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An Introduction of Jameel's Advanced Stressed Economic and Financial Crises Models and to Dramatically Increasing Markets Confidence and Drastically Decreasing Markets Risks

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Abstract

Quantitative Financial Risk Management has tremendously change the way markets' Practitioners, Regulators and Supervisors, Investors, Academics, Economists, Politicians, Policy Makers and Civil Society perceived financial and commodities markets. The generous invention of Black – Scholes – Merton (1973) Formula is of course the advanced turning point. The Normality Assumption (which causes overreliance, overconfidence, overvaluation or undervaluation of assets, overleveraging and underestimation of risks by the market participants) is the fundamental pillar in question, because returns are not normally distributed, returns have fat tails consisting bubbles and crashes for instance like IT-bubble, stock market bubble, housing bubble and commodities bubbles. Nassim N. Taleb (2007) called these Black Swans or Low – Probability, High – Impact events. The formulae in question receives serious criticisms especially in the United States of America to the extent of Tim Harford (2012) published an article entitled 'The Black – Sholes: The Maths Formula linked to the Financial Crash'. Jamilu (2015) using his criterion and Advanced Methods attempted to capture the popular Black Swans (Low – Probability, High – Impact). The aim of this paper is to use Jameel's Advanced Stressed Methods and Criterion to incorporate fat –tailed effects into the existing stochastic Economic and Financial Models thereby tremendously increasing markets confidence and drastically decreasing markets risks. Based on the various presentations of results and graphs obtained, it can be observed, the Jameel's Advanced Stressed Economic and Financial Models can traces the trajectories of the past and future Economic and Financial Crises given reliable, accurate, sophisticated, valid and sufficient models' independent variables.

Keywords: Options, Call, Put, Probability, Black Swans, Jameel

1. Introduction

Economics and Financial Risk Management seriously suffered from the criticisms of Normality Assumption (because returns are not normally distributed, returns have fat tails possesses bubbles and crashes), hence threaten the investors' confidence all over the globe.Right from the popular Black – Scholes – Merton models, Methods of Quantifying Financial Derivatives, Bankruptcy Prediction Models, Stocks, Bonds, ETFs, and Diversification Models dramatically underestimates (overestimates) probability of large shocks especially at the times of Economic and Financial recessions or recoveries and obviously are the fundamental factors that contributed to the late 2000s Energy Crisis, Dot – Com Bubble (1997 – 2000), the United States Subprime Mortgage Crisis (2007 – 2009), United States Housing Bubble (2006 – 2012), United States Housing Correction (2005 – 2006), Greek Government Debt Crisis (2009 – Present), Russian Financial Crisis (2014 – Present) and Chinese Stock Market Crash (2015 – Present).

The overreliance and overconfidence of the Markets practitioners and investors in Normality Assumption has seriously causes the overvaluation of assets, overleveraging and underestimation of risks at huge cost of bail – outs ranging between $3 & 13 trillion of the late 2007 – 2008 crisis (Blyth 2013, 5). Lanchester, 2010, stated that the total cost of the bail outs was amount to $4.6 trillion which is larger the entire cost of NASA including the Moon Landings, the Marshall Plan, the Wars in Korea, Vietnam, Iraq, the Deal, the 1980s Savings and loan Crisis and Louisiana Purchase.

These has become greatest challenge for the Markets Practitioners, Politicians, Policy Makers, Academics, Investors, Economists and Civil Society to fully understand advanced methods of avoiding future occurrences of huge cost of bail outs in the financial systems and systemically to the entire world economy.

The aim of this paper is to present the results obtained using various forms of Jameel's Advanced Stressed Economic and Financial Models and to show clearly 'How Jameel's Advanced Stressed Economic and Financial Crises Models Dramatically Increases Markets Confidence and Drastically Decreases Markets Risks'.

1.1 Literature Review

John M. Moody (1909) was the first to published credit rating grades for publicly traded bonds. In 1941, David Durand applied discriminant analysis proposed by Fisher (1936) to classify prospective borrowers. Attempts have been made in 1950s to merge automated credit decision making with statistical techniques so as to enhance credit decision making. Lack of sophisticated computing tools, the models possessed limitations. Myers and Forgy (1963), compared discrimination analysis with regression in credit scoring application.

Altman (1960), introduced variables in a multivariate discriminant analysis and obtained a function depending on some financial ratios. Beaver in 1966 introduced anunivariate approach of discriminant analysis to assess the individual relationships between predictive variables, and subsequent failure events. In 1968, Altman expanded the work of Beaver (1966) to allow one to assess the relationship between failure and a set of financial characteristics. Martin (1977), presented a logistic regression model to predict probabilities of failure of banks using data obtained from Federal Reserve System. Ohlson (1980), used Logit to predict bankruptcy. Zmijewski (1984) used probit to estimate probability of default and predict bankruptcy.

In 1985, West used factor analysis and logit estimation to assign a probability of a bank being a problematic. In 2001, Shumway introduced dynamic logit or hazard model to predict bankruptcy. Chava &Jarrow (2004), Hillegeist, Keating, Cram, &Lundstedt (2004), and Beaver, McNichols & Rhie (2005) uses Shumway's approach. In 2004, Jones &Hensher introduced a mixed logit model for financial distress prediction and argued that it offers significant improvements compare to binary logit and multinomial logit models. Campbell, Hilscher, &Szilagyi (2008), introduced a dynamic logit model to predict corporate bankruptcies and failures at short and long horizons using accounting and market variables.

In 2011, Altman, Fargler, &Kalotay used accounting – based measures, firm characteristics and industry level expectations of distress conditions to estimate the likehood of default inferred from equity prices. Li, Lee, Zhou, & Sun (2011) introduced a combined random subspace approach (RSB) with binary logit models to generate a so called RSB-L model that takes into account different decision agent's opinions as a matter to enhance results.Sun& Li (2011) tested the feasibility and effectiveness of dynamic modelling for financial distress prediction (FDP) based on the Fisher discriminant analysis model.

Stefan Van der Ploeg (2011) stated that since the seminal work of Martin (1977), the **Logit and Probit Models** has become one of the most commonly applied parametric failure prediction models in the academic literature as well as the banking regulation and supervision. Jamilu (2015) introduced new methods entiled "Jameel's Advanced Stressed Methods uses Jameel's Criterion" to Stress Economic and Financial Stochastic Models, initially using Logit and Probit Models.

2. Method

The methodology adopted in this paper is to use Jameel's Advanced Stressed Economic and Financial Crises Models appeared in Appendix A, B, C and D using Jameel's Advanced Stressed Methods and criterion.

2.1 Jameel's Advanced Stressed Methods

The idea was basically on how to contractionally and expansionally stress Black – Scholes – Merton options pricing model using the respectively geometric volatility σ_A and geometric return μ_A of the arithmetic means of the underlying asset returns and returns of the explained (independent) variables as well as the best fitted fat – tailed effects probability distribution of the underlying asset returns, so as to capture non – normality of financial markets with the effect of small probabilities margin (popularly known as black swan events) reference to the traditional Logit, Probit, Discriminant Function, Mixed Logit, Instantaneous, Multinomial Logistic, Black - Scholes, Kmv – Merton and naïve Kmv – Merton probability of default models.

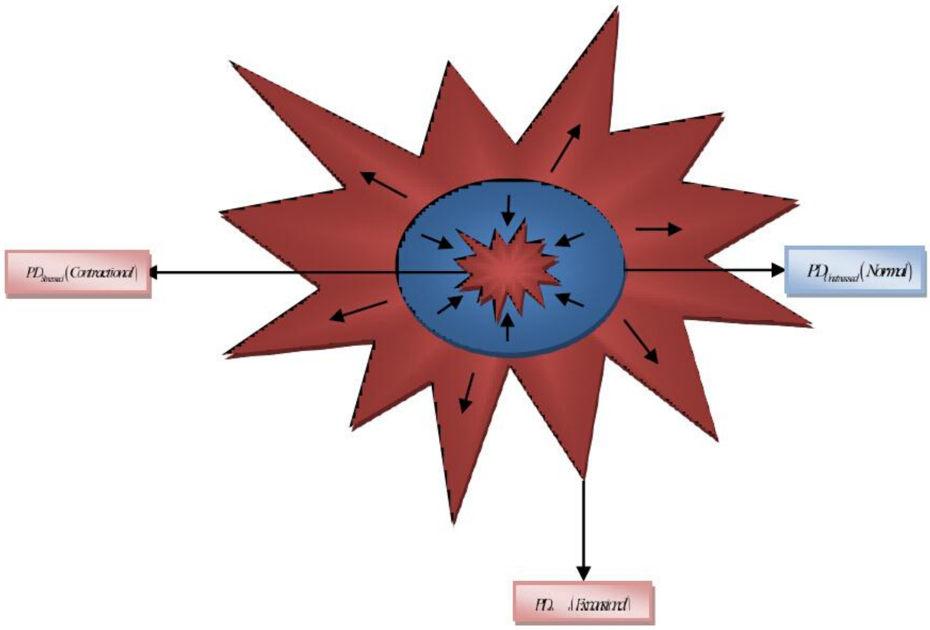


Figure 1. Jameel's Contractional – Expansional Stress Diagram

From the above diagram, the author:

(i) Catastrophically shrink the normal probability of default model $PD_{Unstressed}\left(Normal\right)$ to contractional probability of default models $PD_{stressed}\left(Contractional\right)$ using respectively geometric volatility $\left(\sigma_A\right)$, research company underlying stock returns and returns of the explained (independent) variables $\left(\mu_A\right)$ as well as best fitted fat – tailed probability distribution $f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$; then

(ii) Catastrophically blow the normal probability of default model $PD_{Unstressed}\left(Normal\right)$ to expansional probability of default models $PD_{stressed}\left(Expansional\right)$ using respectively geometric volatility $\left(\sigma_A\right)$, research company underlying stock returns and returns of the explained (independent) variables $\left(\mu_A\right)$ as well as best fitted fat – tailed probability distribution $f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$.

Where, μ_A is the Geometric Return of the Arithmetic Means of the U.S. Macroeconomic Indicators plus Research Company Stock Returns. σ_A is the Geometric Volatility of the Volatilities of the U.S. Macroeconomic Indicators plus Research Company Stock Returns.

2.2 Jameel's Criterion

In this test of Goodness of fit, the author considers the following criterion:

- We accept if the Average of the ranks of Kolmogorov Smirnor, Anderson Darling and Chi-squared is less than or equal to Three (3)
- We must choose the Probability Distribution follows by the data itself regardless of its Rankings
- If there is tie, we include both Probability Distributions in the selection
- At least Two (2) probability distributions must be included in the selection
- We select the most occur probability distribution as the best fitted probability distribution in each case of test of goodness of fit of the stock returns.

2.3 Some Selected Data Sources

- Yahoo Finance
- Google Finance
- Federal Reserve Bank
- Economic Research

2.4 Companies and Fundamental Macroeconomic Indicators used in the Research Work

The Author considers the following:

- Five (5) companies listed on the platform of New York Stock Exchange (NYSE) namely; Chevron Corporation, Honda Motor Corporation, Microsoft Corporation, Exxon Mobil Corporation, and General Electric Corporation for the period of Twenty Five (25) years (2014 – 1991) data.
- The underlying monthly stock returns of the five (5) research companies
- The U.S. GDP
- The U.S. Inflation Rate
- The U.S. Prime Rate
- The U.S. unemployment Rate
- The U.S. USD/GBP Exchange Rate
- The U.S. House Price
- The U.S. Oil Price
- The U.S. Gold Price

Using **QI Macros 2014 Software**, the author obtained the following components:

Multiple Regression Model Component of CHEVRON Corporation (CVX) for calculating Probability of Default:

$$\begin{aligned} Y_{CHEVRON} &= 0.004 + 0.004 \times \Delta P(CHEVRON) - 0.199 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.009 \times \Delta P(INF) \\ &- 0.018 \times \Delta P(UER) + 0.002 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of GENERAL ELECTRIC (GE) for calculating Probability of Default:

$$\begin{aligned} Y_{GE} &= 0.004 - 0.001 \times \Delta P(GE) - 0.207 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.016 \times \Delta P(INF) \\ &- 0.017 \times \Delta P(UER) - 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of MICROSOFT (MSFT) Corporation for calculating Probability of Default:

$$\begin{aligned} Y_{MSFT} &= 0.004 - 0.006 \times \Delta P(MSFT) - 0.189 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.011 \times \Delta P(INF) \\ &- 0.017 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of EXXON MOBIL (XOM) Corporation for calculating Probability of Default:

$$\begin{aligned} Y_{XOM} &= 0.004 + 0.002 \times \Delta P(XOM) - 0.2 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) \\ &- 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of HONDA MOTOR CO., Ltd for calculating Probability of Default:

$$\begin{aligned} Y_{HONDA} &= 0.004 - 0.004 \times \Delta P(HMC) - 0.204 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) \\ &- 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

2.5 Research Companies Calculated Parameters

Chevron Calculated Parameters

$\mu_{GEO}(Chevron) = 0.030383975$, $\sigma_{GEO}(Chevron) = 0.111414539$, $\mu_{STOCK}(Chevron) = 0.004402791$ and $\sigma_{STOCK}(Chevron) = 0.06909299$

General Electric Corporation Calculated Parameters

$\mu_{GEO}(GE) = 0.037067141$, $\sigma_{GEO}(GE) = 0.10009902$, $\mu_{STOCK}(GE) = 0.002163529$ and $\sigma_{STOCK}(GE) = 0.091140157$

Honda Motor Calculated Parameters

$\mu_{GEO}(Honda) = 0.031352397$, $\sigma_{GEO}(Honda) = 0.114001187$, $\mu_{STOCK}(Honda) = 0.005839335$ and $\sigma_{STOCK}(Honda) = 0.084945727$

Microsoft Corporation Calculated Parameters

$\mu_{GEO}(MSFT)=0.031352397$, $\sigma_{GEO}(MSFT)=0.117906073$, $\mu_{STOCK}(MSFT)=0.006798657$ and $\sigma_{STOCK}(MSFT)=0.115022493$

Exxon Mobil Corporation Calculated Parameters

$\mu_{GEO}(XOM)=0.030729517$, $\sigma_{GEO}(XOM)=0.110236167$, $\mu_{STOCK}(XOM)=0.00487448$ and $\sigma_{STOCK}(XOM)=0.062787634$

Using the above data set and the Jameel's Criterion, the following are the Global Economic and Financial Crises Best Fitted Fat – Tailed Probability Distributions in terms of order of hierarchy:

(a) Log – Logistic (3P) Probability Distribution (1st) (b) Cauchy Probability Distribution (2nd) (c) Pearson 5 (3P) (d) Probability Distribution (3rd) (e) Burr (4P) Probability Distribution (4th) (f) Fatique Life (3P) Probability Distribution (5th) (g) Inv.Gaussian (3P) Probability Distribution (6th) (h) Dagum (4P) Probability Distribution (7th) (i) Lognormal (3P) Probability Distribution (8th).

2.5.1 Jameel's - Aish Triangle

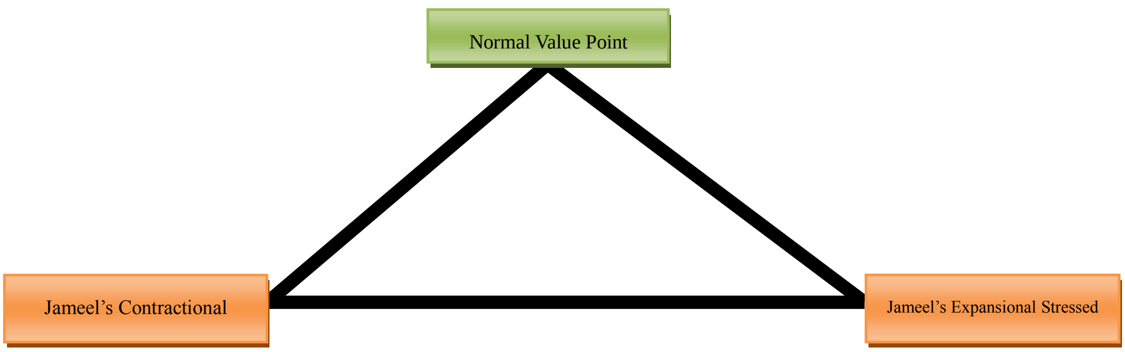


Figure 2. Jameel's - Aish Triangle

2.6 Proposed Theorem (Jameel's Average for Decision Making)

Let $x_1 := Normal\ Value$, $x_2 := Jameel's\ Contractional\ Stressed\ Value$,

And $x_3 := Jameel's\ Expansional\ Stressed\ Value$, Define

$$x_4 := Jameel's\ Arithmetic\ Mean\ Value := \frac{x_1+x_2+x_3}{3}$$ and

$$x_5 := Jameel's\ Geometric\ Mean\ Value := \sqrt[3]{x_1 \,.\, x_2 \,.\, x_3}$$

Then the set $\{x_1, x_2, x_3, x_4, x_5\}$ form a Solution when Making Decision, depending on the financial/non-financial institutions' policies.

2.7 Proposed Jameel's Pentagon for Decision Making

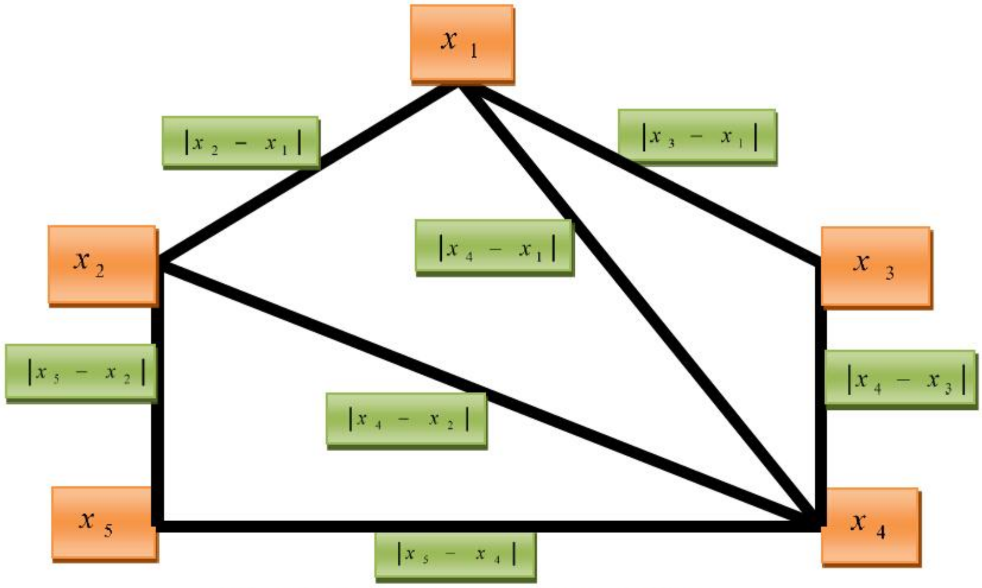


Figure 3. Jameel's Pentagon for Decision Making

3. Results (reference to Jameel's Advanced Stressed Models of Appendix A) Default Probabilities of Chevron Corporation

Under Chevron Corporation, on the month of June, 2014, the probability of default using the existing logit is 0.499097747% and that of probit is 0.501439786%, whereas, using the proposed Jameel's advanced stressed probability of default models I and II are: 0.499976914%, 0.499968258%, 0.500011436%, 0.499933742%, 0.49910206%, 0.499093434%, 0.499136461%, 0.499059039%, 0.501573711%, 0.50160834%, 0.501435622%, 0.501746429%, 0.501422471%, 0.5014571%, 0.501284382%, and 0.501595189% respectively.

Date	M1 TYPE A-	M1 TYPE A+	M1 TYPE B-	M1 TYPE B+	M1 TYPE C-	M1 TYPE C+	M1 TYPE D-	M1 TYPE D+	LOGIT	PROBIT	M2 TYPE A-	M2 TYPE A+	M2 TYPE B-	M2 TYPE B+	M2 TYPE C-	M2 TYPE C+	M2 TYPE D-	M2 TYPE D+
01/12/2014	0.499970779	0.499966686	0.499987104	0.499950363	0.498972963	0.498968885	0.498989222	0.498952627	0.498970924	0.501642165	0.501588987	0.501605363	0.501523683	0.501670667	0.501633977	0.501650353	0.501568673	0.501715657
03/11/2014	0.499988014	0.499988014	0.499988014	0.499988014	0.4996055	0.4996055	0.4996055	0.4996055	0.4996055	0.500629531	0.501566407	0.501566407	0.501566407	0.501566407	0.500629531	0.500629531	0.500629531	0.500629531
01/10/2014	0.499997599	0.499997596	0.49999761	0.499997584	0.499920916	0.499920913	0.499920928	0.499920902	0.499920915	0.500126202	0.501551108	0.50155112	0.501551061	0.501551167	0.500126196	0.500126208	0.500126149	0.500126254
02/09/2014	0.499982294	0.499982294	0.499982294	0.499982294	0.499417245	0.499417245	0.499417245	0.499417245	0.499417245	0.500929942	0.501575535	0.501575535	0.501575535	0.501575535	0.500929942	0.500929942	0.500929942	0.500929942
01/08/2014	0.499981556	0.499981525	0.499981677	0.499981404	0.499392475	0.499392445	0.499392596	0.499392323	0.49939246	0.500969493	0.501576675	0.501576797	0.501576189	0.501577284	0.500969432	0.500969554	0.500968946	0.500970041
01/07/2014	0.49999598	0.499995049	0.49999969	0.499991338	0.499852831	0.499851901	0.499856539	0.499848192	0.499852366	0.50023559	0.501552576	0.501556299	0.501537734	0.501571141	0.500233729	0.500237451	0.500218886	0.500252294
02/06/2014	0.499976914	0.499968258	0.500011436	0.499933742	0.49910206	0.499093434	0.499136461	0.499059039	0.499097747	0.501439786	0.501573711	0.50160834	0.501435622	0.501746429	0.501422471	0.5014571	0.501284382	0.501595189
01/05/2014	0.49997971	0.499976176	0.499993802	0.499962085	0.499275826	0.499272302	0.499289878	0.49925825	0.499274064	0.501158426	0.501575409	0.501589545	0.501519036	0.501645917	0.501151358	0.501165494	0.501094985	0.501221867
01/04/2014	0.499969966	0.499963096	0.499997362	0.499935704	0.498901885	0.498895045	0.498929164	0.498867769	0.498898465	0.501757793	0.501586947	0.501614429	0.501477356	0.50172402	0.501744052	0.501771534	0.501634462	0.501881125
03/03/2014																		
03/02/2014	0.49997319	0.499970675	0.499983219	0.499960647	0.4990775	0.499074994	0.499087493	0.499065001	0.499076247	0.501474095	0.501587038	0.501597098	0.501546918	0.501637218	0.501469065	0.501479125	0.501428945	0.501519245
02/01/2014	0.499992068	0.499992068	0.499992068	0.499992068	0.499738954	0.499738954	0.499738954	0.499738954	0.499738954	0.500416568	0.501559936	0.501559936	0.501559936	0.501559936	0.500416568	0.500416568	0.500416568	0.500416568
02/12/2013	0.499981297	0.499980462	0.499984628	0.499977131	0.499371131	0.499370297	0.499374453	0.499366975	0.499370714	0.501004195	0.50157612	0.501579461	0.501562795	0.501592786	0.501002524	0.501005865	0.500989199	0.50101919
01/11/2013	0.499982482	0.499981596	0.499986013	0.499978065	0.499409304	0.499408421	0.499412828	0.499404897	0.499408862	0.500943319	0.50157417	0.501577712	0.501560043	0.501591839	0.500941547	0.50094509	0.50092742	0.500959217
01/10/2013	0.500002303	0.500000961	0.500007656	0.499995608	0.500054382	0.50005304	0.500059737	0.500047686	0.500053711	0.499914289	0.501541991	0.50154736	0.501520579	0.501568771	0.499911605	0.499916974	0.499890193	0.499938386
03/09/2013	0.499976144	0.499976069	0.49997644	0.499975773	0.499213653	0.499213579	0.499213948	0.499213284	0.499213616	0.501254886	0.501585259	0.501585556	0.501584075	0.50158674	0.501254738	0.501255034	0.501253554	0.501256218

0.499136461%, 0.499059039%, 0.501573711%, 0.50160834%, 0.501435622%, 0.501746429%, 0.501422471%, 0.5014571%, 0.501284382%, and 0.501595189%. While in the case of M2 TYPE A* and M2 TYPE B*, on 10/1/2014, we obtained the stressed probabilities: 0.500003829, 0.50000384, 0.500003782, and 0.500003887 which are clearly lies in between the probabilities of logit and probit.

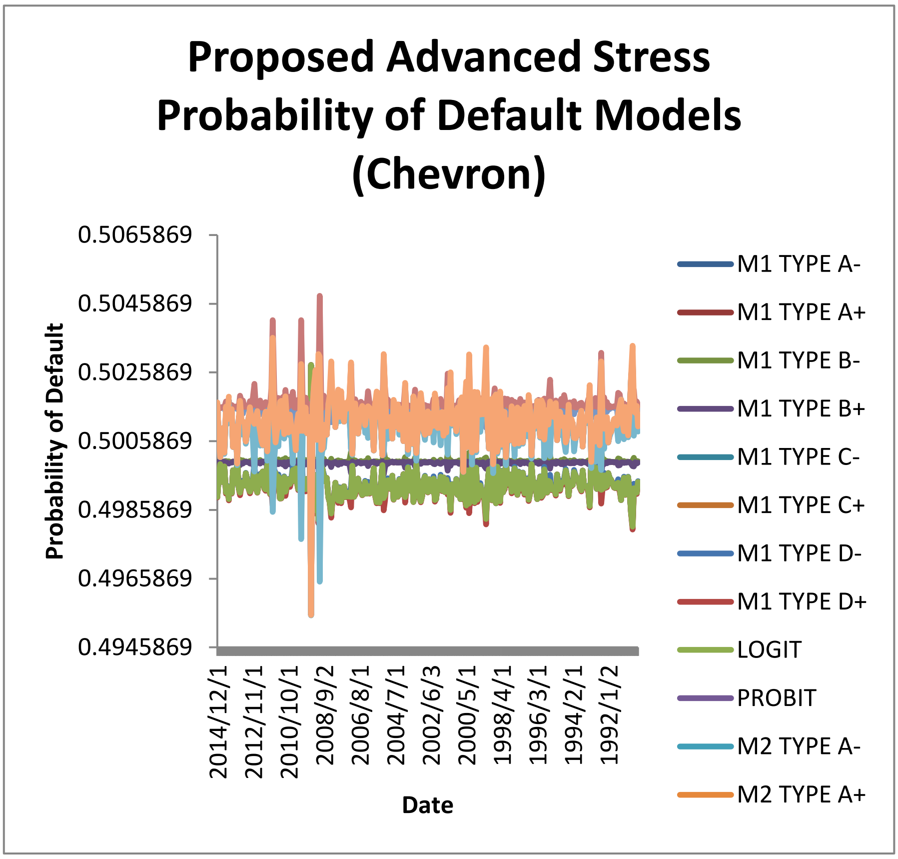


Figure 4. Proposed Jameel's Advanced Stressed Probability of Default Models I and II, Logit and Probit Models for Chevron Corporation, $\mu_{GEO}(Chevron) = 0.030383975$, $\sigma_{GEO}(Chevron) = 0.111414539$, $\mu_{STOCK}(Chevron) = 0.004402791$ and $\sigma_{STOCK}(Chevron) = 0.06909299$

From the above graph, Jameel's Models I and II traces the trajectories of the past historic Financial and Economic crises Company – wise.

Similarly, under General Electric, on the month of September, 2014, the probability of default using the existing logit is 0.499256894% and that of probit is 0.501185825%, whereas for jameel's proposed models I and II in appendix A are respectively: 0.499973123%, 0.499971787%, 0.499979127%, 0.499965783%, 0.49925736%, 0.499256228%, 0.499263546%, 0.499250241%, 0.501608698%, 0.501614042%, 0.50158468%, 0.50163806%, 0.501183154%, 0.501188497%, 0.501159135%, and 0.501212515%. While in the case of M2 TYPE A* and M2 TYPE B*, on 3/2/2014, we obtained the stressed probabilities: 0.500041284, 0.500046627, 0.500017265, and 0.500070645 which are clearly lies in between the probabilities of logit and probit obtained above.

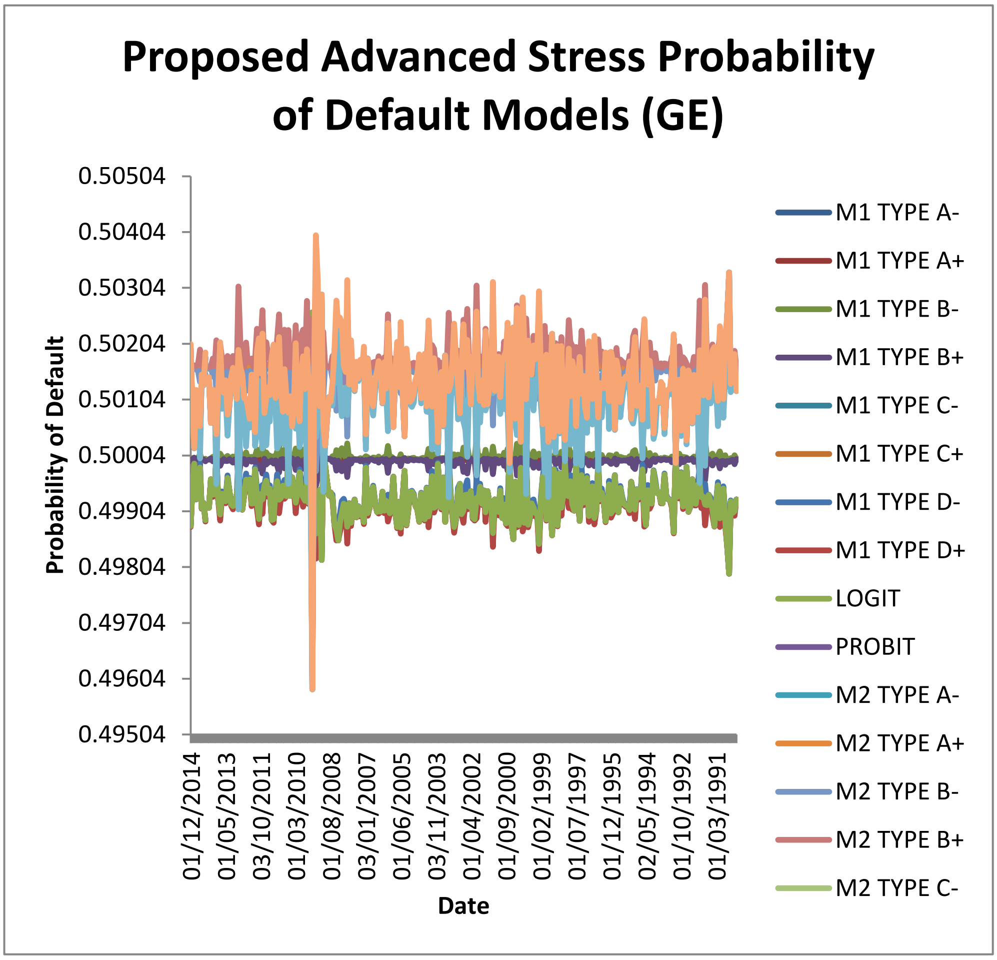


Figure 7. Proposed Jameel's Advanced Stressed Probability of Default Models I and II, Logit and Probit Models for General Electric, $\mu_{GEO}(GE) = 0.037067141$, $\sigma_{GEO}(GE) = 0.10009902$, $\mu_{STOCK}(GE) = 0.002163529$ and $\sigma_{STOCK}(GE) = 0.091140157$

From the above graph, Jameel's Models I and II traces the trajectories of the **PAST** historic Financial and Economic crises Company – wise.

Table 3.1 Chevron Corporation Correlation Matrix

The Matrix below is the Table of Correlations that exists between the Logit, Probit and the Proposed Jameel's Advanced Stressed Probability of Default Models of Chevron Corporation using our data sources from 2014 – 1991.

CORREL	M1 TYPE A-	M1 TYPE A+	M1 TYPE B-	M1 TYPE B+	M1 TYPE C-	M1 TYPE C+	M1 TYPE D-	M1 TYPE D+	LOGIT	PROBIT	M2 TYPE A-	M2 TYPE A+	M2 TYPE B-	M2 TYPE B+	M2 TYPE C-	M2 TYPE C+	M2 TYPE D-	M2 TYPE D+
M1 TYPE A-	1.000	0.217	0.726	-0.510	0.794	0.764	0.895	0.628	0.779	-0.779	-0.907	0.211	-0.666	0.580	-0.816	-0.740	-0.985	-0.383
M1 TYPE A+	0.217	1.000	-0.514	0.729	0.766	0.796	0.630	0.896	0.781	-0.781	0.214	-0.908	0.583	-0.669	-0.742	-0.818	-0.384	-0.985
M1 TYPE B-	0.726	-0.514	1.000	-0.962	0.158	0.110	0.343	-0.080	0.134	-0.134	-0.948	0.826	-0.997	0.981	-0.194	-0.074	-0.595	0.358

M1 TYPE B+	-0.510	0.729	-0.962	1.000	0.117	0.165	-0.074	0.349	0.141	-0.141	0.825	-0.948	0.981	-0.997	-0.081	-0.201	0.353	-0.599
M1 TYPE C-	0.794	0.766	0.158	0.117	1.000	0.999	0.982	0.972	1.000	-1.000	-0.465	-0.426	-0.076	-0.034	-0.999	-0.996	-0.888	-0.865
M1 TYPE C+	0.764	0.796	0.110	0.165	0.999	1.000	0.971	0.982	1.000	-1.000	-0.421	-0.470	-0.028	-0.083	-0.996	-0.999	-0.865	-0.889
M1 TYPE D-	0.895	0.630	0.343	-0.074	0.982	0.971	1.000	0.909	0.977	-0.977	-0.624	-0.247	-0.264	0.156	-0.988	-0.962	-0.959	-0.755
M1 TYPE D+	0.628	0.896	-0.080	0.349	0.972	0.982	0.909	1.000	0.977	-0.977	-0.242	-0.628	0.162	-0.270	-0.962	-0.988	-0.754	-0.959
LOGIT	0.779	0.781	0.134	0.141	1.000	1.000	0.977	0.977	1.000	-1.000	-0.443	-0.448	-0.052	-0.059	-0.998	-0.998	-0.877	-0.877
PROBIT	-0.779	-0.781	-0.134	-0.141	-1.000	-1.000	-0.977	-0.977	-1.000	1.000	0.443	0.448	0.052	0.059	0.998	0.998	0.877	0.877
M2 TYPE A-	-0.907	0.214	-0.948	0.825	-0.465	-0.421	-0.624	-0.242	-0.443	0.443	1.000	-0.603	0.918	-0.869	0.497	0.388	0.820	-0.042
M2 TYPE A+	0.211	-0.908	0.826	-0.948	-0.426	-0.470	-0.247	-0.628	-0.448	0.448	-0.603	1.000	-0.870	0.919	0.393	0.502	-0.038	0.822
M2 TYPE B-	-0.666	0.583	-0.997	0.981	-0.076	-0.028	-0.264	0.162	-0.052	0.052	0.918	-0.870	1.000	-0.994	0.113	-0.009	0.526	-0.434
M2 TYPE B+	0.580	-0.669	0.981	-0.997	-0.034	-0.083	0.156	-0.270	-0.059	0.059	-0.869	0.919	-0.994	1.000	-0.002	0.120	-0.429	0.531
M2 TYPE C-	-0.816	-0.742	-0.194	-0.081	-0.999	-0.996	-0.988	-0.962	-0.998	0.998	0.497	0.393	0.113	-0.002	1.000	0.993	0.904	0.846
M2 TYPE C+	-0.740	-0.818	-0.074	-0.201	-0.996	-0.999	-0.962	-0.988	-0.998	0.998	0.388	0.502	-0.009	0.120	0.993	1.000	0.846	0.905
M2 TYPE D-	-0.985	-0.384	-0.595	0.353	-0.888	-0.865	-0.959	-0.754	-0.877	0.877	0.820	-0.038	0.526	-0.429	0.904	0.846	1.000	0.538
M2 TYPE D+	-0.383	-0.985	0.358	-0.599	-0.865	-0.889	-0.755	-0.959	-0.877	0.877	-0.042	0.822	-0.434	0.531	0.846	0.905	0.538	1.000

From the above correlation matrix, the correlation that exists between (i) LOGIT and M1 TYPE C-, M1 TYPE C+, M1 TYPE D- and M1 TYPE D+ are respectively 1.00, 1.00, 0.977 and 0.977 (ii) PROBIT and M2 TYPE C-, M2 TYPE C+, M2 TYPE D- and M2 TYPE D+ are respectively 0.998, 0.998, 0.877 and 0.877 (iii) M1 TYPE C- and M1 TYPE C+, M1 TYPE D- and M1 TYPE D+ are respectively 0.999, 0.982 and 0.972 are all POSITIVELY and STRONGLY very high correlations compare with the other correlations appeared in the matrix. This shows the level of closeness in terms of economic and financial crises predicative capabilities (performances) and equivalence of the cited models and so on.

Table 3.2 Jameel's - Aish Triangle Reference to Logit Model Chevron Corporation (CVX)

Jameel'sExpansional Value	Normal Logit Value	Jameel'sContractional Value	D(JE - NL)	D(JC - NL)	D(JE - JC)
486170688	476105067.6	486135033.2	10065620.38	10029965.55	35654.82335
486771134.1	475143983.6	486771134.1	11627150.45	11627150.45	0
487078459.3	474666277.1	487078433.6	12412182.13	12412156.45	25.67658478
486587715.5	475429101.3	486587715.5	11158614.25	11158614.25	0
486563700	475466639.3	486563434	11097060.77	11096794.71	266.0573239
487015724.8	474770096.7	487007592.3	12245628.08	12237495.56	8132.514422
486314144.5	475912990.5	486238711.7	10401153.99	10325721.2	75432.78371
486463620.6	475645954.1	486432804.8	10817666.5	10786850.7	30815.79819
486112172.4	476214809.3	486052355.2	9897363.172	9837545.937	59817.23498

On the 12th January, 2014 for instance using *Jameel's - Aish Triangle Reference to Logit Model of* Chevron Corporation (CVX), the author obtained the following Jameel's triangle:

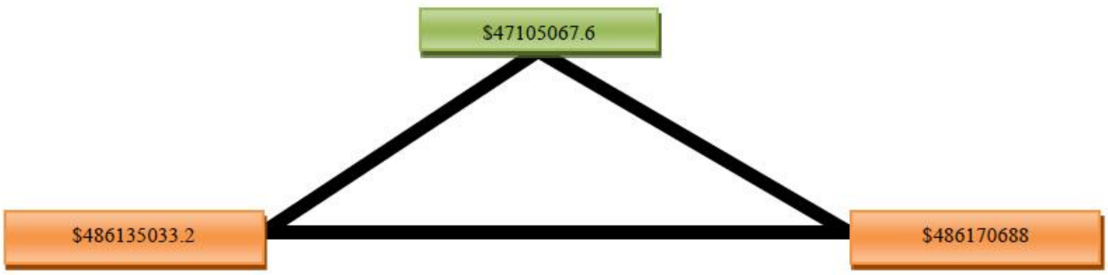


Figure 5. Jameel's – Aish Triangle Values

With the difference between the Jameel's Expansional Stressed Value and Normal Value equal $10065620.38 and that of Jameel's Contractional Stressed Value and Normal Value equal $10029965.55. These are very huge differences between the NORMAL VALUE and Jameel's Stressed Values. This will help to address future global economic and financial crises.

The following is the equivalent Jameel's Pentagon for Decision Making reference to the above Jameel's – Aish Triangle.

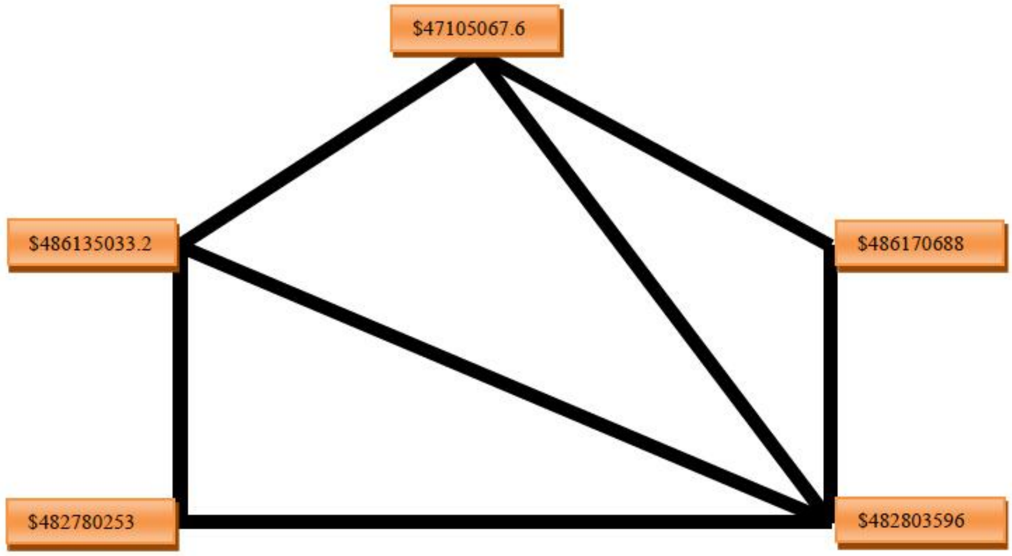


Figure 6. Jameel's Pentagon for Decision Making (Numerical Estimates)

Similar comparisons can be done as in the case of the above Jameel's – Aish Triangle.

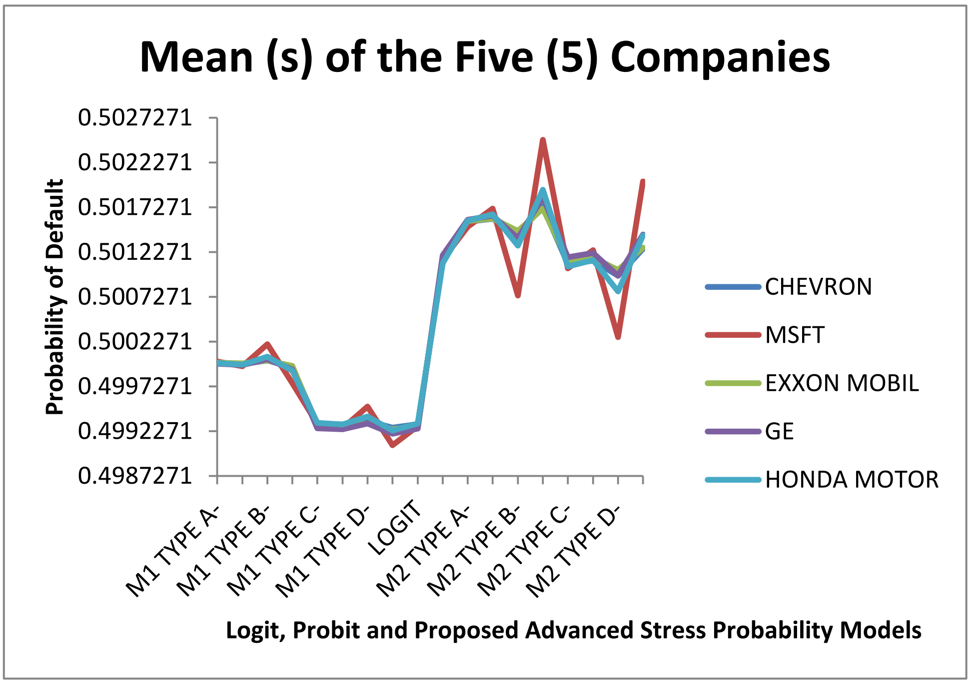


Figure 8. Means of the Jameel's Advanced Stressed Probability of Default Models I and II, Logit and Probit Models of the Five Research Companies

Example 1: (reference to Jameel's Advanced Stressed Models of Appendix C)

Consider an example of Microsoft Corporation (MSFT) option with a term of six months (0.5 years). The current stock price of Microsoft Corporation (MSFT) is $48.14 and the strike of the option is $49.39. The risk-free rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. What is the value of the options using: (1) Black-Scholes - Merton Model; and (2) Jameel's Economic and Financial Crises Advanced Stressed Derivatives Models reference to Black-Scholes - Merton Model?

Using the data of Microsoft Corporation (MSFT) extracted from yahoo finance from 2014 – 1991, we obtained:

$f\left(x,\mu_{underlying},\sigma_{underlying},\xi\right)=0.00000000123523$ (Log – Logistic (3P)), $\mu_A=0.031886784$, $\sigma_A=0.117906073$, $K=\$49.39$, $S=\$48.14$, $\sigma=0.022041976$, $r=0.0392$, $T=0.5$ and $t=0$. Recall that $d_1=\frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$, $d_2=d_1-\sigma\sqrt{T-t}$ then $d_1=\frac{\ln\left(\frac{48.14}{49.39}\right)+\left(0.0392+\frac{0.022041976^2}{2}\right)(0.5-0)}{0.022041976\sqrt{0.5-0}}=-0.18969$

$d_2=-0.18969-0.022041976\sqrt{0.5}=-0.20528$

Therefore, $d_1=-0.18969$ and $d_2=-0.20528$. Using the Microsoft EXCEL, consider the following tables:

Table 1. Black – Scholes –Merton and Jameel Advanced Stressed Call Option Prices

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
-0.1353827	3.6767467	-0.135382736	3.6767467	0.1716279	0.1716279
		BLACK - SCHOLES – MERTON			
		0.171627934			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
-0.1353827	3.6767467	-0.135382735	3.6767467	0.1716279	0.1716279

Table 2: Black – Scholes –Merton and Jameel Advanced Stressed Put Option Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
0.1559984	3.9681278	0.155998418	3.9681278	0.4630091	0.4630091
		BLACK - SCHOLES – MERTON			
		0.463009087			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
0.1559984	3.9681278	0.155998417	3.9681278	0.4630091	0.4630091

Example 2: Assume the interest on loan is at $6.27\%\ p.a$ compounded quarterly. Suppose that this contract is a caplet with notional value of $\$15,000,000$ designed to cap the interest rate for a period of three-month starting six months from now. Assume that the forward rate for three-month period starting on six months is $5.08\%\ p.a.$, compounding quarterly with the volatility of the rate equals $20\%\ p.a$. What are the prices of Caplet and Floolet using: (i) Black Models (1976) (ii) Jameel's Economic and Financial Crises Advanced Stressed Derivatives Models reference to Black (1976) Model? $\sum M=\$15,000,000$, $\Delta=T_i-T_{i-1}=\frac{1}{4}=0.25$, $F=5.08\%=0.0508$, $E=6.27\%=0.0627$, $\sigma=20\%=0.20$, $\sigma_A=0.111414539$, $\mu_A=0.030383975$, $f\left(x,\mu_{Company},\sigma_{Company},\xi\right)=0.000073492$, then

$$D=\frac{1}{(1+\Delta\times F)}=\frac{1}{1+(0.25\times0.0508)}=\frac{1}{1.0127}=0.99$$

$d=\frac{\ln\left(\frac{F}{E}\right)+\frac{1}{2}\times\sigma^2\left(T-t\right)}{\sigma\sqrt{T-t}}=\frac{\ln\left(\frac{0.0508}{0.0627}\right)+\left(0.5\times\left(0.20\right)^2\times0.5\right)}{0.20\times\sqrt{0.5}}=\frac{0.01-0.2105}{0.1414}=-1.4178$. Using **Microsoft EXCEL,** consider the following tables:

Table 3. Merton and Jameel Advanced Stressed Caps Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
-20932.104	77213.465	-20934.98954	77210.58	890.02361	887.13856
		MERTON CAPS PRICE			
		890.3853496			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
-20931.381	77214.189	-20928.49597	77217.074	890.74709	893.63213

Table 4. Merton and Jameel Advanced Stressed Floors Prices

TYPE A+	**TYPE A*+**	**TYPE B+**	**TYPE B*+**	**TYPE C+**	**TYPE D+**
23247.369	**121392.94**	**23250.25403**	**121395.82**	**45069.497**	**45072.382**
		MERTON CAPS PRICE			
		45069.13535			
TYPE A-	**TYPE A*-**	**TYPE B-**	**TYPE B*-**	**TYPE C-**	**TYPE D-**
23246.646	**127872.52**	**23243.76046**	**121389.33**	**45068.774**	**45065.889**

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (10), 11-24.

Example 3 (reference to Black – Scholes – Merton (1973) Default Probability Model):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained: $f\left(x,\mu_{Company},\sigma_{Company},\xi\right)=0.000073492$ (Log-Logistic (3P)), $\mu_A=0.030383975$, and $\sigma_A=0.111414539$. Let $J=0.464641$ then using the proposed Jameel's VII Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS VII AND BLACK - SCHOLES								
FORMULA FOR CALCULATING PROBABILITY OF DEFAULT								
M7 TYPE A+	M7 TYPE A-	M7 TYPE B+	M7 TYPE B-	M7 TYPE C+	M7 TYPE C-	M7 TYPE D+	M7 TYPE D-	BLACK -SCHOLES
0.494376251	0.494359875	0.494441555	0.494294571	0.321102471	0.321086095	0.321167775	0.321020791	0.321094283

Example 4 (reference to Merton (1974) Recovery Rate Model):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained: $f\left(x,\mu_{Company},\sigma_{Company},\xi\right)=0.000073492$ Log – Logistic (3P)), $\mu_A=0.030383975$, and $\sigma_A=0.111414539$. Let $A=0.0508$,

$D = 0.0627$, $T = 0.5$, $\mu_V = 0.00638311$, $d_1 = 0.464641$ and $d_2 = 0.3232196$ then using the proposed Jameel's VIII Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS VIII AND BLACK - SCHOLES

FORMULA FOR CALCULATING RECOVERY RATE

M8 TYPE A+	M8 TYPE A-	M8 TYPE B+	M8 TYPE B-	M8 TYPE C+	M8 TYPE C-	M8 TYPE D+	M8 TYPE D-	BLACK -SCHOLES RR
0.809997025	0.809980649	0.810062329	0.809915345	0.699202953	0.699186577	0.699268257	0.699121273	0.699194765

Example 5 (reference to KMV – Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained: $f(x, \mu_{Company}, \sigma_{Company}, \xi) = 0.00007$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$. Let $DD = 0.3232196$ then using the proposed Jameel's IX Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS IX AND KMV - MERTON

FORMULA FOR CALCULATING PROBABILITY OF DEFAULT

M9 TYPE A+	M9 TYPE A-	M9 TYPE B+	M9 TYPE B-	M9 TYPE C+	M9 TYPE C-	M9 TYPE D+	M9 TYPE D-	KMV - MERTON
0.49609036	0.496073984	0.496155664	0.49600868	0.321102471	0.373256281	0.373337961	0.373190977	0.321094283

Example 6 (reference to Naïve KMV – Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained: $f(x, \mu_{Company}, \sigma_{Company}, \xi) = 0.000073492$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$. Let $Naive\, DD = 0.53636121$ then using the proposed Jameel's X Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS X AND NAIVE KMV - MERTON

FORMULA FOR CALCULATING PROBABILITY OF DEFAULT

M10 TYPE A+	M10 TYPE A-	M10 TYPE B+	M10 TYPE B-	M10 TYPE C+	M10 TYPE C-	M10 TYPE D+	M10 TYPE D-	NAÏVE - KMV - MERTON
0.493506999	0.493490623	0.493572303	0.493425319	0.295862655	0.295846279	0.295927959	0.295780975	0.295854467

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (12), 16 – 34

From the above tables, the eight (8) proposed Jameel's Models in each case gives much close approximation to that of Original Black – Scholes, Merton, KMV – Merton and Naïve KMV - Merton and interestingly captured "fat – tail effect" which is not being captured by the traditional once and has the ability to traces the trajectories of Past and Future Economic and Financial Crises given accurate, valid and reasonable estimations of the models' independent variables.

Example 7: (reference to Jameel's Advanced Stressed Models of Appendix B)

Consider the values obtained in the Proposed Jameel's Models VII (and Black – Scholes) to be Chevron Corporation Probability of Defaut (not for an Option) and Proposed Jameel's Models VIII (and Recovery Rate Black – Scholes). Let $EAD_{Stressed} = \$1,02000$ and $M = 3$ then we can Calculate:

(i) Stressed and Normal Asset Correlations for all exposures;

(ii) Stressed and Normal Capital Requirements;

(iii) Stressed and Normal Risk Weighted Assets;

(iv) Stressed and Normal Regulatory Capital for Credit Risk; and

(v) Stressed and Normal Unexpected Losses.

This can be seen using Microsoft EXCEL in the following table:

Stressed PDs, Rs, bs, RWAs, RCCRs and ULs:

FORMULAS	PD (STRESSED)	R (STRESSED)	b (STRESSED)	K (STRESSED)	$ RWA(STRESSED)	$ RCCR (STRESSED)	UL (STRESSED)	
M7 TYPE A+	0.494376251	0.12	0.024683625	0.119994617	1529931.364	122394.5091	0.071358583	
M7 TYPE A-	0.494359875	0.12	0.024684195	0.120001675	1530021.351	122401.7081	0.071366085	
M7 TYPE B+	0.494441555	0.12	0.024681351	0.119966466	1529572.442	122365.7953	0.071328667	
M7 TYPE B-	0.494294571	0.12	0.024686469	0.120029813	1530380.118	122430.4094	0.071396004	
M7 TYPE C+	0.321102471	0.120000013	0.032670492	0.131097555	1671493.829	133719.5063	0.126248224	
M7 TYPE C-	0.321086095	0.120000013	0.032671502	0.131098763	1671509.23	133720.7384	0.126255275	
M7 TYPE D+	0.321167775	0.120000013	0.032666465	0.131092731	1671432.324	133714.5859	0.126220106	
M7 TYPE D-	0.321020791	0.120000013	0.03267553	0.131103573	1671570.553	133725.6443	0.126283389	
BLACK -SCHOLES	PD (NORMAL)	R (NORMAL)	b (NORMAL)	K (NORMAL)	RWA(NORMAL)	RCCR (NORMAL)	UL (NORMAL)	
	0.321094283	0.120000013	0.032670997	0.131098159	1671501.53	133720.1224	0.12625175	
BLACK -SCHOLES RR	M8 TYPE A+	M8 TYPE A-	M8 TYPE B+	M8 TYPE B-	M8 TYPE C+	M8 TYPE C-	M8 TYPE D+	M8 TYPE D-
0.699194765	0.809997025	0.809980649	0.810062329	0.809915345	0.699202953	0.699186577	0.699268257	0.699121273
BLACK -SCHOLES LGD	LGD TYPE A+	LGD TYPE A-	LGD TYPE B+	LGD TYPE B-	LGD TYPE C+	LGD TYPE C-	LGD TYPE D+	LGD TYPE D-
0.300805235	0.190002975	0.190019351	0.189937671	0.190084655	0.300797047	0.300813423	0.300731743	0.300878727

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (12), 16 – 34

From the above table, considering $(M7\ TYPE\ C-,\ LGD\ TYPE\ C-)$, the values for $R_{Stressed}$, $b_{Stressed}$, $K_{Stressed}$, $RWA_{Stressed}$, $RCCR_{Stressed}$ and $UL_{Stressed}$ are: 0.120000013, 0.032671502, 0.131098763, $1671509.23, $133720.7384 and 0.126255275 respectively, while that of $(M7\ TYPE\ D-,\ LGD\ TYPE\ D-)$ are: 0.120000013, 0.03267553, 0.131103573, $1671570.553, $133725.6443 and 0.126283389 which are all HIGHER or equal in values than corresponding Black – Scholes Formula (Normal) whose values are: 0.120000013, 0.032670997, 0.131098159, $1671501.53, $133720.1224 and 0.12625175. Whereas in case of $M7\ TYPE\ A+,\ M7\ TYPE\ A-, M7\ TYPE\ B+, M7\ TYPE\ B-,\ M7\ TYPE\ C+,\ and\ M7\ TYPE\ D$, the values for $R_{Stressed}$, $b_{Stressed}$, $K_{Stressed}$, $RWA_{Stressed}$, $RCCR_{Stressed}$ and $UL_{Stressed}$ are all LOWER or equal than corresponding values in the case of Black – Scholes Formula (Normal).It would be recalled that Black – Scholes Formula suffered from the criticisms of NORMALITY assumption, that it can either underestimates or overestimate Credit Risks, therefore, from the foregoing, we can deduce the following:

(i) In case of Credit Risk OVERESTIMATION (stress period), we consider Jameel's Models: $(M7\ TYPE\ C-,\ LGD\ TYPE\ C-)$ and $(M7\ TYPE\ D-,\ LGD\ TYPE\ D-)$; whereas,

(ii) In case of Credit Risk UNDERESTIMATION (stress period), we consider Jameel's Models: $(M7\ TYPE\ A+,\ LGD\ TYPE\ A+)$, $(M7\ TYPE\ B+,\ LGD\ TYPE\ B+)$, $(M7\ TYPE\ B-,\ LGD\ TYPE\ B-)$, $(M7\ TYPE\ C+,\ LGD\ TYPE\ C+)$, and $(M7\ TYPE\ D+,\ LGD\ TYPE\ D+)$.

Similarly, we can treat the case of KMV – Merton and Na ïve KMV – Merton in the same manner.

Jameel's – Aish Triangular Solution $\{x_1, x_2, x_3\}$ for:

(i) RWA is given by $\{1671501.53,\ 1671509.23,\ 1671570.553\}$

(ii) RCCR is given by $\{133720.1224,\ 133720.7384,\ 133725.6443\}$

(iii) UL is given by $\{0.12622517,\ 0.126255275,\ 0.126283389\}$

Jameel's Pentagon for Decision Making Solution $\{x_1, x_2, x_3, x_4, x_5\}$ for:

(i) RWA is given by $\{1671501.53,\ 1671509.23,\ 1671570.553, 1671527.1,\ 1671527.1\}$

(ii) RCCR is given by $\{133720.1224,\ 133720.7384,\ 133725.6443,\ 133722.17,\ 133722.17\}$

(iii) UL is given by $\{0.12622517,\ 0.126255275,\ 0.126283389,\ 0.1262635,\ 0.1262635\}$

4.4 Result and Discussion Reference to Jameel's Advanced Stressed Models of Appendix D

Example 4: Consider an example of Microsoft Corporation (MSFT) option with a term of six months (0.5 years). The current stock price of Microsoft Corporation (MSFT) is $\$48.14$ and the strike of the option is $\$49.39$. The risk-free rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. With the above strike prices of 148 months: $\mu_{strike} = 0.008836$, $\sigma_{strike} = 0.188051$, and $f\left(k;\mu_K,\sigma_K,\pi\right) = 1.624231$ (Cauchy) for the current period. Note that, unlike Probability, Probability Distributions Function can take values GREATER THAN ONE at extreme cases since its define as Probability PER UNIT VALUE of a Random Variable, but the integral of this distribution function taken with respect to this value must be exactly equal 1. What is the values of both CALL and PUT the options using: (1) Black-Scholes – Merton (1973) Model; and (2) Proposed Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models reference to Black-Scholes - Merton (1973) Model?

Using the data of Microsoft Corporation (MSFT) extracted from yahoo finance from 2014 – 1991, we obtained: $f\left(x,\mu_{underlying},\sigma_{underlying},\xi\right) = 0.00000000123523$ (Log – Logistic (3P)), $\mu_S = 0.031886784$, $\sigma_S = 0.117906073$, $\mu_K = 0.032829086$, $\sigma_K = 0.124525303$, $\mu_{SK} = 0.028046277$, and $\sigma_{SK} = 0.123540719$ (data available), $K = \$49.39$, $S = \$48.14$, $\sigma = 0.022041976$, $r = 0.0392$, $T = 0.5$ and $t = 0$. Recall that $d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$ then

$$d_1 = \frac{\ln\left(\frac{48.14}{49.39}\right) + \left(0.0392 + \frac{0.022041976^2}{2}\right)(0.5-0)}{0.022041976\sqrt{0.5-0}} = -0.18969 \quad d_2 = -0.18969 - 0.022041976\sqrt{0.5} = -0.20528$$

Therefore, $d_1 = -0.18969$ and $d_2 = -0.20528$. Using the Microsoft EXCEL, consider the following tables:

Table 4.4.1 Call Option Black – Scholes – Merton (1973) and Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models

TYPES (CALL)	TYPE n+	BLACK – SCHOLES	TYPE n-	TYPE n+	BLACK - SCHOLES	TYPE n-	TYPE n+	BLACK - SCHOLES	TYPE n-	TYPE n+	BLACK – SCHOLES	TYPE n-
1, 2, 3, 4	-0.618622475	0.171627934	0.327918129	-0.301642368	0.171627934	0.644898236	0.112693806	0.171627934	0.230562062	-4.236090898	0.171627934	-3.289550294
5, 6, 7, 8,	-0.301642315	0.171627934	0.644898183	68.56648089	0.171627934	-68.22322502	3.315826003	0.171627934	4.262366712	-68.7554295	0.171627934	69.09868537
9, 10, 11, 12	-72.68987798	0.171627934	65.16423679	-4.236090845	0.171627934	-3.289550347	68.56648094	0.171627934	-68.22322507	64.63203236	0.171627934	-72.15767355
13, 14, 15, 16	68.56648084	0.171627934	-68.22322497	-68.7554294	0.171627934	69.09868527	0.112693806	0.171627934	0.230562062	-0.301642315	0.171627934	0.644898183
17, 18, 19, 20	72.18394931	0.171627934	-64.6057566	-4.236090951	0.171627934	-3.289550242	-65.13796103	0.171627934	72.71615374	64.63203236	0.171627934	-72.15767355
21, 22, 23, 24	-65.13796108	0.171627934	72.7161538	64.63203241	0.171627934	-72.1576736	-72.68987803	0.171627934	65.16423684	64.63203231	0.171627934	-72.1576735
25, 26, 27, 28	-72.68987793	0.171627934	65.16423674	-65.13796098	0.171627934	72.71615369	-68.75542945	0.171627934	69.09868532	72.18394926	0.171627934	-64.60575655
29, 30, 31, 32	68.24950078	0.171627934	-68.54020513	72.18394937	0.171627934	-64.60575665	-0.618622422	0.171627934	0.327918076	-3.821754724	0.171627934	-3.703886468
33, 34, 35, 36	0.112693859	0.171627934	0.230562009	3.315826055	0.171627934	4.262366659	-0.618622528	0.171627934	0.327918181	-0.618622422	0.171627934	0.327918076
37, 38, 39, 40	-0.618622528	0.171627934	0.327918181	-3.821754672	0.171627934	-3.703886521	68.24950084	0.171627934	-68.54020518	72.18394931	0.171627934	-64.6057566
41, 42, 43, 44	3.730162177	0.171627934	3.730162177	-69.07240961	0.171627934	68.78170527	-69.07240951	0.171627934	68.78170516	68.24950073	0.171627934	-68.54020508
45, 46, 47, 48	68.24950084	0.171627934	-68.54020518	-0.618622475	0.171627934	0.327918129	-0.204286301	0.171627934	-0.086418045	-0.204286301	0.171627934	-0.086418045
49, 50, 51, 52	-72.68987798	0.171627934	-72.68987798	3.730162282	0.171627934	3.848030432	64.63203236	0.171627934	-72.15767355	3.730162229	0.171627934	3.848030485
53, 54, 55, 56	-3.821754724	0.171627934	-3.703886468	-0.204286248	0.171627934	-0.086418098	68.24950078	0.171627934	-68.54020513	-0.204286301	0.171627934	-0.086418045
57, 58, 59	3.730162177	0.171627934	3.848030538	-65.13796103	0.171627934	72.71615374	0.112693754	0.171627934	0.230562115		0.171627934	

Based on the above CALL option table: Jameel's models 3, 33, and 59 are EXTREMELY recommended at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods) while, LEFT of models 1, 2, 5, 16, 31, 35, 36 and are partially HIGHER values reference to the BSM's price, nevertheless, they also useful. Jameel's models 7, 34, 41, 50, 52 and 57 are EXTREMELY higher values considering BSM but could be useful in other market conditions.

Table 4.4.2 Call Option Black – Scholes – Merton (1973), First and Second Jameel's Proposed Models reference to BSM

CALL	1ST PROPOSED MODEL CLASS (+)	1ST PROPOSED MODEL CLASS (-)	BLACK -SCHOLES	2ND PROPOSED MODEL CLASS (+)	2ND PROPOSED MODEL CLASS (-)
M I, II TYPE 3	0.112693806	0.230562062		0.11315978	0.230096088
M I, II TYPE 33	0.112693859	0.230562009	0.171627934	0.113159833	0.230096035
M I, II TYPE 59	0.112693754	0.230562115		0.112693754	0.23009614

The above table summarized the comparisons between **FIRST** and **SECOND** proposed Jameel's models I. comparing the columns of 1st proposed model class (+) and 2nd proposed model class (+), they ultimately approximates one another with a very strong positive correlation, similarly, 1st proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between BSM Price. These conclude that both **FIRST** and **SECOND** proposed Jameel's models are extremely recommended to be used at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods).

Table 4.4.3 Put Option Black – Scholes – Merton (1973) and Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models

TYPES (PUT)	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK –SCHOLES	TYPE n-
1, 2, 3, 4	0.619299282	**0.463009087**	-0.327241322	0.936279389	**0.463009087**	-0.010261215	0.521943215	**0.463009087**	0.404074959	-2.998169141	**0.463009087**	-3.944709745
5, 6, 7, 8,	0.936279336	**0.463009087**	-0.010261162	-67.93184387	**0.463009087**	68.85786204	4.553747865	**0.463009087**	3.607207156	69.39006653	**0.463009087**	-68.46404835
9, 10, 11, 12	65.45561794	**0.463009087**	-72.39849683	-2.998169194	**0.463009087**	-3.944709692	-67.93184392	**0.463009087**	68.8578621	-71.8662924	**0.463009087**	64.92341351
13, 14, 15, 16	-67.93184382	**0.463009087**	68.85786199	69.39006642	**0.463009087**	-68.46404825	0.521943215	**0.463009087**	0.404074959	0.936279336	**0.463009087**	-0.010261162
17, 18, 19, 20	-64.31437545	**0.463009087**	72.47533047	-2.998169089	**0.463009087**	-3.944709798	73.0075349	**0.463009087**	-64.84657988	-71.8662924	**0.463009087**	64.92341351
21, 22, 23, 24	73.00753495	**0.463009087**	-64.84657993	-71.86629245	**0.463009087**	64.92341357	65.455618	**0.463009087**	-72.39849688	-71.86629235	**0.463009087**	64.92341346
25, 26, 27, 28	65.45561789	**0.463009087**	-72.39849678	73.00753484	**0.463009087**	-64.84657982	69.39006647	**0.463009087**	-68.4640483	-64.31437539	**0.463009087**	72.47533041
29, 30, 31, 32	-68.24882398	**0.463009087**	68.54088194	-64.3143755	**0.463009087**	72.47533052	0.619299229	**0.463009087**	-0.327241269	-3.412505315	**0.463009087**	-3.530373571
33, 34, 35, 36	0.521943163	**0.463009087**	0.404075012	4.553747812	**0.463009087**	3.607207209	0.619299334	**0.463009087**	-0.327241375	0.619299229	**0.463009087**	-0.327241269
37, 38, 39, 40	0.619299334	**0.463009087**	-0.327241375	-3.412505368	**0.463009087**	-3.530373518	-68.24882403	**0.463009087**	68.54088199	-64.31437545	**0.463009087**	72.47533047
41, 42, 43, 44	4.139411691	**0.463009087**	4.02154333	69.07308642	**0.463009087**	-68.78102846	69.07308631	**0.463009087**	-68.78102835	-68.24882392	**0.463009087**	68.54088188
45, 46, 47, 48	-68.24882403	**0.463009087**	68.54088199	0.619299282	**0.463009087**	-0.327241322	0.204963108	**0.463009087**	0.087094852	0.204963108	**0.463009087**	0.087094852
49, 50, 51, 52	65.45561794	**0.463009087**	-72.39849683	4.139411586	**0.463009087**	4.021543435	-71.8662924	**0.463009087**	64.92341351	4.139411638	**0.463009087**	4.021543383
53, 54, 55, 56	-3.412505315	**0.463009087**	-3.530373571	0.204963055	**0.463009087**	0.087094905	-68.24882398	**0.463009087**	68.54088194	0.204963108	**0.463009087**	0.087094852
57, 58, 59	4.139411691	**0.463009087**	4.02154333	73.0075349	**0.463009087**	-64.84657988	0.521943268	**0.463009087**	0.404074907		**0.463009087**	

Based on the above PUT option table: Jameel's models 3, 33, 48, 54 and 59 are EXTREMELY recommended at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods) while, LEFT of models 1, 2, 5, 16, 31, 35, 36, 47 and are partially HIGHER values reference to the BSM's price, nevertheless, they also useful. Jameel's models 7, 34, 41, 50, 52 and 57 are EXTREMELY higher values considering BSM but could be useful in other market conditions.

Table 4.4.4 Put Option Black – Scholes – Merton (1973), First and Second Jameel's Proposed Models reference to BSM

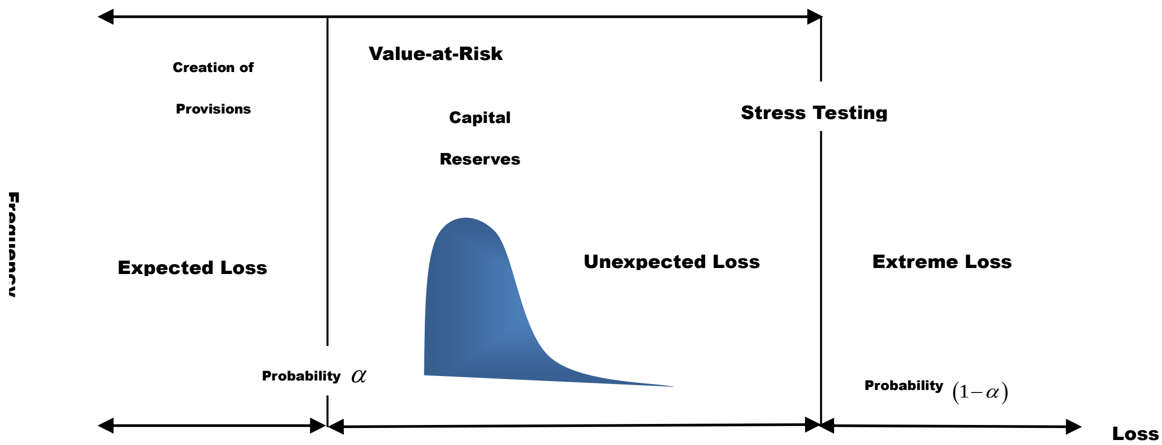
PUT	1ST PROPOSED MODEL CLASS (+)	1ST PROPOSED MODEL CLASS (-)	BLACK –SCHOLES	2ND PROPOSED MODEL CLASS (+)	2ND PROPOSED MODEL CLASS (-)
M I, II TYPE 3	0.521943215	0.404074959		0.521477241	0.404540933
M I, II TYPE 33	0.521943163	0.404075012	0.463009087	0.521477188	0.404540986
M I, II TYPE 59	0.521943268	0.404074907		0.521943268	0.404540881

The above table summarized the comparisons between FIRST and SECOND proposed Jameel's models I. comparing the columns of 1^{st} proposed model class (+) and 2nd proposed model class (+), they ultimately approximates one another with a very strong positive correlation, similarly, 1^{st} proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between BSM Price. These conclude that both FIRST and SECOND proposed Jameel's models are extremely recommended to be used at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods).

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (11), 09 – 19

Proposed Theorem (Bash – Eves – Kali's Sacrifice):

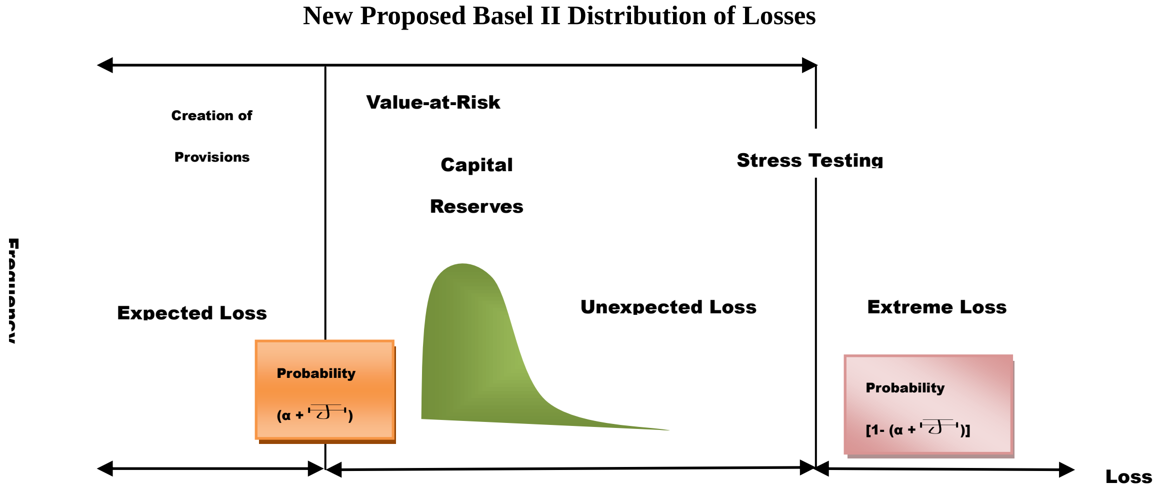
Consider the following diagram: Basel II Distribution of Losses



Let DB be the distribution of losses under BASEL II accord described in the above diagram. Let $J(M)'s$ be all Jameel's Advanced Stressed Stochastic and Deterministic Financial and Economic models obtained as the result of Jameel's CONTRACTIONAL and EXPANSIONAL stress methods, let α and $(1-\alpha)$ be Normal Markets Confidence and Significant levels respectively, let [symbol] $\in(0\ 1)$ be an infinitesimal positive constant then $J(M)'s$ Advanced Stressed Stochastic and Deterministic Financial and Economic models have increases Markets CONFIDENCE LEVEL by (+[symbol]) and decreases Markets SIGNIFICANT LEVEL by (-[symbol]), meaning now, Markets Confidence and Significant levels have consequently became ($\alpha +$ [symbol]) and [1 – ($\alpha +$ [symbol])] respectively. Where, [symbol] is called JAMEEL'S CONSTANT.

Interpretation and Conclusion: Bash – Eves Sacrifice Theorem has increases MARKETS CONFIDENCE dramatically and reduces MARKETS RISKS drastically.

Proposed Eve's Transition Diagram:



5. Conclusion

Based on the available results, there are very huge differences between the Normal Market Value and Jameel's Advanced Stressed Economic and Financial Crises Values. These differences will definitely help in tracing the trajectories of the PAST Crises and in addressing FUTURE Economic and Financial Crises.

For the sake of practitioners, it is believe that the existing Quantitative Risk Management Models underestimates (overestimates) Default Risks especially at the times of Economic and Financial Crises to the extent in which Tim Harford (2012) published an article entitled' Black – Scholes: The Maths Formula linked to the Financial Crash' where he stated that '…It has been argued that one formula known as Black – Scholes, along with its descendants, helped to blow up the financial world'. Many other articles have been published in respect to that. The models here presented will serve as the complimentary but not substitute of the Black – Scholes and its descendants because the models are more robust, holistic and extraordinary, providing better approximations, increasing the probabilities of high losses and above all have the ability to precisely traces the trajectories of the past and future economic and financial crises from the results and graphs shown, since they incorporated fat –tail effects.

Finally, for the sake of future research direction, the models can be improved further to capture more vital information using more macroeconomic indicators and models' independent variables.

Nassim Nicholas Taleb et al (2009) stated that "Black Swan events are almost impossible to predict. Instead of perpetuating the illusion that we can anticipate the future, risk management should try to reduce the impact of the threats we don't understand."

CreditMetrics™ (1997) stated that "We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks. CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results."

"If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one". It is better off improving Credit Risk Models than not.

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APPENDIX A

Proposed Jameel's Models I:

The proposed models considering simple Logistic Regression Model are given by:

Type A:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type A* (Higher Probabilities):

$$PD_{Stressed} = \frac{1}{1 + \mu_A . \exp\left(\sum_{i=0}^{K} \beta_i X_i \right) \mp \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type B:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type B* (Higher Probabilities):

$$PD_{Stressed} = \frac{1}{1+\mu_A.\exp\left(\sum_{i=0}^{K}\beta_i X_i\right) \mp f\left(x;\mu_{company},\sigma_{company},\xi\right)}$$

Type C:

$$PD_{Stressed} = \frac{1}{1+\exp\left(\sum_{i=0}^{K}\beta_i X_i\right) \mp \sigma_A f\left(x;\mu_{company},\sigma_{company},\xi\right)}$$

Type D:

$$PD_{Stressed} = \frac{1}{1+\exp\left(\sum_{i=0}^{K}\beta_i X_i\right) \mp f\left(x;\mu_{company},\sigma_{company},\xi\right)}$$

Where, the Simple **Logistic Regression Model (Logit)** is given by:

$$PD = \frac{1}{1+\exp\left(\sum_{i=0}^{K}\beta_i X_i\right)}$$

$X = \left(X_1, X_2, ..., X_k\right)$ is a vector of explanatory variables (Macro-economic Indicators).

Proposed Jameel's Models II:

The proposed models considering Merton's (Probit) Model are given by:

Type A:

$$PD_{Stressed} = \Phi\left(\beta_0 + \mu_A \sum_{j=1}^{J}\beta_j X_j\right) \pm \sigma_A f\left(x;\mu_{company},\sigma_{company},\xi\right)$$

Type A*:

$$PD_{Stressed} = \Phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm \sigma_A f\left(x;\mu_{company},\sigma_{company},\xi\right)$$

Type B:

$$PD_{Stressed} = \Phi\left(\beta_0 + \mu_A \sum_{j=1}^{J}\beta_j X_j\right) \pm f\left(x;\mu_{company},\sigma_{company},\xi\right)$$

Type B*:

$$PD_{Stressed} = \Phi\left[\mu_A\left(\beta_0 + \sum_{j=1}^{J}\beta_j X_j\right)\right] \pm f\left(x;\mu_{company},\sigma_{company},\xi\right)$$

Type C:

$$PD_{Stressed} = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right) \pm \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type D:

$$PD_{Stressed} = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right) \pm f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Where, **Factor Model based on Merton Model (Probit)** is given by:

$$PD = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right)$$

APPENDIX B

Proposed Jameel's Models VII:

The proposed models considering **Black – Scholes – Merton (1973)** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi\left(-\mu_A J\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **,TYPE B:** $PD_{Stressed} = \Phi\left(-\mu_A J\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

TYPE C: $PD_{Stressed} = \Phi\left(-J\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **, TYPED:** $PD_{Stressed} = \Phi\left(-J\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

Where, $PD = \Phi\left(\frac{-\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t)\right)}{\sigma_V \sqrt{T-t}}\right)$, $J = \frac{\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t)\right)}{\sigma_V \sqrt{T-t}}$ then $PD = \Phi\left(-J\right)$.

Proposed Jameel's Models VIII:

The proposed models considering **Merton (1974)** Recovery Rate Model are given by:

TYPE A: $RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V\, T\right) \frac{\Phi\left(-\mu_A . d_1\right)}{\Phi\left(-\mu_A . d_2\right)} \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **, TYPE B:** $RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V\, T\right) \frac{\Phi\left(-\mu_A . d_1\right)}{\Phi\left(-\mu_A . d_2\right)} \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

TYPE C: $RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V . T\right) \frac{\Phi\left(-d_1\right)}{\Phi\left(-d_2\right)} \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **, TYPE D:** $RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V . T\right) \frac{\Phi\left(-d_1\right)}{\Phi\left(-d_2\right)} \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

Where, $RR = \frac{A_0}{D} \exp\left(\mu_V T\right) \frac{\Phi\left(-d_1\right)}{\Phi\left(-d_2\right)}$, $d_1 = \frac{\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2} T\right)}{\sigma_V \sqrt{T}}$ and $d_2 = d_1 - \sigma_V \overline{T}$

Proposed Jameel's Models IX:

The proposed models considering **KMV – Merton** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi\left(-\mu_A . DD\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **, TYPE B:** $PD_{Stressed} = \Phi\left(-\mu_A . DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

TYPE C: $PD_{Stressed} = \Phi\left(-DD\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$ **, TYPE D:** $PD_{Stressed} = \Phi\left(-DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

Where, the Distance to Default can be calculated as: $DD = \frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}$

Where μ is an estimate of the expected annual return of firm's assets. The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or EDF) is given by: $\pi KMV = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)$ and $\pi KMV = \Phi\left(-DD\right)$

Existing Model (Naïve KMV – Merton Alternative)

Proposed Jameel's Models X:

The proposed models considering **NaïveKMV – Merton** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi\left(-\mu_A . Naive\, DD\right) \pm \sigma_A\, f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$, **TYPE B:** $PD_{Stressed} = \Phi\left(-\mu_A . Naive\, DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

TYPE C: $PD_{Stressed} = \Phi\left(-Naive\, DD\right) \pm \sigma_A\, f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$**, TYPE D:** $PD_{Stressed} = \Phi\left(-Naive\, DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

Where, the Distance to Default can be calculated as: $DD = \frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}$

Where μ is an estimate of the expected annual return of firm's assets.The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or EDF) is given by: $\pi KMV = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)$ and

$\pi KMV = \Phi\left(-DD\right)$ with the substitutions that the volatility of each firm' debt is given by: $Naive\ \sigma_D = 0.05+0.25\sigma_E$ and

$Naive\ \mu = r_{u-1}$

By allowing Naïve estimate of μ todepends on past returns, we incorporate the same information. The Naïve Distance to Default is given by: $Naive\ DD = \frac{\ln\left[\frac{(E+F)}{F}\right]+\left(r_{u-1}-0.5\, Naive\,\sigma_V^2\right)T}{Naive\,\sigma_V\sqrt{T}}$.Naïve Probability estimate is given by:

$$\pi_{Naive} = \Phi\left(-Naive\ DD\right)$$

Proposed Jameel's Models XIII (reference Robert J. Powell and David E. Allen Conditional Probability of Default):

TYPE A:

$$CPD_{Stressed} = \Phi\left(-\mu_A\, CDD\right) \pm \sigma_A\, f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$CPD_{Stressed} = \Phi\left(-\mu_A\, CDD\right) \pm\ f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$CPD_{Stressed} = \Phi\left(-CDD\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$CPD_{Stressed} = \Phi\left(-CDD\right) \pm\ f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Model (Laplace/Gauss – Hermite Default Rate Approximation):

The Laplace/Gauss – Hermite approximation of the likelihood Generalized Linear Mixed Models estimation is given by:

Default Rate Model:

$$PD_{k,t} = \Phi\left(\theta_{r(k)} + X_t\beta + Z_t\right)$$

With the following notation:

Φ is the standard normal cumulative distribution function, $r(k)$ is the rating of obligor k, $\theta_{r(k)}$ is an intercept for rating $r(k)$, X_t is an $1\times p$ vector with the coefficients modeling the impact of the macro – economic variables on the PD, $Z_t \sim \Phi\left(0, \sigma^2\right)$ is a latent factor with variance σ^2. Latent factors can be correlated over time with correlation matrix $Corr\left(Z_1, Z_2, ..., Z_r\right) = C$.

Migration Model:

$$Pdown_{k,t} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right)$$

$$Pup_{k,t} = 1 - \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right)$$

Proposed Jameel's Models 22:

Up Migration Default Rate Models:

The Up Migration Default Rate Models of a Company under stress are given by:

TYPE A:

$$Pdown_{(k,t)stressed} = \Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm\ \sigma_A\ f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$Pdown_{(k,t)stressed} = \Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm\ f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$Pdown_{(k,t)stressed} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right) \pm \sigma_A\, f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$Pdown_{(k,t)stressed} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Down Migration Default Rate Models:

For example, the Down Migration Default Rate Models of a Company under stress using $M22\ TYPE\ A$ are given by:

$$Pup_{(k,t)stressed} = 1 - \left[\Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm \sigma_A\, f\left(x, \mu_{company}, \sigma_{company}, \xi\right)\right]$$

In similar way, we can find the Down Migration Default Rate Models of the remaining types.

APPENDIX C

Proposed Jameel's Models 22:

Recall that the price of CALL OPTION is given by: $C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$ then

The proposed models considering Black – Scholes – Merton (1973) Model are given by:

TYPE A $(\times)$: $C(S,t)_{Stressed} = S\left[\Phi(\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(\mu_A d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE A* $(*)$: $C(S,t)_{Stressed} = S\left[\Phi(\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE B $(\times)$: $C(S,t)_{Stressed} = S\left[\Phi(\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(\mu_A d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE B* $(*)$: $C(S,t)_{Stressed} = S\left[\Phi(\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE C: $C(S,t)_{Stressed} = S\left[\Phi(d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE D: $C(S,t)_{Stressed} = S\left[\Phi(d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - \left[\Phi(d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

The price of PUT OPTION is given by: $P(S,t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)}$ then

TYPE A: $P(S,t)_{Stressed} = -S\left[\Phi(-\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-\mu_A d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE A*: $P(S,t)_{Stressed} = -S\left[\Phi(-\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]Ke^{-r(T-t)}$

TYPE B: $P(S,t)_{Stressed} = -S\left[\Phi(-\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-\mu_A d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] Ke^{-r(T-t)}$

TYPE B*: $P(S,t)_{Stressed} = -S\left[\Phi(-\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] Ke^{-r(T-t)}$

TYPE C: $P(S,t)_{Stressed} = -S\left[\Phi(-d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] Ke^{-r(T-t)}$

TYPE D: $P(S,t)_{Stressed} = -S\left[\Phi(-d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] + \left[\Phi(-d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] Ke^{-r(T-t)}$

Where, the Black – Scholes – Merton (1973) model for CALL and PUT options are given by:

$C(S,t) = \Phi(d_1)S - \Phi(d_2)Ke^{-r(T-t)}$ and $P(S,t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)}$ respectively.

$d_1 = \left(\ln(S/K) + \left(r + \sigma^2/2\right)(T-t)\right) \Big/ \sigma\sqrt{T-t}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$.

Proposed Jameel's Models 24:

Recall that the **CALL PRICE OPTIONS** on foreign exchange rates is given by:

$$C(S_0,t) = e^{-r(T-t)}\left(F\Phi(d_1) - K\Phi(d_2)\right) \text{ then}$$

The proposed models considering Garman - Kohlhagen (1983) Foreign Exchange Rates Options Price are given by:

TYPE A $(\times)$: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(\mu_A d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE A* $(*)$: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE B $(\times)$: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(\mu_A d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE B* $(*)$: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE C: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE D: $C(S_0,t)_{Stressed} = e^{-r(T-t)}\left(F\left[\Phi(d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - K\left[\Phi(d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

The PUT PRICE OPTIONS on foreign exchange rates is given by:

$$P(S_0,t) = e^{-r(T-t)}\left(K\Phi(-d_2) - F\Phi(-d_1)\right) \text{ then}$$

TYPE A $(\times)$: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-\mu_A d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-\mu_A d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE A* $(*)$: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-\mu_A d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE B $\left(\times\right)$: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-\mu_A d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-\mu_A d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE B* $\left(*\right)$: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-\mu_A d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE C: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-d_2) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-d_1) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

TYPE D: $P(S_0,t)_{Stressed} = e^{-r(T-t)}\left(K\left[\Phi(-d_2) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right] - F\left[\Phi(-d_1) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right]\right)$

Proposed Jameel's Models 25:

Recalled that CAPS Price is given by: $Cap(t) = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \left[F(t,T_{i-1},T_i)\Phi(d_i) - E \times \Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right]$

The proposed models considering Black (1976) for CAPS Price is given by:

TYPE A $\left(\times\right)$:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(\mu_A d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(\mu_A\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE A* $\left(*\right)$:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(\mu_A d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE B $\left(\times\right)$:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(\mu_A d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(\mu_A\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE B* $\left(*\right)$:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(\mu_A d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE C:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE D:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left(\Phi(d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -E \times \left(\Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

Proposed Jameel's Models 26:

Recalled that **FLOORS** Price is given by: $Floor(t) = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \left[-F(t,T_{i-1},T_i)\Phi(-d_i) + E \times \Phi\left(-d_i + \sigma_i\sqrt{T_{i-1}-t}\right)\right]$

The proposed models considering **Black (1976) for FLOORS Price** is given by:

TYPE A:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-\mu_A d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(\mu_A\left(-d_i + \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE A*:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-\mu_A d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(-d_i + \sigma_i \sqrt{T_{i-1}-t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE B:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-\mu_A d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(\mu_A\left(-d_i + \sigma_i \sqrt{T_{i-1}-t}\right)\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE B*:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-\mu_A d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(-d_i + \sigma_i \sqrt{T_{i-1}-t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE C:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-d_i) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(-d_i + \sigma_i \sqrt{T_{i-1}-t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

TYPE D:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left(\Phi(-d_i) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ +E \times \left(\Phi\left(-d_i + \sigma_i \sqrt{T_{i-1}-t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{bmatrix}$$

Proposed Jameel's Models 27:

Recalled that the **Payer Swaption**formula is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^{n}\left(F_s(t) \times \Phi(d) - F \times \Phi\left(d - \sigma_s\sqrt{T_0 - t}\right)\right) \times D(t,T_i)$$

The proposed models considering **Black (1976) for Payer Swaption Prices** are given by:

TYPE A $(\times)$:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(\mu_A d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(\mu_A\left(d - \sigma_s\sqrt{T_0 - t}\right)\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

TYPE A* $(*)$:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(\mu_A d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(d - \sigma_s\sqrt{T_0 - t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

TYPE B $(\times)$:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(\mu_A d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(\mu_A\left(d - \sigma_s\sqrt{T_0 - t}\right)\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

TYPE B* $(*)$:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(\mu_A d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(d - \sigma_s\sqrt{T_0 - t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

TYPE C:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(d - \sigma_s\sqrt{T_0 - t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

TYPE D:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n}\begin{pmatrix} F_s(t) \times \left(\Phi(d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \\ -F \times \left(\Phi\left(d - \sigma_s\sqrt{T_0 - t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right)\right) \end{pmatrix} \times D(t,T_i)$$

The pricing formula for the **Receiver Swaptions** is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^{n} \left(-F_s(t) \times \Phi(-d) + F \times \Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \right) \times D(t, T_i)$$ then

TYPE A:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-\mu_A d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\Phi\left(\mu_A \left(-d + \sigma_s \sqrt{T_0 - t}\right)\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

TYPE A*:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-\mu_A d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

TYPE B:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-\mu_A d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\Phi\left(\mu_A \left(-d + \sigma_s \sqrt{T_0 - t}\right)\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

TYPE B*:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-\mu_A d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

TYPE C:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-d) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\times \Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \pm \sigma_A f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

TYPE D:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{pmatrix} -F_s(t) \times \left(\Phi(-d) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \\ +F\left(\times \Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \pm f\left(x, \mu_{underlying}, \sigma_{underlying}, \xi\right) \right) \end{pmatrix} \times D(t, T_i)$$

APPENDIX D (General Form)

First Proposed Jameel's Models I:

The proposed models considering **Black – Scholes – Merton (1973)Call Option Price** are given by:

$$\begin{aligned} C(S,t)_{Stressed} &= S\left[\Phi(\mu_S.\mu_K.d_1) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ &\quad - K\left[\Phi(\mu_S.\mu_K.d_2) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right].e^{-r(T-t)} \end{aligned}$$

The first Type above is the general (initial) proposal not necessarily positive prices (but could also be negative values); however, using Jameel'sContractional and Expansional Stress Methods, we can have the following possible **COMBINATION** of **TERMS** and **SIGNS**.

Combination of Terms and Signs:

Recall that $C_r^n = n!/(n-r)!r!$ then we have $C_1^6 + C_2^6 + C_3^6 + C_4^6 + C_5^6 + C_6^6 = 63$ combination of terms using the general form above and $C_1^8 + C_2^8 + C_3^8 + C_4^8 + C_5^8 + C_6^8 + C_7^8 + C_8^8 = 225$ combination of Signs.

The proposed models considering **Black – Scholes – Merton (1973)Put Option Price** are given by:

$$\begin{aligned} P(S,t)_{Stressed} &= -S\left[\Phi(-\mu_S.\mu_K.d_1) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ &\quad + K\left[\Phi(-\mu_S.\mu_K.d_2) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right].e^{-r(T-t)} \end{aligned}$$

Therefore, we have to further check 4 and 162 remaining combination of Terms and Signs respectively.

Where, $P(S,t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)}$.

Second Proposed Jameel's Models I:

The proposed models considering **Black – Scholes – Merton (1973)Call Option Price** are given by:

$$C(S,t)_{Stressed} = S\left[\Phi\left(\mu_{SK}.d_1\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right] - K\left[\Phi\left(\mu_{SK}.d_2\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right].e^{-r(T-t)}$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CALL** and **PUT** options as in the case of first Proposed Jameel's I Models shown in the table above.

Also, the **GREEKS** of the both **CALL** and **PUT** options can be found in similar ways and patterns.

Therefore, in this section, the author will treat the proposed Jameel's models II to VIII as he treated the First and Second proposed Jameel's models I above. Similarly in the case of Result and Discussion section.

Proposed Jameel's Models II:

The proposed models considering **Garman - Kohlhagen (1983) Foreign Exchange Rates OptionsPrice** are given by:

$$C(S_0,t)_{Stressed} = \begin{bmatrix} F\left[\Phi\left(\mu_S.\mu_K.d_1\right) \pm \sigma_S.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_K.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \\ -K\left[\Phi\left(\mu_S.\mu_K.d_2\right) \pm \sigma_S.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_K.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \end{bmatrix} e^{-r(T-t)}$$

And/or

$$C(S_0,t)_{Stressed} = \begin{bmatrix} F\left[\Phi\left(\mu_{SK}.d_1\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \\ -K\left[\Phi\left(\mu_{SK}.d_2\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \end{bmatrix} e^{-r(T-t)}$$

and that of **PUT** are given by:

$$P(S_0,t)_{Stressed} = \begin{bmatrix} -F\left[\Phi\left(-\mu_S.\mu_K.d_1\right) \pm \sigma_S.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_K.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \\ +K\left[\Phi\left(-\mu_S.\mu_K.d_2\right) \pm \sigma_S.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_K.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \end{bmatrix} e^{-r(T-t)}$$

And/or

$$C(S_0,t)_{Stressed} = \begin{bmatrix} -F\left[\Phi\left(-\mu_{SK}.d_1\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \\ +K\left[\Phi\left(-\mu_{SK}.d_2\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)\right] \end{bmatrix} e^{-r(T-t)}$$

Where, $C(S_0,t) = e^{-r(T-t)}\left(F\Phi(d_1) - K\Phi(d_2)\right)$ and $P(S_0,t) = e^{-r(T-t)}\left(K\Phi(-d_2) - F\Phi(-d_1)\right)$.

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CALL** and **PUT** options as in **ALL** the cases as shown in the tables above.

Proposed Jameel's Models III:

The proposed models considering **Black (1976) for CAPS Price** are given by:

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left[\Phi(\mu_S.\mu_K.d_i) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ -K\left[\Phi\left(\mu_S.\mu_K\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix}$$

And/or

$$Cap(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} F(t,T_{i-1},T_i)\left[\Phi(\mu_{SK}.\mu_K.d_i) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \\ -K\left[\Phi\left(\mu_{SK}\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix}$$

and that of **FLOORLET** are given by:

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left[\Phi(-\mu_S.\mu_K.d_i) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ +K\left[\Phi\left(\mu_S.\mu_K\left(-d_i + \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix}$$

And/or

$$Floor(t)_{Stressed} = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \begin{bmatrix} -F(t,T_{i-1},T_i)\left[\Phi(-\mu_{SK}.d_i) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \\ +K\left[\Phi\left(\mu_{SK}\left(-d_i + \sigma_i\sqrt{T_{i-1}-t}\right)\right) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix}$$

Where, $Cap(t) = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \left[F(t,T_{i-1},T_i)\Phi(d_i) - E \times \Phi\left(d_i - \sigma_i\sqrt{T_{i-1}-t}\right)\right]$ and.

$$Floor(t) = \sum_{i=1}^{n} M \times \Delta \times D(t,T_i) \times \left[-F(t,T_{i-1},T_i)\Phi(-d_i) + E \times \Phi\left(-d_i + \sigma_i\sqrt{T_{i-1}-t}\right)\right]$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CAPS** and **FLOORS** as in **ALL** the cases as shown in the tables above.

Proposed Jameel's Models IV:

The proposed models considering **Black (1976) for PAYER SWAPTIONS Prices** are given by:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{bmatrix} F_s(t) \times \left[\Phi(\mu_S.\mu_K.d) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ -K\left[\Phi\left(\mu_S.\mu_K\left(d - \sigma_s\sqrt{T_0-t}\right)\right) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix} \times D(t,T_i)$$

And/or

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{bmatrix} F_s(t) \times \left[\Phi(\mu_{SK}d) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \\ -K\left[\Phi\left(\mu_{SK}\left(d - \sigma_s\sqrt{T_0-t}\right)\right) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix} \times D(t,T_i)$$

and that of **RECEIVER SWAPTIONS Prices** are given by:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{bmatrix} -F_s(t) \times \left[\Phi(-\mu_S.\mu_K.d) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \\ +K\left[\Phi\left(\mu_S.\mu_K\left(-d + \sigma_s\sqrt{T_0-t}\right)\right) \pm \sigma_S.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_K.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix} \times D(t,T_i)$$

And/or

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^{n} \begin{bmatrix} -F_s(t) \times \left[\Phi(-\mu_{SK}d) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \\ +K\left[\Phi\left(\mu_{SK}\left(-d + \sigma_s\sqrt{T_0-t}\right)\right) \pm \sigma_{SK}.f(s,\mu_S,\sigma_S,\xi) \pm \sigma_{SK}.f(k,\mu_K,\sigma_K,\pi)\right] \end{bmatrix} \times D(t,T_i)$$

Where, Payer Swaption is given by: $Swaption(t) = \sigma \times M \times \sum_{i=1}^{n}\left(F_s(t) \times \Phi(d) - F \times \Phi\left(d - \sigma_s\sqrt{T_0-t}\right)\right) \times D(t,T_i)$

and

Receiver Swaption is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^{n} \left(-F_s(t) \times \Phi(-d) + F \times \Phi\left(-d + \sigma_s \sqrt{T_0 - t}\right) \right) \times D(t, T_i) \cdot$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **PAYER SWAPTION** and **RECEIVER SWAPTION** as in **ALL** the cases as shown in the tables above.

RECOVERY RATES AND DEFAULT PROBABILITIES EXTENDED VERSIONS

Proposed Jameel's Models V:

The proposed models considering **Black – Scholes – Merton (1973) Default Probability Formula** are given by:

$$PD_{Stressed} = \Phi\left(-\mu_S.\mu_K.J\right) \pm \sigma_S.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_K.f\left(k, \mu_K, \sigma_K, \pi\right)$$

And/or $$PD_{Stressed} = \Phi\left(-\mu_{SK}.J\right) \pm \sigma_{SK}.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_{SK}.f\left(k, \mu_K, \sigma_K, \pi\right)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature.

Where, $PD = \Phi\left(\frac{-\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t) \right)}{\sigma_V \sqrt{T-t}} \right)$ with $J = \frac{\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t) \right)}{\sigma_V \sqrt{T-t}}$ then $PD = \Phi(-J)$

Proposed Jameel's Models VI:

The proposed models considering **Black – Scholes – Merton (1973) Recovery Rate Formula** are given by:

$$RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V T\right) \frac{\Phi\left(-\mu_S.\mu_K.d_1\right)}{\Phi\left(-\mu_S.\mu_K.d_2\right)} \pm \sigma_S.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_K.f\left(k, \mu_K, \sigma_K, \pi\right)$$

And/or $$RR_{Stressed} = \frac{A_0}{D} \exp\left(\mu_V T\right) \frac{\Phi\left(-\mu_{SK}.d_1\right)}{\Phi\left(-\mu_{SK}.d_2\right)} \pm \sigma_{SK}.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_{SK}.f\left(k, \mu_K, \sigma_K, \pi\right)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature.

Where, $RR = \frac{A_0}{D} \exp\left(\mu_V T\right) \frac{\Phi(-d_1)}{\Phi(-d_2)}$, $d_1 = \frac{\left(\ln \frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2} T \right)}{\sigma_V \sqrt{T}}$ and $d_2 = d_1 - \sigma_V \overline{T}$

Proposed Jameel's Models VII:

The proposed models considering**KMV – Merton** are given by:

$$PD_{Stressed} = \Phi\left(-\mu_S.\mu_K.DD\right) \pm \sigma_S.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_K.f\left(k, \mu_K, \sigma_K, \pi\right)$$

And/or $$PD_{Stressed} = \Phi\left(-\mu_{SK}.DD\right) \pm \sigma_{SK}.f\left(s, \mu_S, \sigma_S, \xi\right) \pm \sigma_{SK}.f\left(k, \mu_K, \sigma_K, \pi\right)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature. Where,

$\pi KMV = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)$ with $DD = \frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}$ $\pi KMV = \Phi(-DD)$

Proposed Jameel's Models VIII:

The proposed models considering **NaiveKMV – Merton Alternative** are given by:

$$PD_{Stressed} = \Phi\left(-\mu_S.\mu_K.NaiveDD\right) \pm \sigma_S.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_K.f\left(k,\mu_K,\sigma_K,\pi\right)$$

And/or $PD_{Stressed} = \Phi\left(-\mu_{SK}.NaiveDD\right) \pm \sigma_{SK}.f\left(s,\mu_S,\sigma_S,\xi\right) \pm \sigma_{SK}.f\left(k,\mu_K,\sigma_K,\pi\right)$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature. Where,

$Naive\ DD = \frac{\ln\left[\frac{(E+F)}{F}\right] + \left(r_{u-1} - 0.5\,Naive\,\sigma_V^2\right)T}{Naive\,\sigma_V\sqrt{T}}$ with $\pi_{Naive} = \Phi(-Naive\ DD)$. $Naive\ \sigma_D = 0.05 + 0.25\sigma_E$ and $Naive\ \mu = r_{u-1}$