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The Relationship of Mathematical Reasoning to Mathematics Achievement and Academic Excellence

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Abstract

This study aimed to investigate the relationship between mathematical reasoning and achievement in mathematics, and extend this to its correlation as a higher cognitive ability with various academic subjects in general, particularly across different levels of academic performance efficiency. The study employed a descriptive correlational approach with a sample of 106 twelfth-grade secondary school students. The sample was divided into three groups based on academic performance efficiency. The study sought to answer the following questions (1) What is the extent and direction of the correlation between mathematical reasoning as a higher cognitive qualitative ability, and achievement in mathematics as educational content? (2) To what extent does the extent and direction of this correlation vary between mathematical reasoning variables and achievement in mathematics across different levels of academic performance in this subject? (3) What is the extent and direction of the correlation between mathematical reasoning and achievement in different academic courses? (4) To what extent do the extent and direction of correlation relationships vary between mathematical reasoning variables and different academic courses across varying levels of academic performance in these courses? Naturally, these questions were addressed by verifying the validity of the following scientific hypotheses: Hypothesis 1.

There is a strong correlation between mathematical reasoning (as a higher cognitive ability) and achievement in mathematics, as presented to fourth-grade secondary school students. Hypothesis2: The correlation between mathematical reasoning and achievement in mathematics varies in strength and weakness across different levels of academic performance efficiency in this subject.Hypothesis3:There is a strong correlation between the variable of mathematical reasoning and various academic subjects in general. However, the correlation is stronger between this ability and scientific subjects compared to subjects with linguistic or humanistic nature.Hypothesis4:The extent of correlation relationships (both strong and weak) between mathematical reasoning and different academic courses varies across different levels of academic achievement efficiency in general. The results of this study revealed the validation of all these hypotheses except for the third hypothesis, which was partially validated.

The most significant findings from this study have highlighted a set of important indicators that shed light on the nature of non-cognitive factors behind academic excellence in general and specifically in mathematics, as presented in our context previously.

Keywords: awareness of excellence, mathematical reasoning, academic achievement

1. Introduction

Human attitudes towards mathematics and its teaching have varied over time. When the main focus of human interest was his relationship with one or more of the objects in nature, mathematics was considered a sacred invention. And when human attention focused on the surrounding nature, mathematics was used as a precise system to describe this nature.

In our present age, with the tremendous scientific development that encompasses all scientific aspects, mathematics has played an influential role in this progress. It has also enjoyed a substantial share of modernization and innovation, along with the discovery of new theories. As a result of this technological advancement, calculators, computers, and other tools have been invented, facilitating human interaction with his environment and enabling him to harness it to serve his current and future purposes and objectives.

In fact, by looking at the history of the development of mathematics, we can perceive significant changes in concepts and subjects. We also find that new theories have been discovered, replacing the old mathematics. The school of counting and space has been replaced by the school of structure and pattern; The aim of teaching mathematics has shifted towards training in dealing with formal inferential systems or mathematical structures based on axioms according to the rules of logic and logical inference. Hence, the importance of teaching mathematics and establishing its principles among students from the beginning of their learning began to emerge. This cannot be achieved without training in the best suitable teaching methods and precisely defining the educational content. This cannot be achieved without training in the best suitable teaching methods and precisely defining the educational content.

This critical role is entrusted to educational institutions in all their elements (facilities, curricula, teachers, etc.), which are capable of encompassing these advanced mathematical contents, planning for them, and presenting them to students in a way that ensures the attainment of the desired objectives of learning mathematics.

and provide a detailed explanation of what we have summarized above.

1.1 The Nature of Mathematics

Mathematics is an abstract science created and conceived by the human mind. It focuses on ideas, methods, and patterns of thinking. It is not merely the sum of its traditional branches; it is more than arithmetic, which deals with numbers and figures, and exceeds algebra, the language of symbols and relationships. It is more than geometry, which studies shape, size, and space, in addition to trigonometry, statistics, calculus, etc.—the traditional branches that constituted it until recently. According to the modern perspective, it now encompasses more than the sum of these branches, as we will explain below.

1.2 The Method of Mathematical Reasoning

Mathematical reasoning represents the approach followed by mathematicians in their treatment of various contents (symbols, limits, matrices, etc.) and in their derivation of diverse theories from premises assumed to be true.

The Nature of This Deductive Method:

The essence of the deductive method can be summarized by its foundation on a set of basic elements, which are:

- Undefined Terms or Primitive Notions: These are a category of terms whose meanings are understood without the need for definition and are specific to a certain system, acquiring their meaning through it.
- Defined Terms: These are a category of statements that clarify the concepts or terms used within a certain mathematical system, with the undefined terms being components of these statements.
- Axioms: These are a category of statements defined by the properties of the primitive notions (also known as axioms or undefined terms), which govern the relationships between them and are accepted as true without proof. Axioms must satisfy three conditions: independence, completeness, and consistency.

The evolution and changes that mathematics has undergone, as previously mentioned, have in turn affected the development of the perspective on the deductive method and its usage, as represented in the following:

- The deductive method used to begin with several premises, including undefined terms, defined terms, and axioms, which were accepted as true without proof. Therefore, the truth of the results was considered absolute, with the criterion for this absolute truth being conformity to physical reality. However, with the changing nature of mathematics, these premises became mere hypotheses, not necessarily connected to physical reality. The only requirement is that they must be non-contradictory within the framework of the specific mathematical system they are part of, and thus the resulting conclusions are considered true only within that mathematical system.
- The deductive method used to distinguish between axioms and postulates in terms of their self-evident nature. However, it came to view both as mere assumptions of the truth of the statement within the context of the given proof.
- The view of the deductive method was that it relied on the meanings encompassed by mathematical propositions and their properties, using "material implication" to derive propositions from their premises. This imposed certain restrictions on the processes of mathematical reasoning, giving the deductive method a material nature.
- In summary, from all the above, it is emphasized that mathematics is inherently deductive in nature, where true results can be derived from premises assumed to be true, through deductive steps governed by the laws of logic. Based on this, the deductive method is used in deriving its theories and results, making mathematics a deductive structure characterized by abstraction (i.e., it does not carry any material or physical meaning) and gaining its meaning through the mathematical system in which it is used.

1.3 Mathematics as a Science and as a Study Subject

There is a difference between mathematics as a science and mathematics as a study subject taught by teachers to groups of students at various educational stages. These students go through successive stages of development, each characterized by different mental, physical, and psychological traits. Consequently, the approach to mathematics, its teaching methods, and learning strategies vary to suit each stage of development. Hence, it is essential to distinguish between mathematics as a science and mathematics as a study subject. However, before making this distinction, it should be noted that while mathematics as a science differs from mathematics as a study subject in terms of treatment methods, presentation style, and the complexity or focus of the material, both share the fundamental characteristic of being deductive structures.

In differentiating between mathematics as a science and as a study subject, Land (1963) notes that mathematics as a science has developed over long periods due to studies conducted and discoveries made by mathematicians, thus taking its current form. On the other hand, mathematics as a study subject is based on the fundamental concepts of mathematical science but needs to be simplified to match the characteristics of the learner at a specific stage of development and suit their individual readiness.

In the same context, Lamon (1973) adds that mathematics as a science is a deductive structure. However, when it is taught to learners, the focus is not on teaching them how to derive new mathematical information as scientists do. Instead, the emphasis is on equipping them with the ability to perform simple deductive operations that allow them to derive some results from the available mathematical information.

Furthermore, the axioms in mathematical science are abstract, whereas, in the case of the study subject, these axioms need to be clear and understandable for the learner. They should be initially presented with concrete examples before progressing to abstractions and then returning to concreteness through various applications to real-life situations and problems.

1.4 Objectives of Teaching Mathematics

The clarity of the aforementioned distinctions between mathematics as a science and as a study subject has helped crystallize the general educational objectives of teaching mathematics, especially at the secondary level. These objectives can be summarized as follows:

- Acclimating students to using the scientific method: Emphasizing organized observation and inference to enhance their ability for discovery and innovation.
- Equipping students with mathematical principles: Enabling students to comprehend economic and social activities, as well as the evolutionary processes affecting society as a whole.
- Assisting students in understanding aspects of civilization: Keeping abreast of scientific advancements across various fields.
- Facilitating students' comprehension of mathematical methodologies: Encouraging precision in their application, while reinforcing concentration, attention, and perseverance in problem-solving situations.

1.5 Current Study Objectives

From all that has been presented and considering the diverse aspects between mathematics as a science and as a study subject, along with the educational objectives intended through its teaching, our endeavor in this study is to verify the following objectives:

1.5.1 to explore the extent and direction of the correlational relationship between mathematical reasoning (as a higher cognitive ability) and achievement in mathematics as an educational content in general. Additionally, to examine the extent and direction of this relationship across varying levels of proficiency in this specific subject area.

1.5.2 to investigate the differences in the extent and direction of the correlational relationship between mathematical reasoning and achievement in other academic subjects, differing in their nature such as scientific, linguistic, and humanities disciplines, as typically offered to high school students. Also, to delineate the boundaries of change in the extent and direction of this relationship across varying levels of academic performance in these academic courses as a whole.

These details are clarified following our exposition of the key concepts of the study.

2. Concepts of Studies

2.1 Excellence or Giftedness

Regarding excellence in general, the definitions of "gifted" and "talented" explain the nature of this achievement, encompassing multiple aspects that are widely accepted among researchers today ([1]).

The term "gifted and talented" refers to students at various stages of general education, starting from early childhood education through primary, intermediate, and secondary education, who can be identified as possessing latent or

[1] This definition was announced in the Congressional Digest in 1979, under Section 902 of Decree No. 95-561, concerning the education of gifted and talented children (see: 1989, Taylor).

manifest potentials (various capabilities and readiness). Their presence serves as evidence of exceptional or outstanding performance in intellectual, creative, academic, leadership skills, or in visual and performing arts, among others ([2]).

There exists a nuanced difference between the terms "gifted" and "talented." According to Hallahan and Kuffman (1988), "gifted" denotes cognitive and creative excellence accompanied by strong motivation for achievement in these domains. On the other hand, the term "talented" refers to specific ability, readiness, or outstanding achievement in a particular area of performance. Researchers Kirk and Gallagher (1986) affirm a significant positive correlation between cognitive ability and outstanding performance across various domains.

While academic achievement is common among gifted students, some specialists (often in psychology and education, such as Kirk and Gallagher, 1986) estimate that between 15% and 20% of students do not attain the expected levels of academic achievement. This is likely due to a range of factors including lack of interest in academic content, low motivation for achievement, lack of self-confidence, absence of certain indicators of mental health in the learner's personality, or ineffective learning methods employed by the individual in acquiring, producing, or processing various educational content and information.

2.2 Mathematical Reasoning

The concept of mathematical reasoning encompasses two types of reasoning. Firstly, there is induction, or inductive reasoning, which involves generalizing observations and abstract processing of empirical evidence, leading to the formulation of mathematical principles or generalizations that can be applied to similar cases in the future.

On the other hand, deduction involves processing ideas (or accepted mental premises) using logical rules, aiming to formulate generalizations, principles, or rules that can be applied in dealing with similar intellectual issues or components.

In engineering and other branches of mathematics, results can be deduced using logical rules. Proving the validity of theories itself involves deductive reasoning because proving the validity of any theory requires selecting and organizing a set of definitions, premises, and previous theories

Mathematical reasoning, as a capacity within abstract thinking abilities or as a high cognitive ability, can be measured using scientific methods, including psychometric tests such as the Mathematical Reasoning Test, which is utilized in the current study.

2.3 Academic Achievement

Academic achievement generally refers to a student's understanding of various academic contents, their ability to process underlying concepts, and their capacity to solve related problems in educational and real-world contexts.

The effectiveness of academic achievement is typically measured by a student's success grades in any educational subject they are examined in, either orally (in the classroom) or in written form during official examination situations.

Our focus in this study is primarily on academic achievement in mathematics, and exploring the relationship between performance proficiency in mathematics reasoning skills on one hand, and the relationship between mathematical reasoning and achievement in other subjects on the other hand, assuming it to be a high-level cognitive ability underlying academic achievement in all its forms.

3. Previous Studies

The following review of previous studies highlights the extent of researchers' interest in this field regarding mathematical ability (or mathematical reasoning) on one hand, and the characteristics of those excelling in mathematics on the other.

3.1 Studies on the Nature of Mathematical Ability

Researchers have long been conducting numerous studies on mathematical ability, aiming to uncover its nature, its general relationship to mental faculties, and its specific connections to other cognitive variables. Below is an overview of some of these studies.The contemporary Swedish mathematician Werdelin (1958) defines mathematical ability as the capacity to understand, remember, and apply mathematical symbols and methods. He arrived at this definition through his analysis of the structure of this ability in students based on his multifactorial model. He determined the relative role of this ability in learning mathematics and its relationship to mental faculties in general by applying a battery of 36 tests to a sample of 217 students. He concluded that the mathematical tests showed minimal saturation with both the numerical and verbal factors, and no significant saturation with the spatial factor. However, significant saturation was found in the reasoning factor, which he termed the general mathematical reasoning factor. From this, he inferred that this factor plays a crucial role in the structure of mathematical ability. And The Another study by Canisia (1962) applied 36 tests to a sample of 150 female high school students and processed the data factorially using the central

[2] The term "performing arts" here refers to: rhythmic-movement arts, music, drama, etc.

method, resulting in twelve factors. The most important finding of this researcher was that mathematical ability is essentially the ability to engage in mathematical reasoning using symbols, and that mathematical thinking processes are linked to the capacity to draw conclusions, organize structures, and handle relationships. Additional studies attempted to determine the relationship between primary mental abilities, as discovered by Thurstone, and mathematical ability. One such study by Tringham (1971) conducted in Uganda on 58 high school students concluded that there are significant correlations between academic achievement and only two factors in Thurstone's battery: the reasoning factor and the numerical factor.Hamid Al-Abed (1971) used the same correlation matrix from Tringham's previous study and analyzed it factorially using the principal components method. He identified six factors: the numerical factor, fluency factor, spatial factor, deductive reasoning, linguistic reasoning, and writing factor. Mathematical achievement showed the highest saturation with the deductive reasoning factor compared to the other factors.

3.2 Studies on the Characteristics of Mathematics Prodigies

Krutetskii (1976) demonstrated that students who excel in solving mathematical problems are characterized by their rapid generalization of mathematical elements. They have the ability to form certain connections with a minimal number of attempts and show clear flexibility in mental processes. They simplify understanding and comprehension processes to arrive at correct solutions appropriately. They exert their utmost effort to reach clear and simple solutions, arranging their mental processes in understanding and addressing mathematical problems in ways that differ from regular students (See: Sowell, 1990).

Similarly, Osborn (1981) noted that students who excel in mathematics are characterized by uniqueness, the ability for mathematical abstraction, and generalization. Payme (1981) also pointed out that gifted students in mathematics exhibit several traits distinguishing them from non-gifted peers, including a high ability for abstract logical reasoning and problem-solving. They are inclined towards complex problems that stimulate and challenge their thinking. Greens (1981) indicated that mathematically gifted students possess a high ability to organize mathematical data and information, flexibility in handling it, and originality in detailing the data within a mathematical context. They also demonstrate high skill in transferring the effect of learning, possessing keen observational skills, and quickly becoming bored by repetitive explanations from the teacher or routine presentations of material. She also noted that the mathematical giftedness could easily be nurtured towards abstraction, as well as skills related to initiative and creativity. Other researchers (Weaver & Green, 1981, Jung, 1957; and Brawley, 1959) found several distinctive traits of mathematically gifted students compared to regular students. These include the ability to apply previously learned knowledge in other contexts, quickly perceiving relationships between given information in problems to be solved, swiftly transitioning from concrete to abstract thinking, high problem-solving ability, efficiency in mathematical logical reasoning, arriving at multiple solutions, and using various methods or strategies to solve a single mathematical problem. El-Shakhs (1990) pointed to strong evidence that mathematically gifted students exhibit high proficiency in dealing with numbers, shapes, and mathematical concepts. These indicators can identify them early in life. While some mathematically gifted students may have high intelligence, tests involving complex arithmetic problems are necessary to identify those with genuine mathematical talent . Krutetskii (1976) also revealed through a comparison between mathematically gifted students and those of average ability in problem-solving that the gifted students approach problems analytically (i.e., separating different elements of the problem, determining the value of each part individually, then organizing and sequencing them) and then synthetically, deriving mathematical relationships and implications. Finally, a study using the SEMBI educational model for teaching mathematics revealed that most students excelling in this model discover mathematical concepts and principles that were not previously taught to them.

In general, this review of previous studies on mathematical ability (or mathematical reasoning abilities) and its various characteristics, especially regarding the traits of mathematically gifted students, concludes that there is still a need for further research to study the relationship between mathematical reasoning as a higher cognitive ability and achievement in mathematics as an educational content. It also seeks to explore the extent of influence of this cognitive ability (mathematical reasoning) on students' achievement in different subjects and overall academic excellence. This is what the current study aims to achieve, representing an attempt to answer several important questions in this regard, which will be detailed in the context of defining the research problem and the methodology followed in the next chapter.

4. Methodology

In conducting the current study, we rely on the descriptive correlational method. The aim of this study is to explore the extent and direction of the correlational relationship between mathematical reasoning ability on one hand, and academic achievement scores in various subjects, primarily mathematics, on the other hand.

4.1 Research Problem and Study Hypotheses

The research problem addressed in this study is defined by the following questions:

4.1.1 What is the extent and direction of the correlational relationship between mathematical reasoning, as a higher cognitive ability, and achievement in mathematics as an educational content?

4.1.2.To what extent does the magnitude and direction of this correlational relationship between mathematical reasoning and achievement in mathematics vary with different levels of academic performance in this subject?

4.1.3 What is the extent and direction of the correlational relationship between mathematical reasoning and achievement in various academic subjects?

4.1.4 To what extent do the magnitude and direction of the correlational relationships between mathematical reasoning and different academic subjects vary with different levels of academic performance in these subjects?

Naturally, these questions can be answered by attempting to verify the validity of the following scientific hypotheses:

Hypothesis 1: There is a strong correlational relationship between mathematical reasoning (as a higher cognitive ability) and achievement in mathematics, as presented to 12 grade students.

Hypothesis 2 :The correlational relationship between mathematical reasoning and achievement in mathematics varies in strength and weakness, depending on the level of proficiency in mastering this subject.

Hypothesis 3: There is a strong relationship between the variable of mathematical reasoning and various academic subjects in general. However, the correlational relationship is stronger between this ability and scientifically-oriented subjects compared to linguistically or humanistically-oriented subjects.

Hypothesis 4: The extent of the correlational relationships (strength and weakness) between mathematical reasoning and different academic subjects varies with the overall level of academic proficiency.

4.2 Procedural Definition of Variables Included in the Study Hypotheses

- Mathematical Reasoning: Operationally defined as the score obtained by the student in the mathematical reasoning test.
- Achievement in Mathematics: Operationally defined as the grades obtained in the final exam of this subject at the end of the academic year preceding the study.
- Achievement in Various Study Courses: Operationally defined as the grades obtained in the exams of each of these courses at the end of the academic year preceding the study.
- Overall Academic Performance: Operationally defined as the total score in the exams of various study courses at the end of the academic year preceding the study, statistically treated as a percentage.
- Levels of Academic Performance Proficiency: Primarily categorized into three performance levels (High, Average, Low) based on the division of the total sample into three subgroups, according to the mean and standard deviation of grades in mathematics first, and then for the total grades in all courses, as detailed further in the presentation of study procedures later on.

5. Study Procedures

Sample: The study sample consisted of 106 male students at the secondary education level, randomly selected from secondary schools in Kuwait, specifically from the 12 grade of the scientific section. The names of schools and the numbers of selected students from each are detailed in Table (1).

Table (1). Names of schools and the number of students selected from each one

School Name	Division	Total
Urwa Bin Al-Zubair School	Twelfth Scientific (1)	17
	Twelfth Scientific (2)	15
Al-Asmai School	Twelfth Scientific (1)	18
	Twelfth Scientific (2)	16
Hammoud Al-Jaber Al-Sabah School	Twelfth Scientific (1)	20
	Twelfth Scientific (2)	20
Total		106

5.2 Study Tools

The study utilized the Mathematical Reasoning Test from the Differential Aptitude Battery (DAB), which measures the ability to solve mathematical problems through reasoning rather than arithmetic operations and learned mathematical principles. Efficiency in solving mathematical problems on this test is assessed based on logical reasoning processes. Therefore, this test is considered a reliable tool for predicting success in mathematics achievement in subsequent academic years. It also serves as a criterion to evaluate the efficiency of mathematical reasoning abilities, which are crucial for specializing in mathematics and various professions requiring numerical and mathematical symbol processing.

The Arabic version of this test was used in the study, which has been developed and standardized through numerous studies conducted at the Psychological Research and Studies Center at Cairo University.

Regarding grades in mathematics and other academic courses, they were obtained from school records where the study was conducted.

5.3 Steps Followed in Conducting the Study

This study was implemented following the steps below:

- Applying the Mathematical Reasoning Test collectively in classroom sessions during the period from April 12, 2023, to April 18, 2023. This was conducted over six full study periods, each lasting 45 minutes.
- Recording the grades of various subjects in the end-of-year exams from the previous academic year directly from the school records for each student.
- Correcting the Mathematical Reasoning Test according to the designated correction instructions.
- Compiling and entering study variable data (psychometric and academic) into the computer, then verifying afterward.

5.4 Performing Initial Statistical Analyses as Follows

Calculating means and standard deviations for various research variables for all sample individuals (106 students).

5.5 Calculating Correlation Coefficients among the Study Variables

Calculating correlation coefficients among the study variables (inter-variables), which include 8 academic subjects, total academic achievement scores, and Mathematical Reasoning Test scores.

Dividing the total sample into three groups based on the arithmetic mean and standard deviation of mathematics achievement scores, and the arithmetic mean and standard deviation of overall academic performance:

Based on this, the subgroups of the study were defined as follows:

5.5.1 Regarding Achievement in Mathematics

- High Achievement Group: There were 33 students who scored 70 or higher (= Mean + Half Standard Deviation).
- Medium Achievement Group: There were 33 students whose scores ranged between 58 and 70 (= Mean ±Half Standard Deviation around the mean).
- Low Achievement Group: There were 40 students whose scores were below 58 (= Mean - Half Standard Deviation).

5.5.2 Regarding Overall Academic Performance

- High Academic Achievement Group: Consisting of 32 students who scored above 72% of the total exam grades across all subjects combined (= Mean + Half Standard Deviation).
- Medium Academic Achievement Group: Comprising 33 students whose scores ranged between 60% and 71% (= Mean ±Half Standard Deviation around the mean).
- Low Academic Achievement Group: Including 41 students whose scores were below 60% across various subjects (= Mean - Half Standard Deviation).

6. Study Results and Discussion

Through statistical analysis of the study data, the following descriptive indicators were derived regarding the means and standard deviations of the various study variables for the entire sample, as shown in Table 2 below.

Table (2). Means and Standard Deviations for the Sample in Various Study Variables (N = 106)

	Mean	**Standard deviation**
Mathematical reasoning	20	11.9
Mathematics	64.6	12.6
Physics	66.4	12.2
Chemistry	65.5	14
Biology	66.6	12.8
Geology	63.9	13.6
Arabic Language	64.5	12
English Language	65.9	14.3
Islamic Education	74.2	12.6
Total Achievement	66	12

It was also possible to derive the correlation coefficients between mathematical reasoning and various academic subjects for the entire sample (106 students), as shown in Table 3.

Table(3). Correlation Coefficients Between Mathematical Reasoning and Various Academic Subjects

Study variables	Correlation coefficient values
Mathematics	**6222.
Physics	**486.
Chemistry	**504
Biology	**505.
Geology material	**475.
Arabic Language	**531.
English Language	**555.
Islamic Education	**476.
Total Achievement	**567.

The correlation coefficients shown in Table (3) reveal the following:

- The correlation between mathematical reasoning and achievement in mathematics is higher than that in other academic subjects overall.
- The correlation between mathematical reasoning and language-based subjects is higher than that for science-based subjects (excluding mathematics), although the relationship between mathematical reasoning and various academic subjects is generally strong, with a significance level reaching 0.01.

From this result, it is evident that the correlation between mathematical reasoning and different academic subjects confirms the role of mathematical reasoning (as a higher cognitive ability) in academic performance overall, and in academic achievement in mathematics in particular, thus validating the first hypothesis of the study. However, the higher correlation between mathematical reasoning and language subjects differs from what was stated in the third hypothesis of the study.

Regarding the correlation between mathematical reasoning and achievement in mathematics, considering varying levels of proficiency in this subject, as shown in Table (4), the stance from this aspect becomes clear.

Table (4). The Correlation Between Mathematical Reasoning and Achievement in Mathematics Across the Three Levels of Performance Proficiency in This Subject

Performance levels	Number	Correlation coefficient values
High Level	33	**0.750
Intermediate Level	33	**0.130
Low Level	40	0.111

From the correlation coefficients in Table No. (3), it is evident that the relationship between mathematical reasoning and achievement in mathematics varies in strength across different levels of proficiency in this subject. It is strongest at the high proficiency level, with a significance level of (0.01), and this relationship diminishes in the medium and low proficiency levels.

This finding supports the second hypothesis of the study, which suggests that the correlation between mathematical reasoning and achievement in mathematics varies with the level of academic performance proficiency in this subject.

Regarding the fourth hypothesis of the study, concerning the relationship between mathematical reasoning and overall academic achievement across various courses, amidst varying levels of academic performance proficiency, this is illustrated by correlation matrices in Tables (5), (6), and (7) as follows:

Table (5). Correlation matrix of correlation coefficients among study variables at the high level of academic performance overall (n = 32)

	Mathematical reasoning	Math	Physics	Chemistry	Biology	Geology	Arabic Language	English Language	Islamic Education
Mathematical reasoning		**0.538	*0.410	**0.592	**0.503	**0.486	**0.582	**0.575	**0.575
Mathematics			**0.629	**0.706	**0.745	**0.541	**0.570	**0.413	*0.411
Physics				**0.792	**0.728	**0.731	**0.582	**0.461	0.335
Chemistry					**0.694	**0.785	**0.745	**0.618	**0.531
Biology						**0.805	**0.628	**0.525	**0.447
Geology							**0.697	**0.583	**0.630
Arabic Language								**0.775	**0.738
English Language									**0.652
Islamic Education									

Upon examining the correlation coefficients in Table (5), the following becomes clear:

All variables are highly correlated with each other, and mathematics shows the highest overall correlation with scientific subjects. Additionally, mathematical reasoning (as a measure of high cognitive ability) is strongly correlated with all academic courses overall, except for physics, where the correlation is lower. At present, I have no explanation for this. Furthermore, the correlation between similar subjects in nature (scientific and linguistic) is higher than that between different subjects.

In summary, the overall picture of this matrix indicates consistent and high correlations among academic subjects for academically high-achieving students.

Table (6). Correlation coefficients matrix among study variables at the intermediate level of academic performance overall (n = 33)

	Mathematical reasoning	Math	Physics	Chemistry	Biology	Geology	Arabic Language	English Language	Islamic Education
Mathematical reasoning		**0.651	*0.408	0.194	0.33	-0.011	0.274	0.205	0.284
Mathematics			**0.545	*0.352	0.276	0.066	-0.038	-0.063	0.223
Physics				0.229	*0.351	0.059	-0.064	-0.078	0.013
Chemistry					*0.374	0.193	-0.104	-0.203	0.038
Biology						0.281	0.16	-0.22	*0.427
Geology							*0.369	-0.121	*0.346
Arabic Language								0.139	**0.575
English Language									0.054
Islamic Education									

From Table (6), we find that this matrix is very sparse in significant correlation coefficients among all variables with each other, with only 10 correlation coefficients present. These coefficients generally have a low significance level compared to the correlation matrix of academically high-achieving students (Table 5), which clearly featured 35 highly significant correlation coefficients.

We also observe a clear inconsistency in the correlation between mathematical reasoning and mathematics and physics only, with its absence in other scientific courses. Additionally, we find correlations between achievement in mathematics with physics and chemistry only, and also correlations between scientific subjects (geology and biology) with Arabic language and Islamic education, but not with similar scientific subjects.

Overall, the discernible pattern indicates a noticeable decline in academic performance efficiency across various courses, resulting in inconsistent strength of correlation relationships at this intermediate level of academic achievement.

Table (7). Correlation coefficients matrix among study variables at the low level of academic performance overall (n = 41)

	Mathematical reasoning	Math	Physics	Chemistry	Biology	Geology	Arabic Language	English Language	Islamic Education
Mathematical reasoning		**0.374	-0.249	-0.077	-0.123	0.073	-0.037	**0.404	-0.19
Mathematics			-0.099	0.085	-0.188	-0.031	0.107	0.058	-0.115
Physics				0.251	0.244	-0.144	-0.043	0.089	0.027
Chemistry					0.232	0.228	0.041	0.137	0.191
Biology						0.272	0.205	0.051	-0.101
Geology							0.301	0.118	0.088
Arabic Language								*0.311	0.274
English Language									0.147
Islamic Education									

From Table (6), we discover that the correlation relationships among academic courses nearly vanish. The entire matrix contains only three correlation coefficients, two of which are weakly significant. One particular coefficient raises questions—it is between mathematical reasoning and English language, excluding other scientific courses (except mathematics).

Overall, this inconsistency and the disappearance of correlation relationships among study variables, compared to the high and medium levels of academic performance, raise numerous questions about the nature of the underlying factors behind this phenomenon. Nonetheless, the fourth hypothesis of the study is validated, indicating varied correlation relationships between mathematical reasoning and different academic courses, depending on the level of academic achievement proficiency overall.

Moreover, these tables may reveal indicators about the nature of factors contributing to the inconsistency in correlation relationships between mathematical reasoning and other study variables, and among academic subjects themselves.

6.2 From These Indicators, the Following Points Emerge

- There is a clear consistency in academic performance efficiency at the high level of academic achievement, indicating a balance in the attention given by high-achieving students to various study materials, despite their different content.
- Academically high-achieving students appear to possess time and energy management skills, not just cognitive or high intellectual abilities. Possessing these skills impacts their academic performance across all subjects, not just mathematics, and even in pure cognitive tasks, such as performance in mathematical reasoning tests.
- The results also reveal clear mental flexibility in processing or addressing academic content among academically high-achieving individuals. This is evident from the varied nature of these subjects (scientific, linguistic, and humanities), yet all demonstrate high levels of academic achievement.
- It also appears that academically high-achieving students employ effective cognitive strategies and methods in learning, as evidenced by the strong correlations between mathematical reasoning ability and performance in various academic subjects.

In conclusion, it is likely that non-cognitive factors play a role in academic success overall, influenced by time management, balanced allocation of attention and energy to academic content, and mental performances alike. Motivation, use of effective learning strategies, and other non-cognitive factors positively contribute to academic excellence, both specific and general cognitive achievements.

All of the above points highlight the need for further studies on non-cognitive characteristics within the learning and education system for both high-achieving and non-high-achieving individuals. This also calls for more attention to the development of balanced and effective learning methods and skills, complementing the teaching of information based solely on cognitive abilities, as detailed in the following sections.

6.3 Study Recommendations

This study revealed that academic excellence is not solely dependent on cognitive abilities for high-achieving students. Rather, it involves learning strategies, skills, and specific systems that these students follow to achieve academic success. This includes balancing their attention across different subjects, organizing their time effectively, and efficiently distributing their energy in their educational and intellectual efforts. The energy they allocate to mathematical reasoning, for instance, is mirrored in their approach to subjects like mathematics, Arabic language, and other educational content as cognitive knowledge domains. Therefore, it can be said that high-achieving students adopt effective learning strategies that significantly differ from those of non-high achievers.The most significant conclusions drawn from this are summarized in the following recommendations:

1. There is a need to prioritize the development of skills, methods, and effective strategies in teaching and learning environments, focusing on students in general rather than solely relying on cognitive abilities.
2. There is a necessity for further future studies in this field, addressing:

- Exploring appropriate teaching methods for high-achieving students in secondary education, especially in mathematics.
- Comparing mathematical reasoning approaches with problem-solving approaches in teaching mathematics.
- Surveying the opinions of high-achieving students regarding their specific needs in academic achievement situations in general, and particularly in the field of mathematics.
- Conducting studies to identify the characteristics that competent mathematics teachers should possess, which could enhance assessment processes and develop programs aimed at acquiring these characteristics.

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Authors contributions

Dr.Hamdan Alharbi presented the research as an idea, and then the researchers collected the ideas and outlined the research. The researchers read the final manuscript and approved it. Dr.Hamdan Alharbi was responsible for the general formulation of the research methodology, problem, and procedures, and researcher Sara Aldhafairi was responsible for formulating the main concepts of the research, previous studies, and field visits to collect the research results. The two researchers together extracted the results of the study, discussed them, and developed appropriate recommendations for the scientific research. Thus, it can be said that the researchers made equal efforts to produce this joint research.

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The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Obtained.

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