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The Validity of Generalized Modal Syllogisms with the Quantifiers in Square{*most*} and Square{*no*}

Jing Xu[1] & Yuzhen Wang[2]

[1] School of Marxism, Anhui Medical University, Hefei, China

[2] School of Philosophy, Anhui University, Hefei, China

Correspondence: Jing Xu, No. 81, Meishan Road, Shushan District, Anhui Medical University, Hefei, China.

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Abstract

On the basis of the valid generalized modal syllogism □EM◇O-2 containing the quantifiers in Square{*most*} and Square{*no*}, this paper explores the validity of the other generalized modal syllogisms. The reasons of the reducibility between/among valid generalized modal syllogisms are: that there are transformational relationships between/among Aristotelian quantifiers in Square{*no*} or generalized ones in Square{*most*}, and that the necessary modality and the possible one are dual, and that the quantifiers *some* and *no* are symmetric. This method is universal, and helps to study the validity of other kinds of syllogisms. It is hoped that the above results can promote the development of related fields such as computational linguistics.

Keywords: generalized modal syllogisms, validity, Square{*no*}, Square{*most*}, reducibility

1. Introduction

Various quantifiers are used in natural language. In addition to Aristotelian quantifiers (that is, *no*, *all*, *not all* and *some*), there are also a large number of generalized quantifiers, such as: *both*, *many*, *most*, *half of the*, *exactly five*, and so on (Barwise & Cooper, 1981; Peters & Westerståhl, 2006). Any quantifier Q has three negation: outer negative ¬*Q*, inter negative *Q*¬ and dual negative ¬*Q*¬. They build up a modern square{*Q*, ¬*Q*, *Q*¬, ¬*Q*¬}, abbreviated as Square{*Q*} (Zhang, 2014; Lin, 2015). For instance, Square{*no*}={*no*, *some*, *all*, *not all*} and Square{*most*}={*most*, *at most half of the*, *fewer than half of the*, *at least half of the*}. A generalized modal syllogism is obtained by adding one/two/three non-overlapping necessary modality □ or possible modality ◇ to a generalized syllogism (Xu & Zhang, 2023; Yang, 2024). '*most*' is a very common non-trivial generalized quantifier in natural language (Hao, 2024). On the basis of the valid generalized modal syllogism □EM◇O-2 including the quantifiers in Square{*most*} and Square{*no*}, this paper attempts to study the validity of other non-trivial generalized modal syllogisms.

2. Preliminaries

In this paper, let *k*, *n*, *t* be lexical variables, which discussed in domain *D*. The sets that consist of *k*, *n*, and *t* are *K*, *N*, and *T* respectively. '$|K|$' stands for the cardinality of the set *K*, and '$|K \cap T|$' signifies the cardinality of the set that is the intersection of *K* and *T*. Assuming φ, λ, μ, ν be well-formed formulas (often abbreviated as wff). '$\varphi =_{def} \lambda$' shows that φ can be defined as λ. '⊢φ' means that the wff φ can be proved. The meanings of operations (such as ¬, →, ∧, ↔) are the same as that in first order logic (Hamilton, 1978).

The generalized modal syllogisms discussed in this paper merely involve quantifiers from the Square{*most*} and Square{*no*}. Thus they involve the eight types of categorical propositions as shown in Table 1.

Table 1. Eight types of categorical propositions involved in generalized modal syllogisms

Categorical proposition	Tripartite structure	Abbreviation
No *k*s are *t*s	*no*(*k*, *t*)	*E*
Some *k*s are *t*s	*some*(*k*, *t*)	*I*
All *k*s are *t*s	*all*(*k*, *t*)	*A*

Not all *k*s are *t*s	*not all*(*k*, *t*)	*O*
Most *k*s are *t*s	*most*(*k*, *t*)	*M*
At most half of the *k*s are *t*s	*at most half of the*(*k*, *t*)	*H*
Fewer than half of the *k*s are *t*s	*fewer than half of the*(*k*, *t*)	*F*
At least half of the *k*s are *t*s	*at least half of the*(*k*, *t*)	*S*

When adding one necessary modality (□) or one possible modality (◇) to these 8 categorical propositions, one can obtain 16 modal categorical propositions: □*E*, □*I*, □*A*, □*O*, □*M*, □*H*, □*F*, □*S*, ◇*E*, ◇*I*, ◇*A*, ◇*O*, ◇*M*, ◇*H*, ◇*F* and ◇*S*, respectively. A generalized modal syllogism contains at least one modal categorical proposition (Wei & Zhang, 2023). The definitions of its figures are as usual (Chen, 2020). The syllogism □EM◇O-2 is the abbreviation for the second figure syllogism □*no*(*t*, *n*)∧*most*(*k*, *n*)→◇*not all*(*k*, *t*). Many instances in natural language correspond to this syllogism. For example,

Major premise: No apples are necessarily peaches. (Formalized as □*no*(*t*, *n*))

Minor premise: Most fruits in this shop are peaches. (Formalized as *most*(*k*, *n*))

Conclusion: Not all fruits in this shop are possibly apples. (Formalized as ◇*not all*(*k*, *t*))

3. Formal System of Generalized Modal Syllogisms

The formal system of generalized modal syllogisms includes the following components:

3.1 Primitive Symbols

(1) brackets: (,)

(2) operators: ¬, →

(3) modality: □

(4) quantifiers: *no*, *most*

(5) lexical variables: *k*, *n*, *t*

3.2 Formation Rules

(1) Provided that *Q* is a quantifier, *k* and *t* are lexical variables, it follows that *Q*(*k*, *t*) is a wff.

(2) Provided that φ is a wff, it follows that ¬φ and □φ are wffs too.

(3) Provided that λ and μ are wffs, it follows that λ→μ is a wff.

(4) The set of all wffs is generated from (1) to (3).

3.3 Deduction Rules

Rule 1: From ⊢(φ∧λ→μ) and ⊢(μ→ν), ⊢(φ∧λ→ν) can be inferred.

Rule 2: From ⊢(φ∧λ→μ), ⊢(¬μ∧φ→¬λ) can be inferred.

Rule 3: From ⊢(φ∧λ→μ), ⊢(¬μ∧λ→¬φ) can be inferred.

3.4 Relevant Definitions

Definition 1 (truth value of categorical propositions):

(1.1) *all*(*k*, *t*)$=_{\text{def}} K\subseteq T$;

(1.2) *some*(*k*, *t*)$=_{\text{def}} K\cap T\neq\varnothing$;

(1.3) *no*(*k*, *t*)$=_{\text{def}} K\cap T=\varnothing$;

(1.4) *not all*(*k*, *t*)$=_{\text{def}} K\nsubseteq T$;

(1.5) *most*(*k*, *t*)$=_{\text{def}} |K\cap T|>0.5|K|$.

Definition 2 (truth value of modal categorical propositions):

(2.1) □φ is true just in case φ itself is true at every possible world;

(2.2) ◇φ is true just in case φ is true at some possible world.

Definition 3 (inner negation): (*Q*¬)(*k*, *t*)$=_{\text{def}}$ *Q*(*k*, *D*–*t*).

Definition 4 (outer negation): (¬*Q*)(*k*, *t*)$=_{\text{def}}$ It is not that *Q*(*k*, *t*).

Definition 5 (conjunction): $(\varphi\wedge\lambda)=_{def}\neg(\varphi\rightarrow\neg\lambda)$.

Definition 6 (bi-condition): $(\varphi\leftrightarrow\lambda)=_{def}(\varphi\rightarrow\lambda)\wedge(\lambda\rightarrow\varphi)$.

3.5 Relevant Facts

Fact 1 (inner negation):

(1.1) ⊢*all*(*k*, *t*)↔*no*¬(*k*, *t*);

(1.2) ⊢*no*(*k*, *t*)↔*all*¬(*k*, *t*);

(1.3) ⊢*some*(*k*, *t*)↔*not all*¬(*k*, *t*);

(1.4) ⊢*not all*(*k*, *t*)↔*some*¬(*k*, *t*);

(1.5) ⊢*most*(*k*, *t*)↔*fewer than half of the*¬(*k*, *t*);

(1.6) ⊢*fewer than half of the*(*k*, *t*)↔*most*¬(*k*, *t*);

(1.7) ⊢*at least half of the*(*k*, *t*)↔*at most half of the*¬ (*k*, *t*);

(1.8) ⊢*at most half of the*(*k*, *t*)↔*at least half of the*¬ (*k*, *t*).

Fact 2 (outer negation):

(2.1) ⊢¬*all*(*k*, *t*)↔*not all*(*k*, *t*);

(2.2) ⊢¬*not all*(*k*, *t*)↔*all*(*k*, *t*);

(2.3) ⊢¬*no*(*k*, *t*)↔*some*(*k*, *t*);

(2.4) ⊢¬*some*(*k*, *t*)↔*no*(*k*, *t*);

(2.5) ⊢¬*most*(*k*, *t*)↔*at most half of the*(*k*, *t*);

(2.6) ⊢¬*at most half of the*(*k*, *t*)↔*most*(*k*, *t*);

(2.7) ⊢¬*fewer than half of the*(*k*, *t*)↔*at least half of the*(*k*, *t*);

(2.8) ⊢¬*at least half of the*(*k*, *t*)↔*fewer than half of the*(*k*, *t*).

Fact 3 (duality):

(3.1) ⊢¬□*Q*(*k*, *t*)↔◇¬*Q*(*k*, *t*); (3.2) ⊢¬◇*Q*(*k*, *t*)↔□¬*Q*(*k*, *t*).

Fact 4 (symmetry):

(4.1) ⊢*some*(*k*, *t*)↔*some*(*t*, *k*); (4.2) ⊢*no*(*k*, *t*)↔*no*(*t*, *k*).

Fact 5 (subordination):

(5.1) ⊢□*Q*(*k*, *t*)→*Q*(*k*, *t*);

(5.2) ⊢□*Q*(*k*, *t*)→◇*Q*(*k*, *t*);

(5.3) ⊢*Q*(*k*, *t*)→◇*Q*(*k*, *t*).

The above rules, definitions and facts can be proven by first-order logic (Hamilton, 1978), modal logic (Chagrov & Zakharyaschev, 1997) and generalized quantifier theory (Peters & Westerståhl, 2006), the detailed proofs have been omitted for clarity.

4. How to Derive Valid Generalized Modal Syllogisms Based on □EM◇O-2

Our first task is to prove the validity for the syllogism □EM◇O-2 in Theorem 1. Then the remaining 23 valid generalized modal syllogisms can be derived from □EM◇O-2 in Theorem 2.

Theorem 1 (□EM◇O-2): The generalized modal syllogism □*no*(*t*, *n*)∧*most*(*k*, *n*)→◇*not all*(*k*, *t*) is valid.

Proof: Suppose that □*no*(*t*, *n*) and *most*(*k*, *n*) are true, then in the light of Definition (1.3) and (2.1), □*no*(*t*, *n*) is true just in case $T\cap N=\varnothing$ is true at every possible world. Similarly, according to Definition (1.5), *most*(*k*, *n*) is true just in case $|K\cap N|>0.5|K|$ is true at every real world. Because every real world is a possible world. Thus it can be obtained that $K\nsubseteq T$ is true. Now proving it by reductio ad absurdum. Assuming $K\nsubseteq T$ is not true, that is to say, $K\subseteq T$ is true. As we have got $T\cap N=\varnothing$, so $K\cap N=\varnothing$, which conflicts with the previous $|K\cap N|>0.5|K|$. This indicates that the assumption doesn't hold. It means that $K\nsubseteq T$ is true. Hence *not all*(*k*, *t*) is true in virtue of Definition (1.4). According to Fact (5.3), ◇*not all*(*k*, *t*) can be obtained immediately.

Theorem 2: The validity of the following 23 generalized modal syllogisms can be inferred from □EM◇O-2:

(2.1) ⊢□EM◇O-2→□EM◇O-1

(2.2) ⊢□EM◇O-2→□E□AH-1

(2.3) ⊢□EM◇O-2→□E□AH-1→□E□AH-2

(2.4) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1

(2.5) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1→□E□A◇H-2

(2.6) ⊢□EM◇O-2→M□A◇I-3

(2.7) ⊢□EM◇O-2→M□A◇I-3→□AM◇I-3

(2.8) ⊢□EM◇O-2→□AF◇O-2

(2.9) ⊢□EM◇O-2→□EM◇O-1→□AM◇I-1

(2.10) ⊢□EM◇O-2→□EM◇O-1→□AM◇I-1→M□A◇I-4

(2.11) ⊢□EM◇O-2→□E□AH-1→□A□AS-1

(2.12) ⊢□EM◇O-2→□E□AH-1→□A□AS-1→□A□A◇S-1

(2.13) ⊢□EM◇O-2→M□A◇I-3→F□A◇O-3

(2.14) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□A□EH-2

(2.15) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□A□EH-2→□A□E◇H-2

(2.16) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□A□EH-2→□A□EH-4

(2.17) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□A□EH-2→□A□E◇H-2→□A□E◇H-4

(2.18) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□EM◇O-3

(2.19) ⊢□EM◇O-2→□EM◇O-1→A□M◇I-1→□EM◇O-3→□EM◇O-4

(2.20) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1→□M□A◇I-3

(2.21) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1→□M□A◇I-3→□A□M◇I-3

(2.22) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1→□E□M◇O-2

(2.23) ⊢□EM◇O-2→□E□AH-1→□E□A◇H-1→□E□M◇O-2→□E□M◇O-1

Proof:

[1] ⊢□*no*(*t*, *n*)∧*most*(*k*, *n*)→◇*not all*(*k*, *t*) (i.e. □EM◇O-2, Theorem 1)

[2] ⊢□*no*(*n*, *t*)∧*most*(*k*, *n*)→◇*not all*(*k*, *t*) (i.e. □EM◇O-1, by [1], Fact (4.2))

[3] ⊢¬◇*not all*(*k*, *t*)∧□*no*(*t*, *n*)→¬*most*(*k*, *n*) (by [1], Rule 2)

[4] ⊢□¬*not all*(*k*, *t*)∧□*no*(*t*, *n*)→¬*most*(*k*, *n*) (by [3], Fact (3.2))

[5] ⊢□*all*(*k*, *t*)∧□*no*(*t*, *n*)→*at most half of the*(*k*, *n*) (i.e. □E□AH-1, by [4], Fact (2.2) and (2.5))

[6] ⊢□*all*(*k*, *t*)∧□*no*(*n*, *t*)→*at most half of the*(*k*, *n*) (i.e. □E□AH-2, by [5], Fact(4.2))

[7] ⊢□*all*(*k*, *t*)∧□*no*(*t*, *n*)→◇*at most half of the*(*k*, *n*) (i.e. □E□A◇H-1, by [5], Fact (5.3), Rule 1)

[8] ⊢□*all*(*k*, *t*)∧□*no*(*n*, *t*)→◇*at most half of the*(*k*, *n*) (i.e. □E□A◇H-2, by [7], Fact (4.2))

[9] ⊢¬◇*not all*(*k*, *t*)∧*most*(*k*, *n*)→¬□*no*(*t*, *n*) (by [1], Rule 3)

[10] ⊢□¬*not all*(*k*, *t*)∧*most*(*k*, *n*)→◇¬*no*(*t*, *n*) (by [9], Fact (3.1) and (3.2))

[11] ⊢□*all*(*k*, *t*)∧*most*(*k*, *n*)→◇*some*(*t*, *n*) (i.e. M□A◇I-3, by [10], Fact (2.2) and (2.3))

[12] ⊢□*all*(*k*, *t*)∧*most*(*k*, *n*)→◇*some*(*n*, *t*) (i.e. □AM◇I-3, by [11], Fact (4.1))

[13] ⊢□*all*¬(*t*, *n*)∧*fewer than half of the*¬(*k*, *n*)→◇*not all*(*k*, *t*) (by [1], Fact (1.2) and (1.5))

[14] ⊢□*all*(*t*, *D*–*n*)∧*fewer than half of the*(*k*, *D*–*n*)→◇*not all*(*k*, *t*) (i.e. □AF◇O-2, by [13], Definition 3)

[15] ⊢□*all*¬(*n*, *t*)∧*most*(*k*, *n*)→◇*some*¬(*k*, *t*) (by [2], Fact (1.2) and (1.4)

[16] ⊢□*all*(*n*, *D*–*t*)∧*most*(*k*, *n*)→◇*some*(*k*, *D*–*t*) (i.e. □AM◇I-1, by [15], Definition 3)

[17] ⊢□*all*(*n*, *D*–*t*)∧*most*(*k*, *n*)→◇*some*(*D*–*t*, *k*) (i.e. M□A◇I-4, by [16], Fact (4.1))

[18] ⊢□*all*(*k*, *t*)∧□*all*¬(*t*, *n*)→*at least half of the*¬ (*k*, *n*) (by [5], Fact (1.2) and (1.8))

[19] ⊢□*all*(*k*, *t*)∧□*all*(*t*, *D*–*n*)→*at least half of the*(*k*, *D*–*n*) (i.e. □A□AS-1, by [18], Definition 3)

[20] ⊢□*all*(*k*, *t*)∧□*all*(*t*, *D*–*n*)→◇*at least half of the*(*k*, *D*–*n*) (i.e. □A□A◇S-1, by [19], Rule 1, Fact (5.3))

[21] ⊢□*all*(*k*, *t*)∧*fewer than half of the*¬(*k*, *n*)→◇*not all*¬(*t*, *n*) (by [11], Fact (1.3) and (1.5))

[22] ⊢□*all*(*k*, *t*)∧*fewer than half of the*(*k*, *D*–*n*)→◇*not all*(*t*, *D*–*n*) (i.e. F□A◇O-3, by [21], Definition 3)

[23] ⊢¬◇*some*(*k*, *D*–*t*)∧□*all*(*n*, *D*–*t*)→¬*most*(*k*, *n*) (by [16], Rule 2)

[24] ⊢□¬*some*(*k*, *D*–*t*)∧□*all*(*n*, *D*–*t*)→¬*most*(*k*, *n*) (by [23], Fact (3.2))

[25] ⊢□*no*(*k*, *D*–*t*)∧□*all*(*n*, *D*–*t*)→*at most half of the*(*k*, *n*) (i.e. □A□EH-2, by [24], Fact (2.4) and (2.5))

[26] ⊢□*no*(*k*, *D*–*t*)∧□*all*(*n*, *D*–*t*)→◇*at most half of the*(*k*, *n*) (i.e. □A□E◇H-2, by [25], Rule 1, Fact (5.3))

[27] ⊢□*no*(*D*–*t*, *k*)∧□*all*(*n*, *D*–*t*)→*at most half of the*(*k*, *n*) (i.e. □A□EH-4, by [25], Fact (4.2))

[28] ⊢□*no*(*D*–*t*, *k*)∧□*all*(*n*, *D*–*t*)→◇*at most half of the*(*k*, *n*) (i.e. □A□E◇H-4, by [26], Fact (4.2))

[29] ⊢¬◇*some*(*k*, *D*–*t*)∧*most*(*k*, *n*)→¬□*all*(*n*, *D*–*t*) (by [16], Rule 3)

[30] ⊢□¬*some*(*k*, *D*–*t*)∧*most*(*k*, *n*)→◇¬*all*(*n*, *D*–*t*) (by [29], Fact (3.1) and (3.2))

[31] ⊢□*no*(*k*, *D*–*t*)∧*most*(*k*, *n*)→◇*not all*(*n*, *D*–*t*) (i.e. □EM◇O-3, by [30], Fact (2.1) and (2.4))

[32] ⊢□*no*(*D*–*t*, *k*)∧*most*(*k*, *n*)→◇*not all*(*n*, *D*–*t*) (i.e. □EM◇O-4, by [31], Fact (4.2))

[33] ⊢¬◇*at most half of the*(*k*, *n*)∧□*all*(*k*, *t*)→¬□*no*(*t*, *n*) (by [7], Rule 2)

[34] ⊢□¬*at most half of the*(*k*, *n*)∧□*all*(*k*, *t*)→◇¬*no*(*t*, *n*) (by [33], Fact (3.1) and (3.2))

[35] ⊢□*most*(*k*, *n*)∧□*all*(*k*, *t*)→◇*some*(*t*, *n*) (i.e. □M□A◇I-3, by [34], Fact (2.3) and (2.6))

[36] ⊢□*most*(*k*, *n*)∧□*all*(*k*, *t*)→◇*some*(*n*, *t*) (i.e. □A□M◇I-3, by [35], Fact (4.1))

[37] ⊢¬◇*at most half of the*(*k*, *n*)∧□*no*(*t*, *n*)→¬□*all*(*k*, *t*) (by [7], Rule 3)

[38] ⊢□¬*at most half of the*(*k*, *n*)∧□*no*(*t*, *n*)→◇¬*all*(*k*, *t*) (by [37], Fact (3.1) and (3.2))

[39] ⊢□*most*(*k*, *n*)∧□*no*(*t*, *n*)→◇*not all*(*k*, *t*) (i.e. □E□M◇O-2, by [38], Fact (2.1) and (2.6))

[40] ⊢□*most*(*k*, *n*)∧□*no*(*n*, *t*)→◇*not all*(*k*, *t*) (i.e. □E□M◇O-1, by [39], Fact (4.2))

Up to this point, the above reasoning processes show that 23 valid generalized modal syllogisms can be inferred from the syllogism □EM◇O-2 on the basis of some definitions, rules and facts, etc.

5. Conclusion

With the help of set theory, generalized quantifier theory and modal logic, this paper first formalizes the categorical propositions containing quantifiers within Square{*most*} and Square{*no*} and modalities (□ and ◇), then proves the validity of the generalized modal syllogism □EM◇O-2. On the basis of this syllogism, other 23 valid generalized modal syllogisms are deduced by means of some reducible operations. All proofs in this paper are deductive reasoning, and therefore their results have logical consistency.

The reasons of the reducibility between/among the generalized modal syllogisms are: that there are transformational relationships between/among Aristotelian quantifiers in Square{*no*} or generalized ones in Square{*most*}, and that the necessary modality and the possible one are dual, and that the quantifiers *some* and *no* are symmetric. This method is universal, and helps to study the validity of other kinds of syllogisms. It is hoped that the above results can promote the development of related fields such as computational linguistics.

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Authors' contributions

Jing Xu was responsible for study design and writing the paper. Yuzhen Wang was responsible for revising the manuscript. All authors read and approved the final manuscript.

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We declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Obtained.

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The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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