chart converts discrete data into a kind of serialized format, as well as extends dichotomous data to continuous data. In addition, the scale value of GSP Chart is general called Gamma value distributed between 0 and 1, and in specific, LGRG-*S* is called Gamma value for student and LGRG-*P* is called Gamma value for problem (Sheu, Pham, Nguyen, & Nguyen, 2013).

Table 1. GSP Chart

S \ P	Problem-number $P_j, j = 1,2,\cdots,n$	Total score	LGRA- S
Student-number $S_i, i = 1,2,\cdots,m$	$X = [x_{ij}]_{m\times n}$	$SS_i = \sum_{j=1}^{n} x_{ij}$	High ↕ GS_i Low
Number of correct answer	$PP_j = \sum_{i=1}^{m} x_{ij}$	$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}$	
LGRA- P	More ⟷ Less GP_j		

Note. Adapted from "Rasch GSP toolbox for assessing academic achievement," by Sheu, Pham, Tsai, Nguyen, Nguyen, & Nagai, 2014.

Gamma value for student LGRG-*S*:

$$GS_i = \Gamma_{S0i} = \Gamma(x_{S0}, x_{Si}) = \frac{\overline{\Delta}_{S\max} - \overline{\Delta}_{S0i}}{\overline{\Delta}_{S\max} - \overline{\Delta}_{S\min}},$$

where, $x_{Si} = (x_{i1}, x_{i2}, \cdots, x_{in})$, $x_{S0} = (\max_{\forall i}(x_{i1}), \max_{\forall i}(x_{i2}), \cdots, \max_{\forall i}(x_{in}))$, $\overline{\Delta}_{S0i}$ is the Minkowski distance between target point which is characterized by reference vector x_{S0} and considered point which is characterized by inspected vector x_{Si}.

Gamma value for problem LGRG-*P*:

$$GP_j = \Gamma_{P0j} = \Gamma(x_{P0}, x_{Pj}) = \frac{\overline{\Delta}_{P\max} - \overline{\Delta}_{P0j}}{\overline{\Delta}_{P\max} - \overline{\Delta}_{P\min}},$$

where, $x_{Pj} = (x_{1j}, x_{2j}, \cdots, x_{mj})$, $x_{P0} = (\max_{\forall j}(x_{1j}), \max_{\forall j}(x_{2j}), \cdots, \max_{\forall j}(x_{mj}))$, $\overline{\Delta}_{P0j}$ is the Minkowski distance between target point which is characterized by reference vector x_{P0} and considered point which is characterized by inspected vector x_{Pj}.

2.2 RaschGSP Curve (Rasch Model GSP Curve)

RaschGSP Curve is a new mathematical modeling used to analyze the student item responses proposed by Nagai (2010) (Tzeng, Sheu, Liang, Wang, & Nagai, 2012), which is a mixture of GSP Chart and Rasch Model, and constructed with Nagai's formula of localized grey relational grade and Rasch logistic regression. RaschGSP Curve function has since been extended: (1) To measure examinees' competence and calibrate items response quality effectively, (2) To visualize characteristic curve of sets of students and problems accurately, (3) To problem-solve small sample sizes with nonparametric methods mathematically.

The logistic regression model used in RaschGSP method is the three parameters logistic model. Its mathematical formula is described as follows:

$$y = f(x) = \gamma + \frac{1-\gamma}{1+e^{-\alpha(x-\beta)}}$$

where *x* is the order value of student (or the order of item difficulty), *y* is the localized grey relational grade, and α, β and γ featured as parameters, in specific, α means student or item discrimination between group of high gamma and group of low gamma, β means partitioned between high difficulty group and low difficulty group, and γ means guessing distributed between lowest score and highest score.

2.3 Grey Structural Modeling (GSM)

GSM is other method proposed by Nagai (2005), which draws a digraph from *m* elements and a standard of modeling in given systems. GSM considered as succeeding to Interpretive Structural Modeling (ISM) (Warfield, 1973) is used to analyze structure of system, even decision making (Nagai, Yamaguchi, & Li, 2005; Yamaguchi, Li, Akabane, & Kitaoka, 2007). GSM procedure is based on GRA, included localized GRA, globalized GRA and grey relational ordinal. Namely, obtaining grey relational grade for the ordered pair by grey relational analysis, whereas one of the parties had a larger value, it was identified as more important item, and became criteria of structural system arrangement.

In grey system theory (Deng, 1989), GRA is an effective mathematical tool to treat the uncertain, multiple, discrete and incomplete information. This study refers the localized grey relational grade and globalized grey relational grade proposed by Nagai, which is based on Minkowski distance (Yamaguchi, Li, & Nagai, 2007).

The reference vector x_0 and inspected vectors of original data x_i are established as followed:

$$x_0 = (x_0(1), x_0(2), \cdots, x_0(n))$$
$$x_1 = (x_1(1), x_1(2), \cdots, x_1(n))$$
$$x_2 = (x_2(1), x_2(2), \cdots, x_2(n))$$
$$\vdots$$
$$x_m = (x_m(1), x_m(2), \cdots, x_m(n)), \text{ where } i = 1,2,\cdots,m;\ m \in N$$

The formula of localized grey relational grade is:

$$\gamma_{oi} = \frac{\max_{\forall i}\|x_0 - x_i\|_\rho - \|x_0 - x_i\|_\rho}{\max_{\forall i}\|x_0 - x_i\|_\rho - \min_{\forall i}\|x_0 - x_i\|_\rho}$$

And, formula of globalized grey relational grade is:

$$\gamma_{ij} = 1 - \frac{\|x_i - x_j\|_\rho}{\max_{\forall i}\max_{\forall j}\|x_i - x_j\|_\rho}$$

As the above equations, the digraph shows a relationship between given elements with an arrow and their position with a hierarchy visually according to three parameters: distinguish coefficient ρ which decides the basic composition of digraph, class coefficient θ which gives the hierarchy, and path coefficient ψ which gives an ordered pair of element arrows. In this paper, $\rho = 2$ is used that means Euclidean distance is applied.

2.4 Matrix Based Structure Modeling (MSM)

MSM was also proposed by Nagai (2013) (Nagai, Tsai, & Chen, 2013; Nagai, & Tsai, 2013), it was intended to generate an overall association between matrices and reconfigure multiple matrices into square design, in order to construct a new relationship between the respective sets of generation, and visualize the entirety of matrix and the logical relationship of local clusters.

Based on the multi-matrix method of interpretative structural modeling, it is simply known as multi-matrix interpretative structural model.

Let the combined set

$$M = \bigcup M_i$$

where M_i is the concept set matrix, it can be all set of material objects, it can also be the set of all attributes such as set of students, set of questions, set of learning concept, set of goods, set of merchandise category, set of commodity components, set of commodity production conditions, etc.

The structure system theory of MSM is contracted as the following:

Establish the structure system theory of MSM

$$W = MSM\,(M, T, f)$$

where i. $M = \bigcup M_i$ is a combined matrix of factor set,

ii. $T = \bigcup_{r \in M} T_r$ is a structure matrix,

iii. $f : M \times M \rightarrow T$ is reachability function,

$$M \times M = \{(m_i, m_j) \mid m_i, m_j \in M\}$$

$$f(m_i, m_j) = m_{ij} \in T_r .$$

Because f is a reachability function, so f satisfies the following:

(1) Reflexive law: $f(m_i, m_i) = 1 \Leftrightarrow m_i \rightarrow m_i, \forall i$

(2) Anti-symmetric law:

$$f(m_i, m_j) = f(m_j, m_i) = 1$$

$$\Leftrightarrow m_i \rightarrow m_j, m_j \rightarrow m_i \Rightarrow m_i \leftrightarrow m_j \quad , \forall i \neq j$$

$$\Leftrightarrow m_i = m_j$$

(3) Transitive law:

$$\left.\begin{array}{l} f(m_i, m_j) = 1,\ f(m_j, m_k) = 1 \\ \Rightarrow f(m_i, m_k) = 1 \\ \Leftrightarrow m_i \rightarrow m_j,\ m_j \rightarrow m_k \\ \Rightarrow m_i \rightarrow m_k \end{array}\right. \quad , \forall i \neq j \neq k$$

In practice, the MSM cluster matrix calculates amount and scale size using structural new relation matrix as purpose based on the generating of multiple matrix relation and the creating of organization structure, the first one is derived from the relationship between the various elements of attributes, and second one, it is the cluster related structure of the purpose orientation (Ho, Chiang, Liao, Kung, & Nagai, 2013; Ho, Sheu, & Nagai, 2013).

3. Analysis of Practical Example

"Introduction to Education" is 2 credit semester hours course, and enrolled students are 36 sophomores, who study in the different academic major. In this paper designed a mid-term test of 35 problems with 4 choose 1 choice question is to assess the learning outcomes.

3.1 Research Process

The research process is composed of two steps: qualitative and quantitative analysis (See Figure 1), the former focusses basically on the contents of "Introduction to Education" covering philosophy, sociology, psychology and pedagogy, and the terms of acquired knowledge, skill or values; the latter is concerned with: (1) Assigning the question-concept (See Table 2), (2) Proceeding data and analyzing captured structure, (3) Illustrating structural characterization and accomplishing remedial work. Note worth, quantitative analysis is optimizing based on mathematical modeling. First of all to proceed the study is to inspect the raw data whether to be reliable and consistent.

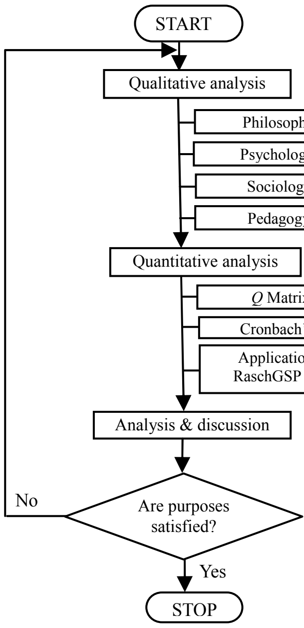


Figure 1. Research process

3.2 Test Design

At its simplest, test design and its process comprises the following: (1) To identify categories, (2) To ensure testing completeness, (3) To fix concept issues and follows a methodical approach. This approach consists of two steps, to capture academic related concepts (See Table 2) on the one side, to illustrate the terms' relationship based on mathematical modeling (See Figure 2, & Figure 3) on the other side. Then, to carry out testing and analyzing assessment results.

3.2.1 Academic Concepts Chart

Academic concepts chart here means to converge and organize textual elements of "Introduction to Education", general speaking, on the one side composed of disciplinary concepts and course objectives have been generated and aligned to textbooks and lectures, on the other side test design characterized with Schön's proposal (Schön, 1987) to educate the reflective practitioner by, of, and in the experience. Moreover, academic concepts chart is used to distribute and formulate in adherence to: (1) Bloom, Krathwohl, and Harrow respectively proposed three general domains of learning (Anderson, & Krathwohl, et al, 2001 eds.), (2) Tatsuoka's *Q*-matrix (Tatsuoka, 1985).

On account of higher education dealing more with reasoning, teaching and learning of "Introduction to Education" have characterized that cognitive education is shared foundation of affective and psychomotor domains development (See Figure 2, & Figure 8). Affective domain implies empathy, salience, meaning and value, psychomotor claims carry out, action, practice and tactics, and cognitive not naïve understanding and memory, more contextual and structural featured. In short, cognitive, affective and psychomotor.

Table 2 designed shows as follows: (1) Concepts assigned to academic branches according to wielding and disciplinary attributes, the former like interactive approach, learning by doing etc., the latter like doctrine of evil human nature, monkey comb behavior etc., (2) Framework built on explaining selecting and professionalism in view of psychology and sociology (See Figure 2, & Figure 3), (3) Answering the questions lying on comprehension of context and discourse analysis.

Table 2. Disciplinary Concepts Chart

P\C	pedagogy	philosophy	sociology	psychology	cognitive	affective	psychomotor
P1			Imitation				v
P2	interactive approach					v	
P3	learning by doing						v
P4		tabula rasa			v		
P5				motive theory		v	
P6		greek philosophy			v		
P7			family education school education media education		v		
P8		willingness				v	
P9		epistemology metaphysics axiology			v		
P10			class shift				v
P11		metaphysics			v		
P12			living norm				v
P13		knowledge paradigm			v		
P14	professionalism						v
P15		doctrine of evil human nature				v	
P16		epistemology			v		
P17			monkey comb behabior			v	
P18				belongingness & love needs		v	
P19				individual differences	v		
P20				schemata assimilation accommodation equilibration	v		
P21	adaptive education						v
P22				gestalt psychology			v
P23				critical period	v		
P24				operant conditioning			v
P25				positive psychology		v	
P26				heredity Gene	v		
P27			micro hermeneutics		v		
P28			educational media				v
P29			equities			v	
P30			group identity			v	
P31		axiology				v	
P32	selection function						v
P33				physiological psychology	v		
P34			educational sociology			v	
P35			economy				v

3.2.2 Concepts ISM

Based on ISM, the above academic concepts chart has been establishing relationship illustrated: (1) Ranking the relationship among concepts, (2) Defining the topic issue, (3) Interpreting the issue's knowledge and the related context. It is noteworthy that concepts chart is converted from isolated diagram to digraph structure (See Figure 2, Figure 3, & Table 3), and delivered concrete and dynamical issues, which have been denoted more or less subjectively.

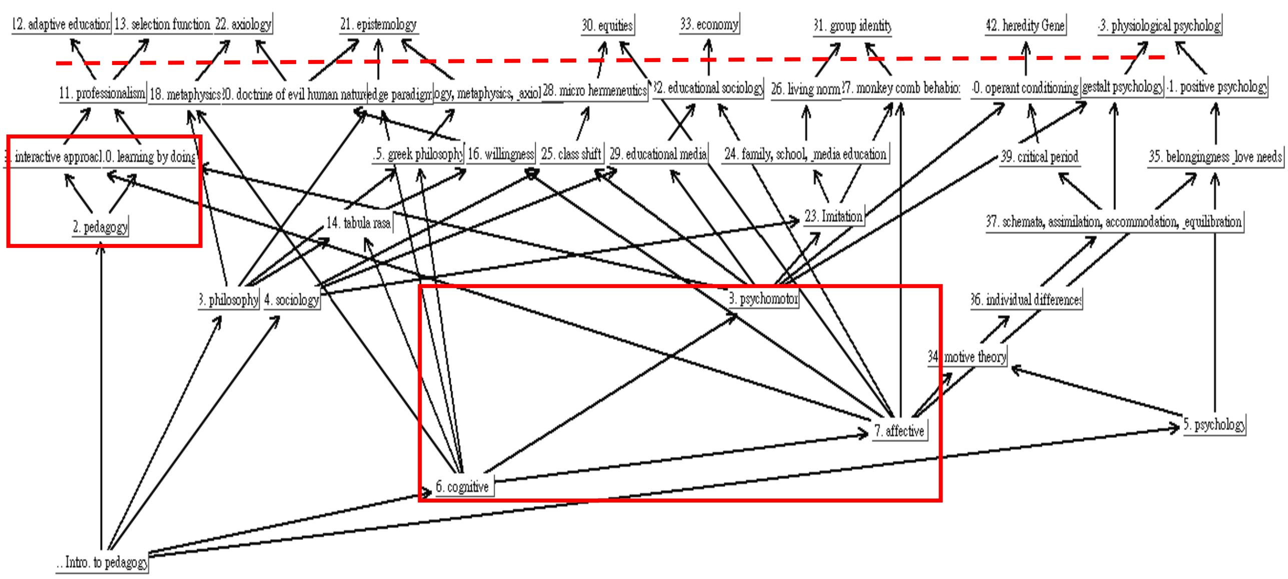


Figure 2. Domain Concepts ISM

Table 3. Relation Matrix of Core Concepts

C\C	1	2	3	4	5	6	7	8	9	10	11
1.intro. to pedagogy	0	1	0	0	0	0	0	0	0	0	0
2.professionalism	0	0	1	1	0	0	0	0	0	0	0
3.adaptive education	0	0	0	0	0	1	1	0	0	0	0
4.selection function	0	0	0	0	1	0	1	0	0	0	0
5.heredity Gene	0	0	0	0	0	0	1	0	0	0	0
6.physiological psychology	0	0	0	0	0	0	0	1	0	0	0
7.epistemology	0	0	0	0	0	0	0	0	0	0	1
8.axiology	0	0	0	0	0	0	0	0	1	0	0
9.equities	0	0	0	0	0	0	0	0	0	1	0
10.group identity	0	0	0	0	0	0	0	0	0	0	0
11.economy	0	0	0	0	0	0	0	0	0	0	0

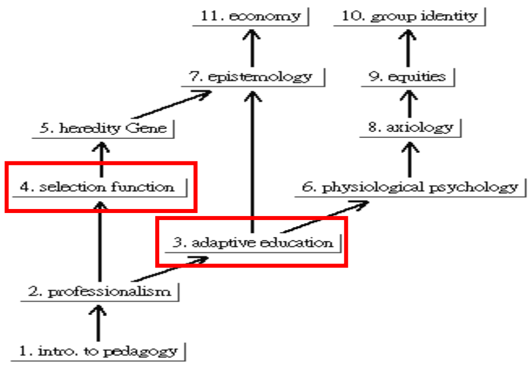


Figure 3. Domain Core Concepts ISM

As indicated above, assessment process in social science doesn't view right or wrong answer, nor compare whether better than before, in contrast, examine how well been comprehensive and interpreted.

3.3 Assessment

Assessment analysis expected to deliver predictability in process and reliability in quality optimizes substantial teaching and learning. Therefore, to choose eligible methodology for assessment analysis has become important. Here is to apply Nagai's proposed GSP Chart, GSM and MSM as research tools, which have been explored and executed usefully, accessibly and effectively.

3.3.1 Data Processing

On account of measurement reliability and validity, consistency of data is central to the concept of reliability. Cronbach (Cronbach, 1951) proposed α coefficient as criteria to measure how closely related internal consistency or reliability of a set of items that serve to estimate whether to pertain to a certain construct. Doubtlessly, Cronbach's α is alternative methods for assessing reliability in cross-sectional studies, but widely accessible. In the case of psychometric tests, most fall within the range of 0.75 to 0.83 (Nunnally, 1978). The data of 36 examinees and 35 test questions viewed as the reliability, of whose Cronbach's α coefficient is 0.834 in SPSS Statistics (See Appendix A, & Appendix B). It means all of items are allegedly clean and eligible data.

3.3.2 GSP Chart and RaschGSP Curve

GSP Chart and RaschGSP Curve as mathematical modeling used to analyze and display assessment results, note worth that, RaschGSP Curve has been extended to be a kind of three parameter logistic model, inclusive difficulty, discrimination and pseudo-guessing.

Firstly, S-P Chart's disparity index is 0.37, indicated the non-homogeneity degree of the test data. Secondly, Table 4 displays assessment results as follows: (1) To order students and problems according to larger-the-better of right answers and localized grey relational analysis, (2) To generate correlated gamma numerical value sequences for the correlation analysis instead of discrete sorting sequences, plotted lines into curve (See Figure 4-1), (3) To display the scope of paradoxical misconcepts and boundary line of target students in view of statistical graphic and gamma numerical value.

Indeed, either sensorial misconcepts or alternative conceptions, even cognitive comprehension can be erroneous, illogical or misinformed interfering with being able to correctly grasp new concepts and thinking (Burgoon, Heddle, & Duran, 2010), and have usually been removed by remedial or second teaching. Accordingly, needed at least attention to, the one is vertical top right corner and bottom left corner, which involves the same question with right and wrong answers; the other one is horizontal gamma numerical value 0.405-0.368, who require extra help to better.

Finally, based on GSP Chart and RaschGSP Curve method plotting assessment results of students and problems (See Figure 4-1, & Figure 4-2), is to realize a new research model effectively and efficiently. Moreover, from Figure 4-1 and Figure 4-2 shown, as is well known: (1) Both given with numerical value as preferable sets, (2) RaschGSP Curve formulated discrete sorting sets of students and problems into continuous data sets, (3) Nonparametric inferential statistical methods warranted by mathematical procedures as an applicable research approach.

Table 4. GSP Chart

S\P		P21	P8	P16	P31	P20	P1	P33	P18	P9	P13	P19	P7	P34	P2	P29	P17	P25	P6	P10	P22	P32	P4	P15	P11	P24	P27	P12	P3	P30	P26	P23	P14	P35	P5	P28	Σ	LGRA
	MAX	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
S3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	33	1.000
S1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	32	0.914
S19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	31	0.841
S12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	0	1	0	31	0.841
S2	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	1	0	0	29	0.719
S27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	0	1	0	1	0	0	1	1	1	0	1	0	27	0.616
S18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	0	1	1	1	1	0	1	0	1	0	0	0	26	0.570
S29	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	1	1	26	0.570
S6	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	25	0.526
S15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	1	0	0	0	1	1	0	1	1	1	1	0	1	0	24	0.484
S16	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	1	0	0	0	1	1	0	1	1	0	0	0	1	24	0.484
S26	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	1	0	0	0	1	1	0	0	1	1	0	0	0	23	0.444
S10	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	23	0.444
S8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	23	0.444
S28	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	0	1	1	0	0	1	0	0	1	0	1	0	0	0	23	0.444
S17	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	1	1	22	0.405
S4	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1	1	1	0	1	1	1	0	0	0	0	0	0	1	0	0	22	0.405
S11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	21	0.368
S14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	21	0.368
S23	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	0	0	1	0	0	1	1	1	0	1	0	1	0	0	0	0	1	1	21	0.368
S9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	20	0.333
S24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0.333
S5	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	20	0.333
S34	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	20	0.333
S7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	0.298
S13	1	1	1	1	1	1	1	1	0	1	0	1	1	0	0	1	1	1	1	1	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	19	0.298
S31	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	0	1	0	0	0	0	0	1	0	1	0	0	1	0	0	1	1	1	0	0	0	19	0.298
S21	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	1	0	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0	1	0	0	18	0.265
S20	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	18	0.265
S22	1	1	1	1	0	1	1	1	1	1	1	1	0	1	0	0	1	0	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	0	0	18	0.265
S25	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	18	0.265
S30	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	18	0.265
S32	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	14	0.140
S36	1	1	1	0	1	1	0	1	1	1	1	1	0	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	13	0.111
S35	1	1	1	0	1	1	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0.000
S33	1	0	1	1	0	1	1	0	1	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0.000
Σ		35	34	34	34	33	33	33	33	33	32	32	28	27	27	27	26	25	24	23	20	18	18	16	14	14	14	14	11	11	11	11	11	9	8	6	779	
LGRA		1.000	0.908	0.908	0.908	0.837	0.837	0.837	0.837	0.837	0.777	0.777	0.592	0.553	0.553	0.553	0.517	0.483	0.450	0.418	0.330	0.276	0.276	0.225	0.176	0.176	0.176	0.176	0.107	0.107	0.107	0.107	0.107	0.063	0.042	0.000		

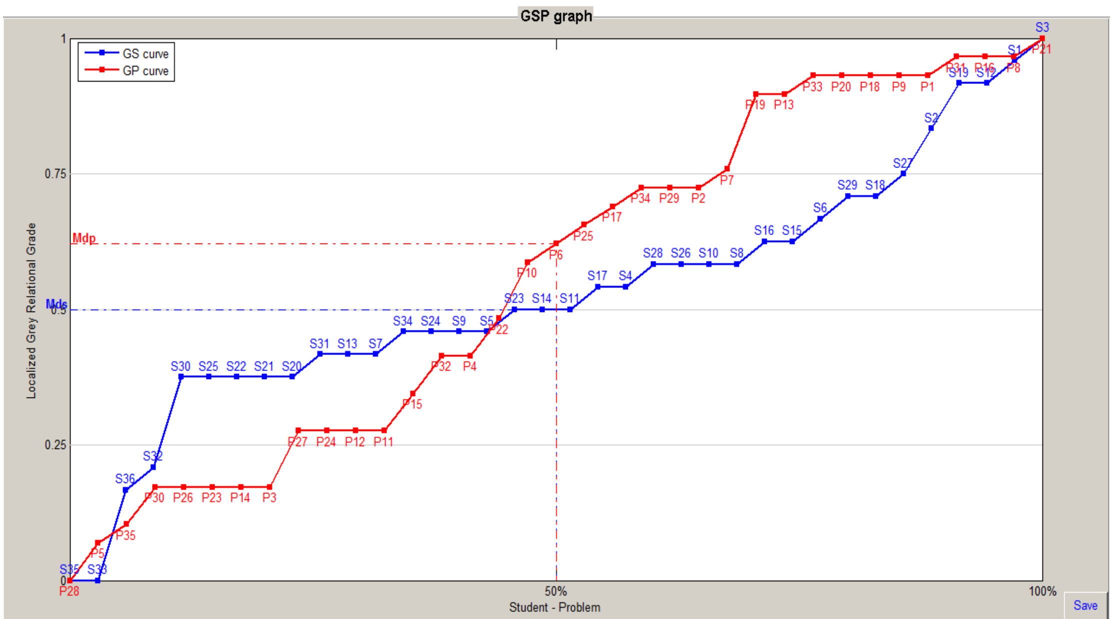


Figure 4-1. GS & GP Line Graph

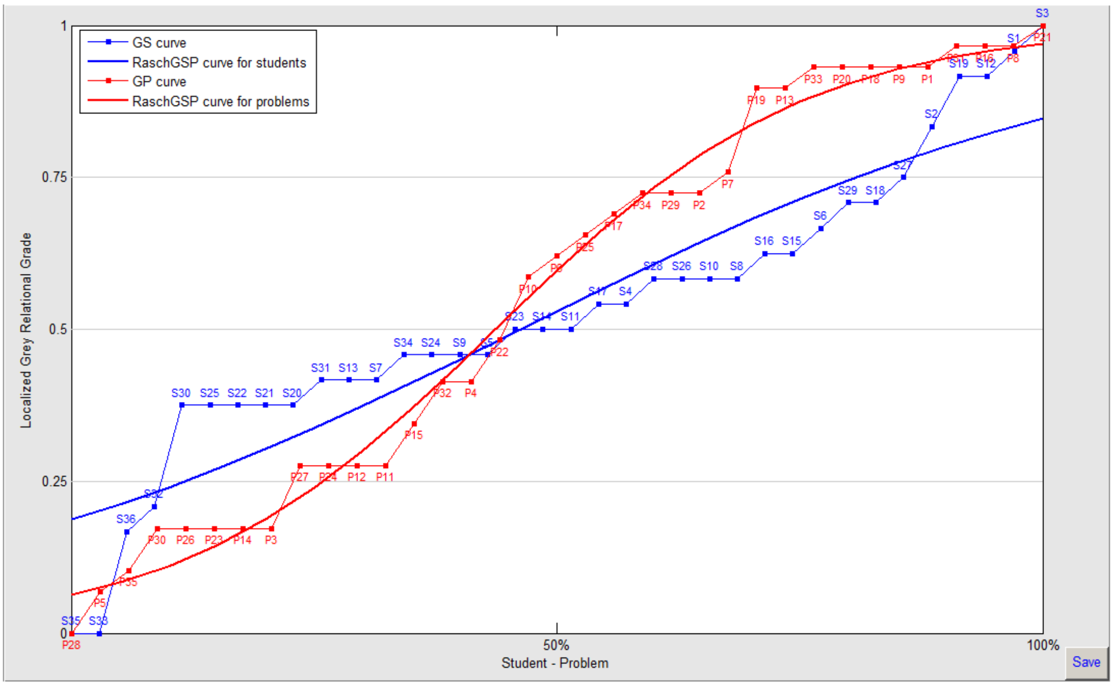


Figure 4-2. GSP Curve

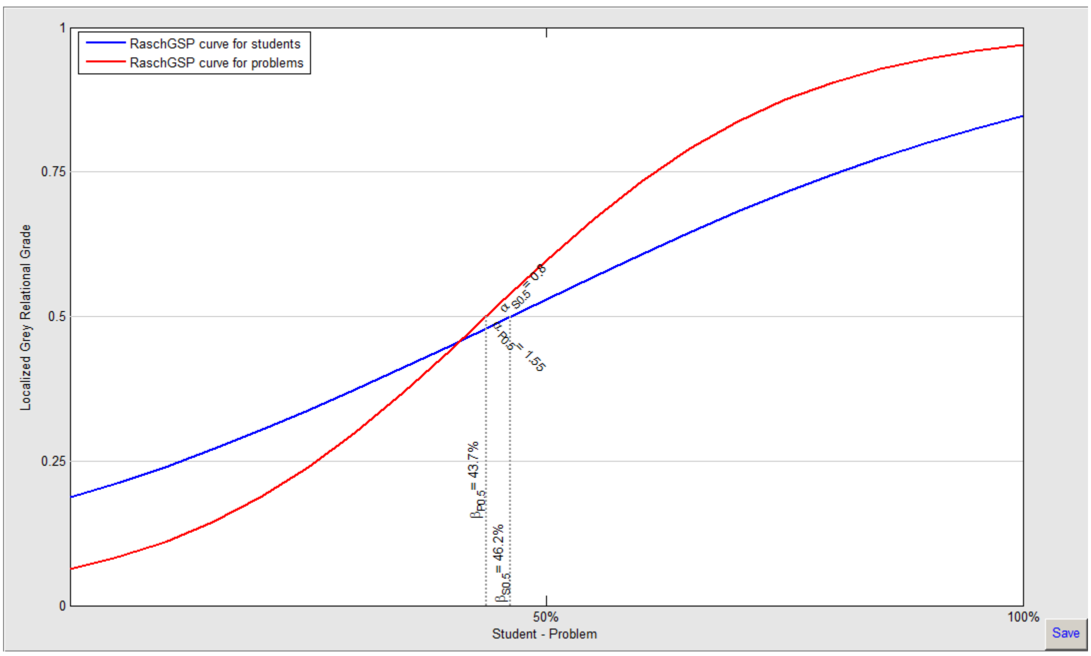


Figure 4-3. RaschGSP Curve

Figure 4-3 shows gamma 0.5 is the slope of RaschGSP curve for students at the point (α , 0.8) and for problems at the point (α , 1.55), the former β is 46.2%, the latter β is 43.7%.

3.3.3 Structural Analysis

Applying Nagai's localized grey relational analysis gamma value, globalized grey relational analysis gamma value and graph theory create relationship structure of assessment results (Pham, Sheu, Tsai, Nguyen, Nguyen, & Nagai, 2014), whereby to conduct structural analysis. Structural analysis aims here to: (1) Identify and define students' and problems' respective relationship structure of assessment results, (2) Verify and interpret the structure's fitness and unfitness, (3) Carry out and revalue multiple remedial tactics.

As indicated above, relationship structure of assessment results of students and problems has been presented in Figure 5-1 and Figure 6-1. Among nets correlation between students or problems, Figure 5-2 and Figure 6-2 character the special case, allegedly Reachable GSM, which has been cultivating based on cut array and correlation relationship transitivity. Moreover, GSM and RGSM purvey visible graph in benefit to diagnose.

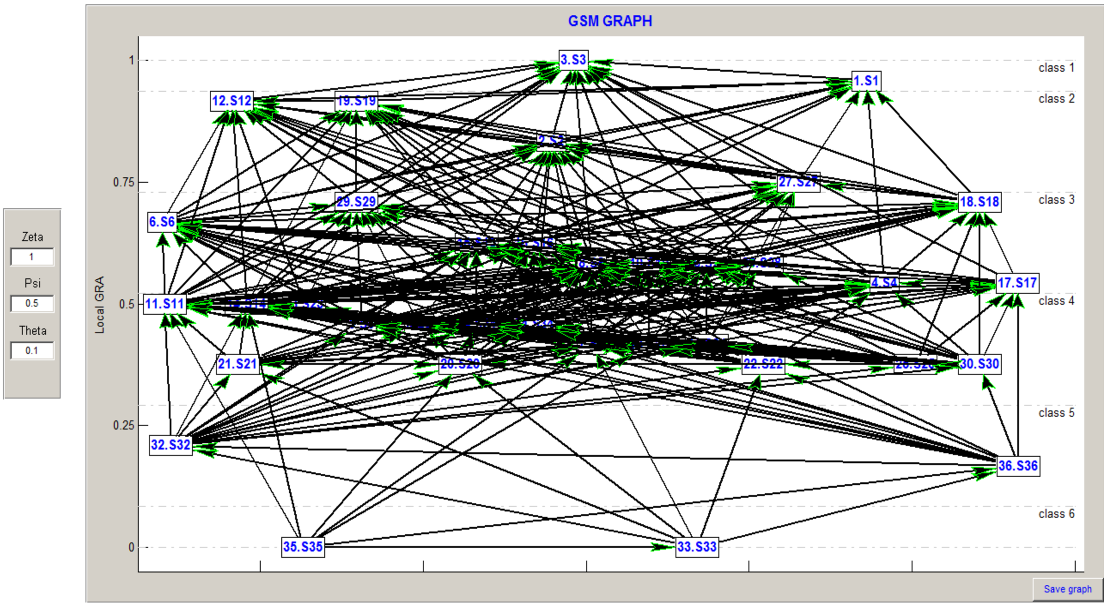


Figure 5-1. Assessment Results Students GSM

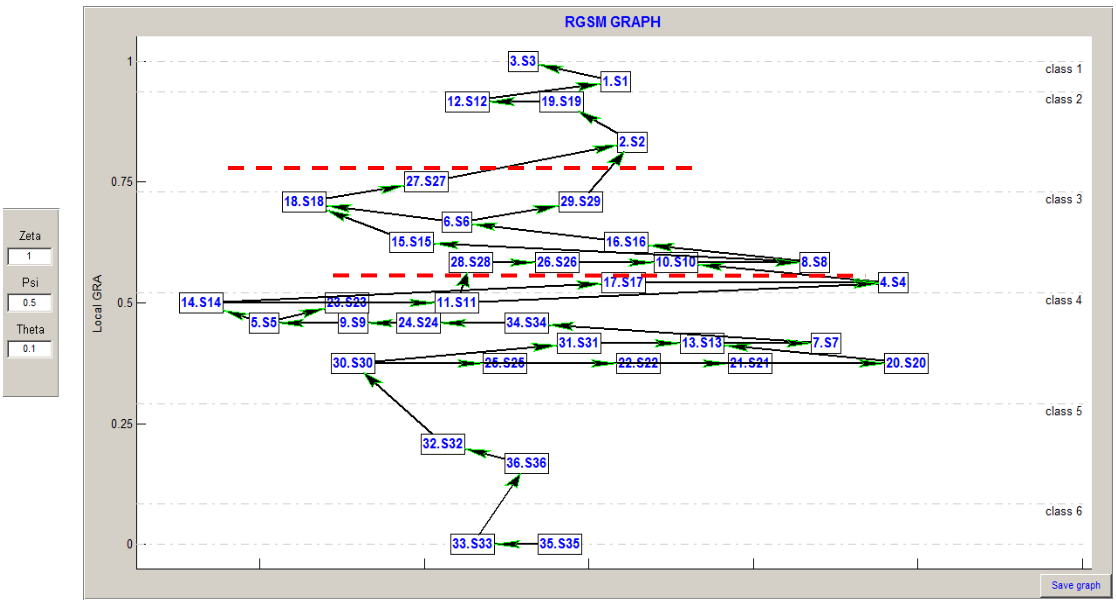


Figure 5-2. Assessment Results Students RGSM

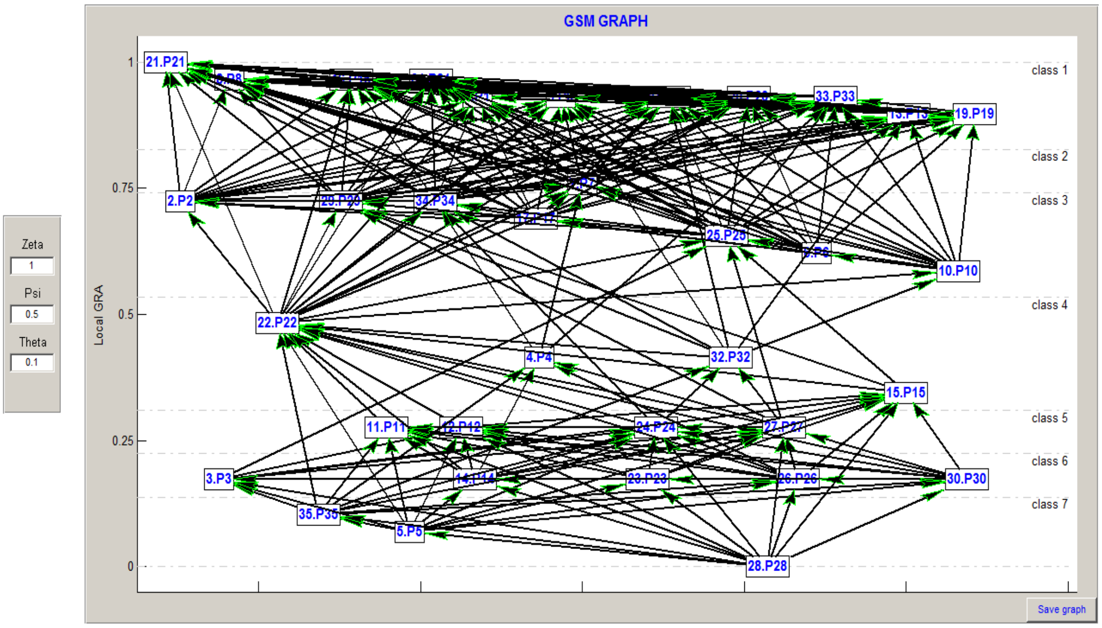


Figure 6-1. Assessment Results Problems GSM

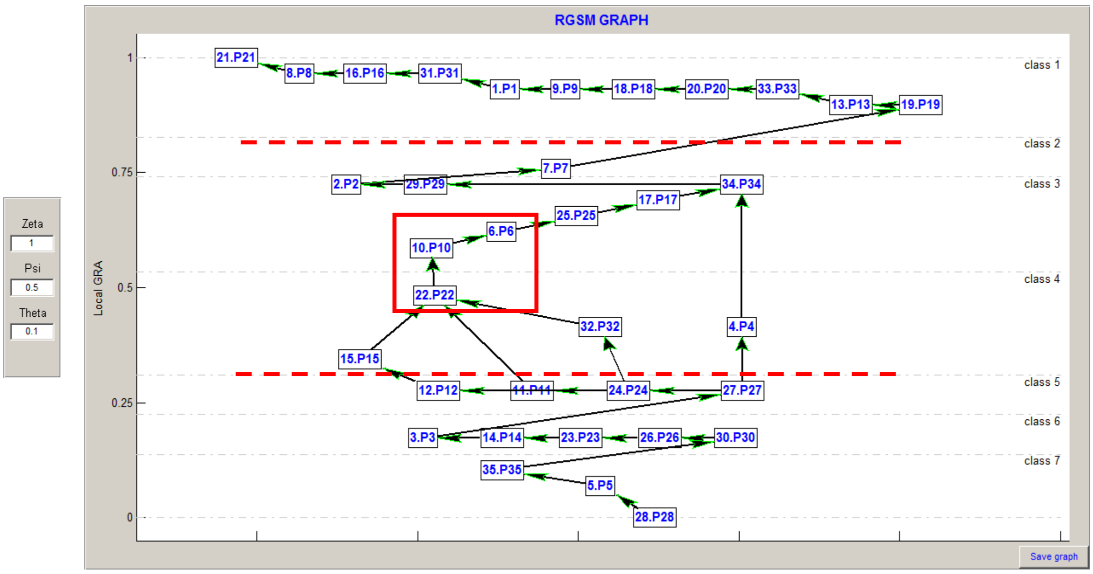


Figure 6-2. Assessment Results Problems RGSM

In short, according to Figure 5-1 or mainly Figure 5-2 shown, student correlation structure and their capitals have been divided into: (1) Class 1-3 as tutors for class 3-4, (2) Class 4-6 accept teacher's second teaching, (3) Both for a limited time. According to Figure 6-1 or mainly Figure 6-2 shown, problem difficulty correlation structure and their items have been assigned: (1) Problems in class 1 tagged as well understood, (2) Problems in class 6-7 tagged as little understood, (3) P6, P10, & P22 tagged as key concepts to advance for teaching and learning. Note worthily, GSP and GSM or RGSM are like a two sides of the coin, which better-off each other.

3.4 Structural Characterization

Applying MSM to profile assessment feature of students and problems has been available to read, communicate and discuss assessment results by visualization.

3.4.1 DSP MSM

Firstly, DSP is abbreviation for Domain/disciplinary branches, Student and Problem. Secondly, Figure 7 shown: (1) Introduction to pedagogy divided into D1-pedagogy, D2-philosophy, D3-sociolohy, and D4-psychology, (2) 35 Problems assigned into 4 Domains, (3) 35 Problems answered positive by 36 Students. Note worth, Figure 7 is substantially 3 layers structure digraph, namely, introduction to pedagogy on layer 1, domain1-4 and students on layer 2, and problems on layer 3. Modification is for reading and displaying MSM alternative. Finally, Figure 7 is used to verify as individual or the whole.

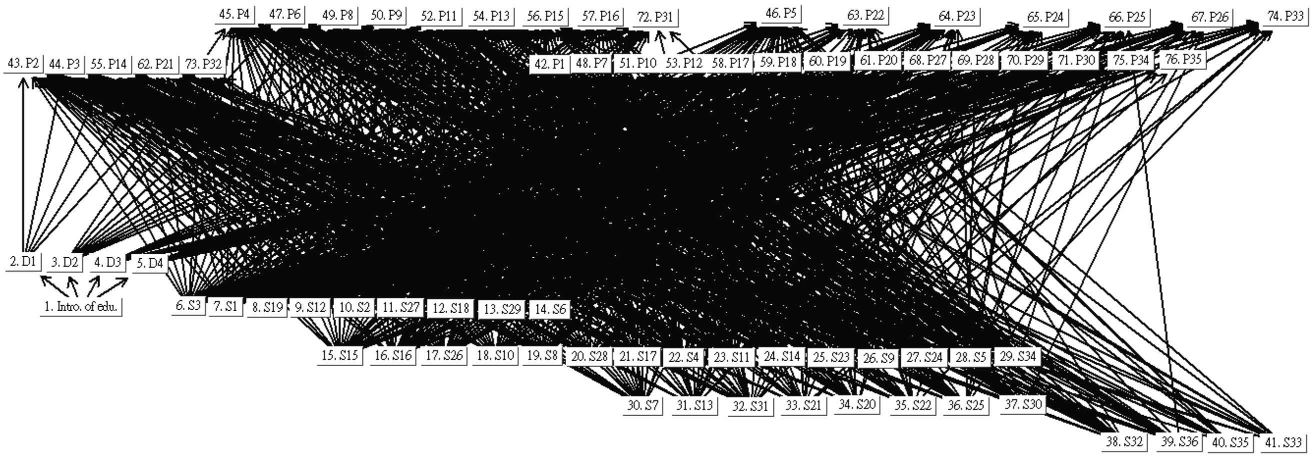


Figure 7. DSP MSM

3.4.2 DPC MSM

As set out above, DPC is similarly shorthand for Domain, Problem and Problems' Concept. In Figure 8 is shown: (1) Domain, problem and concept correlation-based built up (See Table 2), (2) Partial concepts assigned more than one domain among pedagogy, philosophy, sociology and psychology, (3) Disciplinary branches constituting educational context and content.

Accordingly, teaching and learning not only regarded as a process, but also as contextual activity, indeed as one lesson with multiple design, psychology, sociology, ethics and professionalism have turned into indispensable foundations of teaching and learning. Nevertheless, this is not in contradiction with respective knowledge structure and special features (See Figure 9).

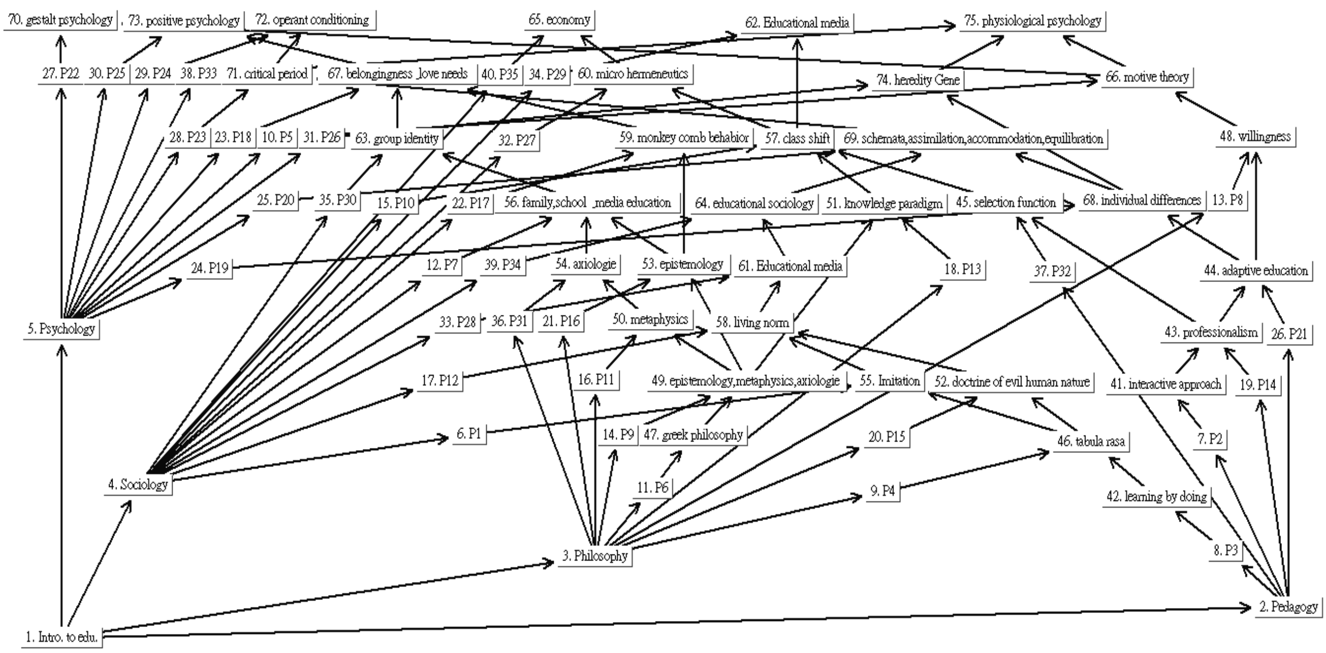


Figure 8. DPC MSM

3.4.3 Educational Foundation Concepts MSM

In terms of professional, pedagogy is 1950s advanced to academic discipline, of which objectives, features and implications are different from philosophy, sociology and psychology. Though, pedagogy is allegedly applied science, which combines philosophy as guide line, sociology as background, psychology as principle, and professionalism as tool converging to learn by, in and for doing.

Educational Foundation theory shown in Figure 8: (1) Philosophy, sociology and psychology deemed either respective or consolidated, (2) Respective discipline loaded with knowledge paradigm, and consolidated knowledge structure based on special interpretation, (3) Vertical and horizontal route-set up starting with philosophy, surrounding in sociology and steering by psychology.

Finally, Figure 9 signifies to: (1) Knowledge structure of "Introduction to Education", or not less than test content, (2) Benchmark of remedial or second teaching, (3) Viewing of Figure 8 and Figure 9, Figure 6-2 based on integrated topic teaching.

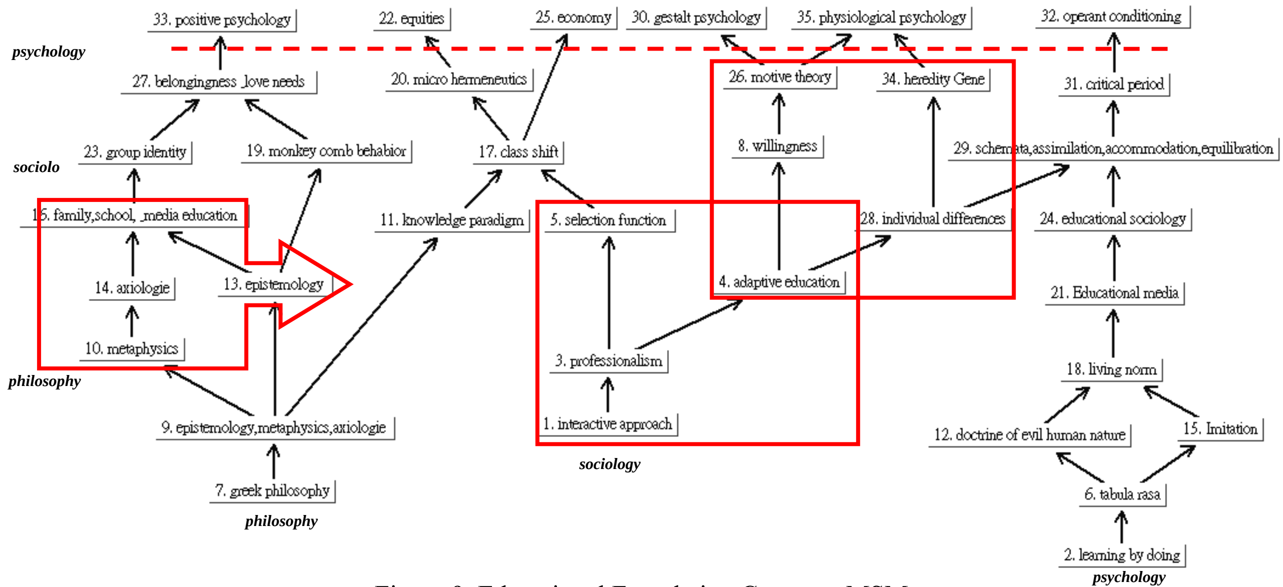


Figure 9. Educational Foundation Concepts MSM

4. Conclusions

Toward an increased efficient future, assessment is perceived as function instrument, rather as the last defense line professionally to determine teaching and learning success or not. To advance quality and equity in education, learning outcomes are indeed proposed criteria to an effective education, and assessment results are essential to inform diagnosis and furnish remedial collaborative tactic. In short, education ecology has been converting either from human-center to cause-effect, or from learning by doing, reflective practitioner to effective realization.

Secondly, assessment is not only activity but also process, which connects before, during, and after teaching and learning plan. Therefore, standard test design, testing and analyze, and feedback and redesign altogether as the complex whole has become more important. It is noteworthy that assessment complex based on mathematical modeling has explored and evidenced eligibly and indispensably. Nonetheless, humanities education and social science education aim not only at right/wrong answer, rather beyond understanding, also at judging value, commitment etc.

Finally, learning outcomes assessment analysis as a public good, to strengthen interdisciplinary cooperation and to blend mathematical modeling in education has become important and necessary. Applying Nagai's GSP Chart, RaschGSP Curve, GSM and MSM to analyze learning outcomes assessment serves following purposes: (1) To analyze data from test scores to improve teaching and learning systematically, (2) To convert educational tests into visualization and tactic output methodically, (3) To chain qualitative and quantitative binary research as the whole mathematically.

References

Anderson, L. W., & Krathwohl, D. R., et al (2001 eds.). *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. Boston, MA: Allyn & Bacon.

Burgoon, J. N., Heddle, M. L., & Duran, E. (2010). Re-examining the similarities between teacher and student conceptions about physical science. *Journal of Science Teacher Education, 21*(7), 859-872. http://dx.doi.org/10.1007/s10972-009-9177-0

Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika, 16*(3), 297-334. http://dx.doi.org/10.1007/BF02310555

Deng, J. L. (1989). Introduction to Grey System Theory. *Journal of Grey System, 1*(1), 1-24.

Earl, L. M. (2003). *Assessment as Learning: Using Classroom Assessment to Maximize Student Learning*. Thousand Oaks, CA: Corwin.

Ho, H. C. (2014). The Cardinal Analysis of Philosophy of Education via Soft Computing Method. *Journal of Grey System, 17*(2), 101-112.

Ho, H. C., Chiang, H. J., Liao, S. L., Kung, K. D., & Nagai, M. (2013). A Study of Assessment on Educational Philosophy with FCA and MSM - A Case Study on Selected Units of Philosophy. *Proceedings of 2013 International Conference on Grey System Theory and Kansei Engineering Conference*, 212-221. (Written in Chinese)

Ho, H. C., Fann, W. J. D., Chiang, H. J., Nguyen, P. T., Pham, D. H., Nguyen, P. H., & Nagai, M. (2016). Application of Rough Set, GSM and MSM to Analyze Learning Outcome - An Example of Introduction to Education. *Journal of Intelligent Learning Systems and Applications, 8*(1), 23-38. http://dx.doi.org/10.4236/jilsa.2016.81003

Ho, H. C., Pham, D. H., Shih, P. C., Sheu, T. W., & Nagai, M. (2014). Based GSP chart, Rasch model GSP curve and GSM to analyze math test - an example on 6th graders' math subject test. *Journal of Educational Measurement and Statistics, 22*(1), 25-65. (Written in Chinese)

Ho, H. C., Sheu, T. W., & Nagai, M. (2013). A Study of Constructing an Epistemological Entity and Cognition Patterns with FCA, Rough Sets and MSM – A Case Study on an Educational Philosophical Assessment. *Chinese Association of Psychological and Educational Testing 2013 Annual Conference*. (Written in Chinese)

Ho, H. C., Shiu, S. J., Pham, D. H., Nguyen, P. T., & Nagai, M. (2015). Applying Entropy-LGRA and GSM to Analyze Learning Outcome - An Example of Classroom Management. *Journal of Taiwan Kansei Information, 6*(2), 41-52.

Nagai, M., & Tsai, C. P. (2013). Matrix based interpretative structural modeling. *International Journal of Kansei Information, 4*(3), 159-174.

Nagai, M., Tsai, C. P., & Chen, T. L. (2013). Matrix based interpretative structural modeling. *Proceedings of 2013 the 5th International Conference on Technology and Mathematics Education and Workshop of Mathematics Teaching*, 509-518. (Written in Chinese)

Nagai, M., Yamaguchi, D., & Li, G. D. (2005). Grey structural modeling. *Journal of Grey System, 8*(2), 119-130.

Nunnally, J. C. (1978). Assessment of Reliability. In: *Psychometric Theory* (2nd ed.). New York: McGraw-Hill.

Pham, D. H., Sheu, T. W., Tsai, C. P., Nguyen, P. T., Nguyen, P. H., & Nagai, M. (2014). GSM-RGSM Software and Its Application in Educational Measurement. *Education Practice and Innovation, 1*(3), 35-48. http://dx.doi.org/10.15764/EPI.2014.03005

Sato, T. (1985). *Introduction to student - problem curve theory analysis and evaluation*. Tokyo: Meiji Tosho.

Schön, D. (1987). *Educating the reflective practitioner*. San Francisco, CA: Josey Bass.

Sheu, T. W., Pham, D. H., Nguyen, P. T., & Nguyen, P. H. (2013). A Matlab Toolbox for Student-Problem Chart and Grey Student-Problem Chart and Its Application. *International Journal of Kansei Information, 4*(2), 75-86.

Sheu, T. W., Pham, D. H., Tsai, C. P., Nguyen, P. T., Nguyen, P. H., & Nagai, M. (2014). Rasch GSP toolbox for assessing academic achievement. *Journal of Software, 9*(7), 1903-1913. http://dx.doi.org/10.4304/jsw.9.7.1903-1913

Tatsuoka, K. K. (1985). A probabilistic model for diagnosing misconceptions by the pattern classification approach. *Journal of Educational Statistics, 10*, 55-73. http://dx.doi.org/10.2307/1164930

Tzeng, J. W., Sheu, T. W., Liang, J. C., Wang, B. T., & Nagai, M. (2012). A New Proposal Based on Rasch Model GSP Chart and Grey Structural Model with Analysis and Discussion. *International Journal of Advancements in Computing Technology, 4*(9), 111-121. http://dx.doi.org/10.4156/ijact.vol4.issue9.13

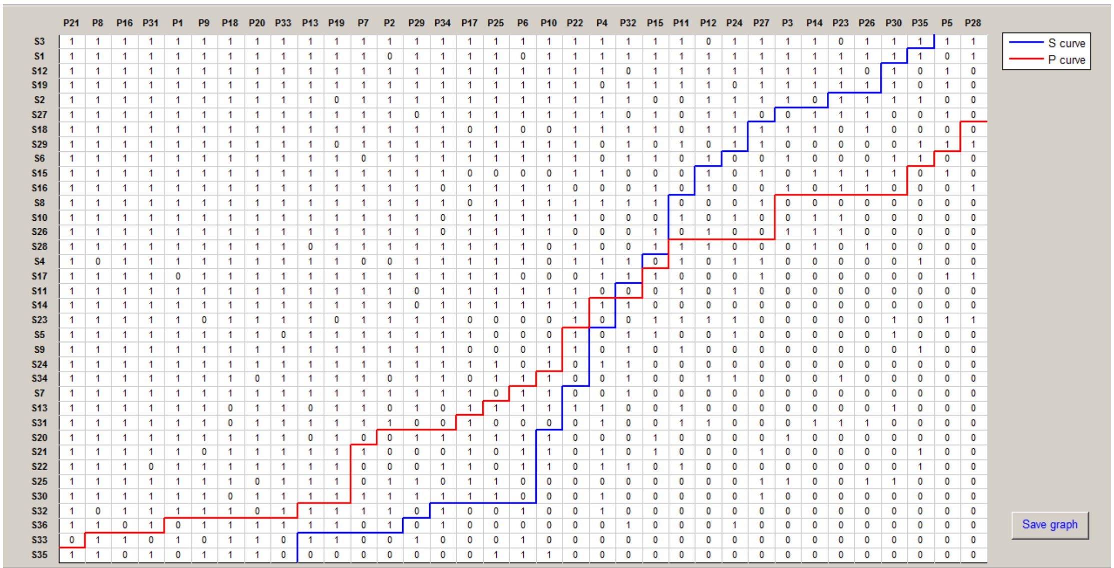
Warfield, J. N. (1973). Binary Matrices in System Modeling. *IEEE Trans. Syst. Man. and Cybern., SMC-3*(5), 441-449. http://dx.doi.org/10.1109/TSMC.1973.4309270

Yamaguchi, D., Li, G. D., & Nagai, M. (2007). Verification of effectiveness for grey relational analysis models. *Journal of Grey System, 10*(3), 169-181.

Yamaguchi, D., Li, G. D., Akabane, K. M. T., & Kitaoka, M. (2007). A realization algorithm of grey structural modeling. *Journal of Grey System, 10*(1), 33-40.

Appendix A

S-P Chart



Appendix B

Cronbach's α coefficient

Case Processing Summary		N	%
Cases	Valid	36	100
	Exclude	0	0
	Total	36	100
a	Listwise deletion based on all variables in the procedure		

Reliability Statistics	
Cronbach's Alpha Value	N of Items
0.834204603	35