An Introduction of Jameel’s Advanced Stressed Economic and Financial Crises Models and to Dramatically Increasing Markets Confidence and Drastically Decreasing Markets Risks

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Abstract
Quantitative Financial Risk Management has tremendously change the way markets’ Practitioners, Regulators and Supervisors, Investors, Academics, Economists, Politicians, Policy Makers and Civil Society perceived financial and commodities markets. The generous invention of Black – Scholes – Merton (1973) Formula is of course the advanced turning point. The Normality Assumption (which causes overreliance, overconfidence, overvaluation or underestimation of assets, overleveraging and underestimation of risks by the market participants) is the fundamental pillar in question, because returns are not normally distributed, returns have fat tails consisting bubbles and crashes for instance like IT-bubble, stock market bubble, housing bubble and commodities bubbles. Nassim N. Taleb (2007) called these Black Swans or Low – Probability, High – Impact events. The formulae in question receives serious criticisms especially in the United States of America to the extent of Tim Harford (2012) published an article entitled ‘The Black – Sholes: The Maths Formula linked to the Financial Crash’. Jamilu (2015) using his criterion and Advanced Methods attempted to capture the popular Black Swans (Low – Probability, High – Impact). The aim of this paper is to use Jameel’s Advanced Stressed Methods and Criterion to incorporate fat –tailed effects into the existing stochastic Economic and Financial Models thereby tremendously increasing markets confidence and drastically decreasing markets risks. Based on the various presentations of results and graphs obtained, it can be observed, the Jameel’s Advanced Stressed Economic and Financial Models can traces the trajectories of the past and future Economic and Financial Crises given reliable, accurate, sophisticated, valid and sufficient models’ independent variables.

Keywords: Options, Call, Put, Probability, Black Swans, Jameel

1. Introduction

The overreliance and overconfidence of the Markets practitioners and investors in Normality Assumption has seriously causes the overvaluation of assets, overleveraging and underestimation of risks at huge cost of bail – outs ranging between $3 & 13 trillion of the late 2007 – 2008 crisis (Blyth 2013, 5). Lanchester, 2010, stated that the total cost of the bail outs was amount to $4.6 trillion which is larger the entire cost of NASA including the Moon Landings, the Marshall Plan, the Wars in Korea, Vietnam, Iraq, the Deal, the 1980s Savings and loan Crisis and Louisiana Purchase.

These has become greatest challenge for the Markets Practitioners, Politicians, Policy Makers, Academics, Investors, Economists and Civil Society to fully understand advanced methods of avoiding future occurrences of huge cost of bail outs in the financial systems and systemically to the entire world economy.
The aim of this paper is to present the results obtained using various forms of Jameel’s Advanced Stressed Economic and Financial Models and to show clearly ‘How Jameel’s Advanced Stressed Economic and Financial Crises Models Dramatically Increases Markets Confidence and Drastically Decreases Markets Risks’.

1.1 Literature Review

John M. Moody (1909) was the first to publish credit rating grades for publicly traded bonds. In 1941, David Durand applied discriminant analysis proposed by Fisher (1936) to classify prospective borrowers. Attempts have been made in 1950s to merge automated credit decision making with statistical techniques so as to enhance credit decision making. Lack of sophisticated computing tools, the models possessed limitations. Myers and Forgy (1963), compared discrimination analysis with regression in credit scoring application.


In 1985, West used factor analysis and logit estimation to assign a probability of a bank being a problematic. In 2001, Shumway introduced dynamic logit or hazard model to predict bankruptcy. Chava &Jarrow (2004), Hillegeist, Keating, Cram, &Lundstedt (2004), and Beaver, McNichols & Rhie (2005) uses Shumway’s approach. In 2004, Jones &Hensher introduced a mixed logit model for financial distress prediction and argued that it offers significant improvements compare to binary logit and multinomial logit models. Campbell, Hilscher, & Szilagyi (2008), introduced a dynamic logit model to predict corporate bankruptcies and failures at short and long horizons using accounting and market variables.

In 2011, Altman, Fargler, & Kalotay used accounting – based measures, firm characteristics and industry level expectations of distress conditions to estimate the likelihood of default inferred from equity prices. Li, Lee, Zhou, & Sun (2011) introduced a combined random subspace approach (RSB) with binary logit models to generate a so called RSB-L model that takes into account different decision agent’s opinions as a matter to enhance results. Sun & Li (2011) tested the feasibility and effectiveness of dynamic modelling for financial distress prediction (FDP) based on the Fisher discriminant analysis model.

Stefan Van der Ploeg (2011) stated that since the seminal work of Martin (1977), the Logit and Probit Models has become one of the most commonly applied parametric failure prediction models in the academic literature as well as the banking regulation and supervision. Jamilu (2015) introduced new methods entitled “Jameel’s Advanced Stressed Methods uses Jameel’s Criterion” to Stress Economic and Financial Stochastic Models, initially using Logit and Probit Models.

2. Method

The methodology adopted in this paper is to use Jameel’s Advanced Stressed Economic and Financial Crises Models appeared in Appendix A, B, C and D using Jameel’s Advanced Stressed Methods and criterion.

2.1 Jameel’s Advanced Stressed Methods

The idea was basically on how to contractionally and expansionally stress Black – Scholes – Merton options pricing model using the respectively geometric volatility \( \sigma_A \) and geometric return \( \mu_A \) of the arithmetic means of the underlying asset returns and returns of the explained (independent) variables as well as the best fitted fat – tailed effects probability distribution of the underlying asset returns, so as to capture non – normality of financial markets with the effect of small probabilities margin (popularly known as black swan events) reference to the traditional Logit, Probit, Discriminant Function, Mixed Logit, Instantaneous, Multinomial Logistic, Black - Scholes, Kmv – Merton and naïve Kmv – Merton probability of default models.
(i) Catastrophically shrink the normal probability of default model \( PD_{\text{Unstressed}} (\text{Normal}) \) to contractional probability of default models \( PD_{\text{stressed}} (\text{Contractional}) \) using respectively geometric volatility \( (\sigma_{\text{A}}) \), research company underlying stock returns and returns of the explained (independent) variables \( (\mu_{\text{A}}) \) as well as best fitted fat – tailed probability distribution \( f \left( x, \mu_{\text{Company}}, \sigma_{\text{Company}}, \xi \right) \); then

(ii) Catastrophically blow the normal probability of default model \( PD_{\text{Unstressed}} (\text{Normal}) \) to expansional probability of default models \( PD_{\text{stressed}} (\text{Expansional}) \) using respectively geometric volatility \( (\sigma_{\text{A}}) \), research company underlying stock returns and returns of the explained (independent) variables \( (\mu_{\text{A}}) \) as well as best fitted fat – tailed probability distribution \( f \left( x, \mu_{\text{Company}}, \sigma_{\text{Company}}, \xi \right) \).

Where, \( \mu_{\text{A}} \) is the Geometric Return of the Arithmetic Means of the U.S. Macroeconomic Indicators plus Research Company Stock Returns. \( \sigma_{\text{A}} \) is the Geometric Volatility of the Volatilities of the U.S. Macroeconomic Indicators plus Research Company Stock Returns.

2.2 Jameel’s Criterion

In this test of Goodness of fit, the author considers the following criterion:

- We accept if the Average of the ranks of Kolmogorov Smirnор, Anderson Darling and Chi-squared is less than or equal to Three (3)
- We must choose the Probability Distribution follows by the data itself regardless of its Rankings
- If there is tie, we include both Probability Distributions in the selection
- At least Two (2) probability distributions must be included in the selection
- We select the most occur probability distribution as the best fitted probability distribution in each case of test of goodness of fit of the stock returns.

2.3 Some Selected Data Sources

- Yahoo Finance
- Google Finance
- Federal Reserve Bank
- Economic Research
2.4 Companies and Fundamental Macroeconomic Indicators used in the Research Work

The Author considers the following:

- Five (5) companies listed on the platform of New York Stock Exchange (NYSE) namely; Chevron Corporation, Honda Motor Corporation, Microsoft Corporation, Exxon Mobil Corporation, and General Electric Corporation for the period of Twenty Five (25) years (2014 – 1991) data.
- The underlying monthly stock returns of the five (5) research companies
- The U.S. GDP
- The U.S. Inflation Rate
- The U.S. Prime Rate
- The U.S. unemployment Rate
- The U.S. USD/GBP Exchange Rate
- The U.S. House Price
- The U.S. Oil Price
- The U.S. Gold Price

Using QI Macros 2014 Software, the author obtained the following components:

**Multiple Regression Model Component of CHEVRON Corporation (CVX) for calculating Probability of Default:**

\[
Y_{CVX} = 0.004 + 0.004 \times \Delta P(CHEVRON) - 0.199 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.009 \times \Delta P(INF) - 0.018 \times \Delta P(UER) + 0.002 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD / GBP)
\]

**Multiple Regression Model Component of GENERAL ELECTRIC (GE) for calculating Probability of Default:**

\[
Y_{GE} = 0.004 - 0.001 \times \Delta P(GE) - 0.207 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.016 \times \Delta P(INF) - 0.017 \times \Delta P(UER) - 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD / GBP)
\]

**Multiple Regression Model Component of MICROSOFT (MSFT) Corporation for calculating Probability of Default:**

\[
Y_{MSFT} = 0.004 - 0.006 \times \Delta P(MSFT) - 0.189 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.011 \times \Delta P(INF) - 0.017 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD / GBP)
\]

**Multiple Regression Model Component of EXXON MOBIL (XOM) Corporation for calculating Probability of Default:**

\[
Y_{XOM} = 0.004 + 0.002 \times \Delta P(XOM) - 0.2 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) - 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD / GBP)
\]

**Multiple Regression Model Component of HONDA MOTOR CO., Ltd for calculating Probability of Default:**

\[
Y_{HONDA} = 0.004 - 0.004 \times \Delta P(HMC) - 0.204 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) - 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD / GBP)
\]

2.5 Research Companies Calculated Parameters

**Chevron Calculated Parameters**

\[
\begin{align*}
\mu_{geo}(Chevron) &= 0.030383975, \quad \sigma_{geo}(Chevron) = 0.111414539, \quad \mu_{stock}(Chevron) = 0.004402791 \quad \text{and} \quad \sigma_{stock}(Chevron) = 0.06909299 \\
\end{align*}
\]

**General Electric Corporation Calculated Parameters**

\[
\begin{align*}
\mu_{geo}(GE) &= 0.037067141, \quad \sigma_{geo}(GE) = 0.10009902, \quad \mu_{stock}(GE) = 0.002163529 \quad \text{and} \quad \sigma_{stock}(GE) = 0.100140157 \\
\end{align*}
\]

**Honda Motor Calculated Parameters**

\[
\begin{align*}
\mu_{geo}(Honda) &= 0.031352397, \quad \sigma_{geo}(Honda) = 0.11414539, \quad \mu_{stock}(Honda) = 0.005839335 \quad \text{and} \quad \sigma_{stock}(Honda) = 0.084945727 \\
\end{align*}
\]
Microsoft Corporation Calculated Parameters

\[
\begin{align*}
\mu_{GEO}(MSFT) &= 0.031352397, & \sigma_{GEO}(MSFT) &= 0.117906073, & \mu_{STOCK}(MSFT) &= 0.006798657 \quad \text{and} \quad \sigma_{STOCK}(MSFT) &= 0.115022493 \\
\end{align*}
\]

Exxon Mobil Corporation Calculated Parameters

\[
\begin{align*}
\mu_{GEO}(XOM) &= 0.030729517, & \sigma_{GEO}(XOM) &= 0.110236167, & \mu_{STOCK}(XOM) &= 0.00487448 \quad \text{and} \quad \sigma_{STOCK}(XOM) &= 0.062787634
\end{align*}
\]

Using the above data set and the Jameel’s Criterion, the following are the Global Economic and Financial Crises Best Fitted Fat – Tailed Probability Distributions in terms of order of hierarchy:


2.5.1 Jameel’s - Aish Triangle

![Jameel’s - Aish Triangle](image)

2.6 Proposed Theorem (Jameel’s Average for Decision Making)

Let \( x_1 := \text{Normal Value} \), \( x_2 := \text{Jameel’s Contractional Stressed Value} \), and \( x_3 := \text{Jameel’s Expansional Stressed Value} \). Define

\[
\begin{align*}
x_4 &:= \text{Jameel’s Arithmetic Mean Value} := \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \\
x_5 &:= \text{Jameel’s Geometric Mean Value} := \sqrt[3]{x_1 \cdot x_2 \cdot x_3}
\end{align*}
\]

Then the set \( \{x_1, x_2, x_3, x_4, x_5\} \) form a Solution when Making Decision, depending on the financial/non-financial institutions’ policies.
2.7 Proposed Jameel’s Pentagon for Decision Making

3. Results (reference to Jameel’s Advanced Stressed Models of Appendix A) Default Probabilities of Chevron Corporation

Under Chevron Corporation, on the month of June, 2014, the probability of default using the existing logit is 0.499097747% and that of probit is 0.501439786%, whereas, using the proposed Jameel’s advanced stressed probability of default models I and II are: 0.499976914%, 0.4999968258%, 0.500011436%, 0.499933742%, 0.49910206%, 0.499093434%, 0.499136461%, 0.499059039%, 0.501573711%, 0.50160834%, 0.501456622%, 0.501746429%, 0.501422471%, 0.5014571%, 0.501284382%, and 0.501595189% respectively.
0.499136461%, 0.499059039%, 0.501573711%, 0.50160834%, 0.501435622%, 0.501746429%, 0.501422471%, 0.5014571%, 0.501284382%, and 0.501595189%. While in the case of M2 TYPE A* and M2 TYPE B*, on 10/1/2014, we obtained the stressed probabilities: 0.500003829, 0.50000384, 0.500003782, and 0.500003887 which are clearly lies in between the probabilities of logit and probit.

![Proposed Advanced Stress Probability of Default Models](image)

Figure 4. Proposed Jameel’s Advanced Stressed Probability of Default Models I and II, Logit and Probit Models for Chevron Corporation, \( \mu_{z_0}(Chevron) = 0.030383975 \), \( \sigma_{z_0}(Chevron) = 0.111414539 \), \( \mu_{stock}(Chevron) = 0.004402791 \) and \( \sigma_{stock}(Chevron) = 0.06999299 \). From the above graph, Jameel’s Models I and II traces the trajectories of the past historic Financial and Economic crises Company – wise.

Similarly, under General Electric, on the month of September, 2014, the probability of default using the existing logit is 0.499256894% and that of probit is 0.501185825%, whereas for jameel’s proposed models I and II in appendix A are respectively: 0.499973123%, 0.499971787%, 0.499979127%, 0.499965783%, 0.49925736%, 0.499256228%, 0.499263546%, 0.499250241%, 0.501608698%, 0.501614042%, 0.50158468%, 0.50163806%, 0.501183154%, 0.501188497%, and 0.501212515%. While in the case of M2 TYPE A* and M2 TYPE B*, on 3/2/2014, we obtained the stressed probabilities: 0.500003829, 0.50000384, 0.500003782, and 0.500003887 which are clearly lies in between the probabilities of logit and probit obtained above.
Figure 7. Proposed Jameel’s Advanced Stressed Probability of Default Models I and II, Logit and Probit Models for General Electric, \( \mu_{GEO}(GE) = 0.037067141 \), \( \sigma_{GEO}(GE) = 0.10009902 \), \( \mu_{STOCK}(GE) = 0.002163529 \) and \( \sigma_{STOCK}(GE) = 0.091140157 \).

From the above graph, Jameel’s Models I and II traces the trajectories of the PAST historic Financial and Economic crises Company – wise.

Table 3.1 Chevron Corporation Correlation Matrix

The Matrix below is the Table of Correlations that exists between the Logit, Probit and the Proposed Jameel’s Advanced Stressed Probability of Default Models of Chevron Corporation using our data sources from 2014 – 1991.

| CORREL | M1 TYPE A- | M1 TYPE A+ | M1 TYPE B- | M1 TYPE B+ | M1 TYPE C- | M1 TYPE C+ | M1 TYPE D- | M1 TYPE D+ | LOGIT | PROBIT | M2 TYPE A- | M2 TYPE A+ | M2 TYPE B- | M2 TYPE B+ | M2 TYPE C- | M2 TYPE C+ | M2 TYPE D- | M2 TYPE D+ |
|--------|------------|------------|------------|------------|------------|------------|------------|------------|-------|--------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| M1     | 1.000      | 0.217      | 0.736      | 0.510      | 0.764      | 0.989      | 0.628      | 0.936      | 0.997 | 0.646  | 0.500      | 0.814      | 0.740      | 0.818      | 0.384      | 0.985      | 0.393      | 0.985      |
| M1     | 0.217      | 1.000      | 0.729      | 0.766      | 0.858      | 0.894      | 1           | 0.321      | 0.908 | 0.580  | 0.814      | 0.740      | 0.818      | 0.384      | 0.985      | 0.393      | 0.985      |
| M1     | 0.736      | 0.729      | 1.000      | 0.729      | 0.766      | 0.858      | 0.894      | 1           | 0.321      | 0.908 | 0.580  | 0.814      | 0.740      | 0.818      | 0.384      | 0.985      | 0.393      | 0.985      |
| M1     | 0.510      | 0.766      | 0.729      | 1.000      | 0.736      | 0.510      | 0.764      | 0.766      | 1     | 0.321  | 0.908      | 0.580      | 0.818      | 0.384      | 0.985      | 0.393      | 0.985      |
| M1     | 0.989      | 0.989      | 0.989      | 0.989      | 1           | 0.989      | 0.989      | 0.989      | 0.989   | 1     | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      |
| M1     | 0.628      | 0.894      | 0.858      | 0.894      | 0.989      | 1           | 0.628      | 0.894      | 0.858   | 0.989  | 1           | 0.628      | 0.894      | 0.858      | 0.989      | 0.989      | 0.989      |
| M1     | 0.936      | 1           | 0.766      | 0.858      | 0.894      | 0.628      | 1           | 0.936      | 0.766   | 0.858  | 0.989      | 0.628      | 1           | 0.936      | 0.766      | 0.858      | 0.989      |
| M1     | 0.997      | 0.321      | 0.858      | 0.894      | 0.894      | 0.989      | 0.989      | 1           | 0.997   | 0.321  | 0.858      | 0.894      | 0.894      | 0.989      | 0.989      | 1           | 0.997      |
| M1     | 0.646      | 0.580      | 0.580      | 0.580      | 0.814      | 0.740      | 0.818      | 0.858      | 1     | 0.580  | 0.814      | 0.740      | 0.818      | 0.858      | 0.989      | 0.989      | 1           |
| M1     | 0.500      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814   | 1     | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 0.814      | 1           |
| M1     | 0.814      | 0.740      | 0.740      | 0.740      | 0.814      | 0.740      | 0.740      | 0.740      | 0.740   | 0.814  | 1           | 0.740      | 0.740      | 0.740      | 0.740      | 0.740      | 0.740      | 0.814      |
| M1     | 0.814      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818   | 0.818  | 0.818      | 1           | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      | 0.818      |
| M1     | 0.858      | 0.894      | 0.894      | 0.894      | 0.894      | 0.894      | 0.894      | 0.894      | 0.894   | 0.894  | 0.894      | 0.894      | 1           | 0.894      | 0.894      | 0.894      | 0.894      | 0.894      | 0.894      |
| M1     | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989   | 0.989  | 0.989      | 0.989      | 0.989      | 1           | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      | 0.989      |
From the above correlation matrix, the correlation that exists between (i) LOGIT and M1 TYPE C-, M1 TYPE C+, M1 TYPE D- and M1 TYPE D+ are respectively 1.00, 1.00, 0.977 and 0.977 (ii) PROBIT and M2 TYPE C-, M2 TYPE C+, M2 TYPE D- and M2 TYPE D+ are respectively 0.998, 0.998, 0.877 and 0.877 (iii) M1 TYPE C- and M1 TYPE C+, M1 TYPE D- and M1 TYPE D+ are respectively 0.999, 0.982 and 0.972 are all POSITIVELY and STRONGLY very high correlations compare with the other correlations appeared in the matrix. This shows the level of closeness in terms of economic and financial crises predicative capabilities (performances) and equivalence of the cited models and so on.
Table 3.2 Jameel’s - Aish Triangle Reference to Logit Model Chevron Corporation (CVX)

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</table>

On the 12th January, 2014 for instance using *Jameel’s - Aish Triangle Reference to Logit Model of Chevron Corporation (CVX)*, the author obtained the following Jameel’s triangle:

![Figure 5. Jameel’s – Aish Triangle Values](image)

With the difference between the Jameel’s Expansional Stressed Value and Normal Value equal $10065620.38 and that of Jameel’s Contractional Stressed Value and Normal Value equal $10029965.55. These are very huge differences between the NORMAL VALUE and Jameel’s Stressed Values. This will help to address future global economic and financial crises.

The following is the equivalent Jameel’s Pentagon for Decision Making reference to the above Jameel’s – Aish Triangle.
Figure 6. Jameel’s Pentagon for Decision Making (Numerical Estimates)

Similar comparisons can be done as in the case of the above Jameel’s – Aish Triangle.

Figure 8. Means of the Jameel’s Advanced Stressed Probability of Default Models I and II, Logit and Probit Models of the Five Research Companies

Example 1: (reference to Jameel’s Advanced Stressed Models of Appendix C)

Consider an example of Microsoft Corporation (MSFT) option with a term of six months (0.5 years). The current stock price of Microsoft Corporation (MSFT) is $48.14 and the strike of the option is $49.39. The risk-free rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. What is the value of the options using: (1) Black-Scholes - Merton Model; and (2) Jameel’s Economic and Financial Crises Advanced Stressed Derivatives Models reference to Black-Scholes - Merton Model?

Using the data of Microsoft Corporation (MSFT) extracted from yahoo finance from 2014 – 1991, we obtained:
Underlying 

\[ f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) = 0.00000000123523 \text{ (Logistic (3P)), } \mu_A = 0.031886784, \sigma_A = 0.117906073, \]

\[ K = 49.39, \quad S = 48.14, \quad \sigma = 0.022041976, \quad r = 0.0392, \quad T = 0.5 \quad \text{and} \quad t = 0. \quad \text{Recall that} \]

\[ d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}}, \quad d_2 = d_1 - \sigma \sqrt{T-t} \]

\[ d_1 = \frac{\ln(48.14/49.39) + \left(0.0392 + 0.022041976^2\right)(0.5 - 0)}{0.022041976 \sqrt{0.5}} = -0.18969 \]

\[ d_2 = -0.18969 - 0.022041976 \sqrt{0.5} = -0.20528 \]

Therefore, \( d_1 = -0.18969 \) and \( d_2 = -0.20528 \). Using the Microsoft EXCEL, consider the following tables:

**Table 1. Black–Scholes–Merton and Jameel Advanced Stressed Call Option Prices**

<table>
<thead>
<tr>
<th>TYPE A+</th>
<th>TYPE A*+</th>
<th>TYPE B+</th>
<th>TYPE B*+</th>
<th>TYPE C+</th>
<th>TYPE D+</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1353827</td>
<td>3.6767467</td>
<td>-0.1353827</td>
<td>3.6767467</td>
<td>0.1716279</td>
<td>0.1716279</td>
</tr>
</tbody>
</table>

**Table 2: Black–Scholes–Merton and Jameel Advanced Stressed Put Option Prices**

<table>
<thead>
<tr>
<th>TYPE A+</th>
<th>TYPE A*+</th>
<th>TYPE B+</th>
<th>TYPE B*+</th>
<th>TYPE C+</th>
<th>TYPE D+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1559984</td>
<td>3.9681278</td>
<td>0.1559984</td>
<td>3.9681278</td>
<td>0.4630091</td>
<td>0.4630091</td>
</tr>
</tbody>
</table>

\[ \sum M = 15,000,000, \quad \Delta = T_i - T_{i-1} = \frac{1}{4} = 0.25, \quad F = 5.08\% = 0.0508, \quad E = 6.27\% = 0.0627, \]

\[ \sigma = 20\% = 0.20, \quad \sigma_A = 0.111414539, \quad \mu_A = 0.030383975, \quad f(x, \mu_{\text{Company}}, \sigma_{\text{Company}}, \xi) = 0.0000073492, \]

\[ D = \frac{1}{(1 + \Delta \times F)} = \frac{1}{1 + (0.25 \times 0.0508)} = \frac{1}{1.0127} = 0.99 \]

**Example 2:** Assume the interest on loan is at 6.27\% p.a. compounded quarterly. Suppose that this contract is a caplet with notional value of $15,000,000 designed to cap the interest rate for a period of three-month starting six months from now. Assume that the forward rate for three-month period starting on six months is 5.08\% p.a. compounding quarterly with the volatility of the rate equals 20\% p.a. What are the prices of Caplet and Floolet using: (i) Black Models (1976) (ii) Jameel’s Economic and Financial Crises Advanced Stressed Derivatives Models reference to Black (1976) Model?
Using Microsoft EXCEL, consider the following tables:

Table 3. Merton and Jameel Advanced Stressed Caps Prices:

<table>
<thead>
<tr>
<th>TYPE A+</th>
<th>TYPE A++</th>
<th>TYPE B+</th>
<th>TYPE B++</th>
<th>TYPE C+</th>
<th>TYPE D+</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20932.104</td>
<td>77213.465</td>
<td>-20934.98954</td>
<td>77210.58</td>
<td>890.02361</td>
<td>887.13856</td>
</tr>
</tbody>
</table>

Table 4. Merton and Jameel Advanced Stressed Floors Prices

<table>
<thead>
<tr>
<th>TYPE A+</th>
<th>TYPE A++</th>
<th>TYPE B+</th>
<th>TYPE B++</th>
<th>TYPE C+</th>
<th>TYPE D+</th>
</tr>
</thead>
<tbody>
<tr>
<td>23247.369</td>
<td>121392.94</td>
<td>23250.25403</td>
<td>121395.82</td>
<td>45069.497</td>
<td>45072.382</td>
</tr>
</tbody>
</table>

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (10), 11-24.

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

\[ f(x, \mu_{\text{Company}}, \sigma_{\text{Company}}, \xi) = 0.000073492 \left(\log - \text{Logistic (3P)}\right), \mu_A = 0.030383975, \text{ and } \sigma_A = 0.111414539. \]  Let \( J = 0.464641 \) then using the proposed Jameel’s VII Models, we obtained the following table:

<p>| PROPOSED JAMEEL’S MODELS VII AND BLACK - SCHOLES FORMULA FOR CALCULATING PROBABILITY OF DEFAULT |</p>
<table>
<thead>
<tr>
<th>M7 TYPE A+</th>
<th>M7 TYPE A-</th>
<th>M7 TYPE B+</th>
<th>M7 TYPE B-</th>
<th>M7 TYPE C+</th>
<th>M7 TYPE C-</th>
<th>M7 TYPE D+</th>
<th>M7 TYPE D-</th>
<th>BLACK -SCHOLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.494376251</td>
<td>0.494359875</td>
<td>0.494294571</td>
<td>0.321102471</td>
<td>0.321167775</td>
<td>0.321020791</td>
<td>0.321094283</td>
<td>0.321058206</td>
<td>0.321058206</td>
</tr>
</tbody>
</table>

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

\[ f(x, \mu_{\text{Company}}, \sigma_{\text{Company}}, \xi) = 0.000073492 \left(\log - \text{Logistic (3P)}\right), \mu_A = 0.030383975, \text{ and } \sigma_A = 0.111414539. \]  Let \( A = 0.0508 \),
\[ D = 0.0627, \quad T = 0.5, \quad \mu_e = 0.00638311, \quad d_i = 0.464641 \quad \text{and} \quad d_2 = 0.3232196 \] then using the proposed Jameel’s VIII Models, we obtained the following table:

**Proposed Jameel’s Models VIII and Black-Scholes Formula for Calculating Recovery Rate**

<table>
<thead>
<tr>
<th>M8 Type A⁺</th>
<th>M8 Type A⁻</th>
<th>M8 Type B⁺</th>
<th>M8 Type B⁻</th>
<th>M8 Type C⁺</th>
<th>M8 Type C⁻</th>
<th>M8 Type D⁺</th>
<th>M8 Type D⁻</th>
<th>Black-Scholes RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.809997025</td>
<td>0.809980649</td>
<td>0.810063239</td>
<td>0.809915345</td>
<td>0.699202953</td>
<td>0.699186577</td>
<td>0.699268257</td>
<td>0.699121273</td>
<td>0.699194765</td>
</tr>
</tbody>
</table>

Example 5 (reference to KMV–Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

\[ f(x, \mu_e, \sigma_e, \sigma_f, \mu_f, \sigma_f) = 0.000073492 \quad \text{(Log–Logistic (3P))}, \quad \mu_A = 0.030383975 \quad \text{and} \quad \sigma_A = 0.111414539 \] . Let \( DD = 0.3232196 \) then using the proposed Jameel’s IX Models, we obtained the following table:

**Proposed Jameel’s Models IX and KMV-Merton Formula for Calculating Probability of Default**

<table>
<thead>
<tr>
<th>M9 Type A⁺</th>
<th>M9 Type A⁻</th>
<th>M9 Type B⁺</th>
<th>M9 Type B⁻</th>
<th>M9 Type C⁺</th>
<th>M9 Type C⁻</th>
<th>M9 Type D⁺</th>
<th>M9 Type D⁻</th>
<th>KMV-Merton</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49609036</td>
<td>0.496073984</td>
<td>0.496155664</td>
<td>0.49600868</td>
<td>0.321102471</td>
<td>0.373256281</td>
<td>0.373337961</td>
<td>0.373190977</td>
<td>0.321094283</td>
</tr>
</tbody>
</table>

Example 6 (reference to Naïve KMV–Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

\[ f(x, \mu_e, \sigma_e, \sigma_f, \mu_f, \sigma_f) = 0.000073492 \quad \text{(Log–Logistic (3P))}, \quad \mu_A = 0.030383975 \quad \text{and} \quad \sigma_A = 0.111414539 \] . Let \( Naive DD = 0.53636121 \) then using the proposed Jameel’s X Models, we obtained the following table:

**Proposed Jameel’s Models X and Naïve KMV-Merton Formula for Calculating Probability of Default**

<table>
<thead>
<tr>
<th>M10 Type A⁺</th>
<th>M10 Type A⁻</th>
<th>M10 Type B⁺</th>
<th>M10 Type B⁻</th>
<th>M10 Type C⁺</th>
<th>M10 Type C⁻</th>
<th>M10 Type D⁺</th>
<th>M10 Type D⁻</th>
<th>Naïve-KMV-Merton</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.493506999</td>
<td>0.493490623</td>
<td>0.493572303</td>
<td>0.493425319</td>
<td>0.295862655</td>
<td>0.295846279</td>
<td>0.295927959</td>
<td>0.295780975</td>
<td>0.295854467</td>
</tr>
</tbody>
</table>

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (12), 16 – 34.

From the above tables, the eight (8) proposed Jameel’s Models in each case gives much close approximation to that of Original Black – Scholes, Merton, KMV – Merton and Naïve KMV – Merton and interestingly captured “fat-tail effect” which is not being captured by the traditional once and has the ability to traces the trajectories of Past and Future Economic and Financial Crises given accurate, valid and reasonable estimations of the models’ independent variables.

Example 7: (reference to Jameel’s Advanced Stressed Models of Appendix B)

Consider the values obtained in the Proposed Jameel’s Models VII (and Black – Scholes) to be Chevron Corporation Probability of Defaut (not for an Option) and Proposed Jameel’s Models VIII (and Recovery Rate Black – Scholes). Let \( EAD_{Stressed} = $1,02000 \) and \( M = 3 \) then we can Calculate:

(i) Stressed and Normal Asset Correlations for all exposures;

(ii) Stressed and Normal Capital Requirements;

(iii) Stressed and Normal Risk Weighted Assets;

(iv) Stressed and Normal Regulatory Capital for Credit Risk; and

(v) Stressed and Normal Unexpected Losses.

This can be seen using Microsoft EXCEL in the following table:
From the above table, considering the values for Stressed R, Stressed b, Stressed K, Stressed RWA, Stressed RCCR and Stressed UL are: 0.120000013, 0.032671502, 0.131098763, $1671509.23, $133720.7384 and 0.126255275 respectively, while that of M TYPE D LGD TYPE D are: 0.120000013, 0.03267553, 0.131103573, $1671570.553, $133725.6443 and 0.126283389 which are all HIGHER or equal in values than corresponding Black–Scholes Formula (Normal) whose values are: 0.120000013, 0.032670997, 0.131098159, $1671501.53, $133720.1224 and 0.12625175.

Whereas in case of M TYPE A M TYPE A M TYPE B M TYPE B, M TYPE C and M TYPE D, the values for Stressed R, Stressed b, Stressed K, Stressed RWA, Stressed RCCR and Stressed UL are all LOWER or equal than corresponding values in the case of Black–Scholes Formula (Normal). It would be recalled that Black–Scholes Formula suffered from the criticisms of NORMALITY assumption, that it can either underestimates or overestimate Credit Risks, therefore, from the foregoing, we can deduce the following:

(i) In case of Credit Risk OVERESTIMATION (stress period), we consider Jameel’s Models:

(\text{M}^7 \text{ TYPE C}^-, \text{LDG TYPE C}^-) and (\text{M}^7 \text{ TYPE D}^-, \text{LDG TYPE D}^-); whereas.

(ii) In case of Credit Risk UNDERESTIMATION (stress period), we consider Jameel’s Models:

(\text{M}^7 \text{ TYPE A}^+, \text{LDG TYPE A}^+), (\text{M}^7 \text{ TYPE B}^+, \text{LDG TYPE B}^+), (\text{M}^7 \text{ TYPE C}^+, \text{LDG TYPE C}^+), and (\text{M}^7 \text{ TYPE D}^+, \text{LDG TYPE D}^+).

Similarly, we can treat the case of KMV–Merton and Naïve KMV–Merton in the same manner.

Jameel’s–Aish Triangular Solution \{x_1, x_2, x_3\} for:

(i) RWA is given by \{1671501.53, 1671509.23, 1671570.553\}
(ii) RCCR is given by \{133720.1224, 133720.7384, 133725.6443\}
(iii) UL is given by \{0.12622517, 0.126255275, 0.126283389\}

Jameel’s Pentagon for Decision Making Solution \{x_1, x_2, x_3, x_4, x_5\} for:

(i) RWA is given by \{1671501.53, 1671509.23, 1671570.553, 1671527.1, 1671527.1\}
(ii) RCCR is given by \{133720.1224, 133720.7384, 133725.6443, 133722.17, 133722.17\}
(iii) UL is given by \{0.12622517, 0.126255275, 0.126283389, 0.1262635, 0.1262635\}

4.4 Result and Discussion Reference to Jameel’s Advanced Stressed Models of Appendix D

Example 4: Consider an example of Microsoft Corporation (MSFT) option with a term of six months (0.5 years). The current stock price of Microsoft Corporation (MSFT) is $48.14 and the strike of the option is $49.39. The risk-free
rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. With the above strike prices of 148 months: 

\[ \mu_{\text{strike}} = 0.008836, \quad \sigma_{\text{stock}} = 0.188051, \quad \text{and} \quad f(k; \mu_k, \sigma_k, \pi) = 1.624231 \text{ (Cauchy)} \]

for the current period. Note that, unlike Probability, Probability Distributions Function can take values GREATER THAN ONE at extreme cases since its defined as Probability PER UNIT VALUE of a Random Variable, but the integral of this distribution function taken with respect to this value must be exactly equal 1. What is the values of both CALL and PUT the options using: (1) Black-Scholes – Merton (1973) Model; and (2) Proposed Jameel’s Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models reference to Black-Scholes - Merton (1973) Model?

Using the data of Microsoft Corporation (MSFT) extracted from yahoo finance from 2014 – 1991, we obtained:

\[ f(x; \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) = 0.000000012352 \text{ (Log – Logistic (3P))}, \quad \mu_k = 0.031886784, \quad \sigma_k = 0.117906073, \quad \mu_k = 0.032829086, \]
\[ \sigma_k = 0.124525303, \quad \mu_{sk} = 0.028046277, \quad \text{and} \quad \sigma_{sk} = 0.123540719 \text{ (data available)} \]
\[ K = 54.39, \quad S = 548.14, \quad \sigma = 0.22041976, \quad r = 0.0392, \quad T = 0.5 \quad \text{and} \quad t = 0. \]

Recall that

\[ d_1 = \frac{\ln \frac{S}{K} + \left( r - \frac{\sigma^2}{2} \right) (T - t)}{\sigma \sqrt{T - t}} \]
\[ d_2 = d_1 - \sigma \sqrt{T - t} \]

then

\[ d_1 = \frac{\ln (48.14 / 49.39) + (0.0392 + 0.22041976^2) (0.5 - 0)}{0.22041976 \sqrt{0.5 - 0}} = -0.18969 \]
\[ d_2 = -0.18969 - 0.22041976 \sqrt{0.5} = -0.20528 \]

Therefore, \( d_1 = -0.18969 \) and \( d_2 = -0.20528 \). Using the Microsoft EXCEL, consider the following tables:

**Table 4.4.1 Call Option Black – Scholes – Merton (1973) and Jameel’s Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models**

<table>
<thead>
<tr>
<th>Types</th>
<th>Type n</th>
<th>Black Scholes</th>
<th>Type n</th>
<th>Black Scholes</th>
<th>Type n</th>
<th>Black Scholes</th>
<th>Type n</th>
<th>Black Scholes</th>
<th>Type n</th>
<th>Black Scholes</th>
<th>Type n</th>
<th>Black Scholes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 4</td>
<td>0.01492272</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>9, 10, 12</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>13, 14, 16, 18</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>15, 16, 18</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>21, 22, 24, 26</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>27, 28, 30, 32</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>33, 34, 36, 38</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>37, 39, 41, 43</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>44, 45, 47, 49</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>50, 51, 53, 55</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>56, 58, 60, 62</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
<tr>
<td>63, 65, 67, 69</td>
<td>0.03184315</td>
<td>0.17147984</td>
<td>0.23918840</td>
<td>0.25015406</td>
<td>0.17147984</td>
<td>0.16409234</td>
<td>0.11597686</td>
<td>0.27802689</td>
<td>0.17147984</td>
<td>0.22002962</td>
<td>0.24009187</td>
<td>0.17147984</td>
</tr>
</tbody>
</table>

Based on the above CALL option table: Jameel’s models 3, 33, and 59 are EXTREMELY recommended at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods) while, LEFT of models 1, 2, 5, 16, 31,
35, 36 and are partially HIGHER values reference to the BSM’s price, nevertheless, they also useful. Jameel’s models 7, 34, 41, 50, 52 and 57 are EXTREMELY higher values considering BSM but could be useful in other market conditions. Table 4.4.2 Call Option Black – Scholes – Merton (1973), First and Second Jameel’s Proposed Models reference to BSM

<table>
<thead>
<tr>
<th>CALL</th>
<th>1ST PROPOSED MODEL CLASS (+)</th>
<th>1ST PROPOSED MODEL CLASS (-)</th>
<th>BLACK -SCHOLES</th>
<th>2ND PROPOSED BLACK -SCHOLES</th>
<th>2ND PROPOSED MODEL CLASS (+)</th>
<th>2ND PROPOSED MODEL CLASS (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M I, II TYPE 3</td>
<td>0.112693806</td>
<td>0.230562062</td>
<td>0.11315978</td>
<td>0.230096088</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M I, II TYPE 33</td>
<td>0.112693859</td>
<td>0.230562009</td>
<td>0.171627934</td>
<td>0.113159833</td>
<td>0.230096035</td>
<td></td>
</tr>
<tr>
<td>M I, II TYPE 59</td>
<td>0.112693754</td>
<td>0.230562115</td>
<td>0.112693754</td>
<td>0.23009614</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The above table summarized the comparisons between FIRST and SECOND proposed Jameel’s models I. comparing the columns of 1st proposed model class (+) and 2nd proposed model class (+), they ultimately approximates one another with a very strong positive correlation, similarly, 1st proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between BSM Price. These conclude that both FIRST and SECOND proposed Jameel’s models are extremely recommended to be used at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods).

Table 4.4.3 Put Option Black – Scholes – Merton (1973) and Jameel’s Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models

Based on the above PUT option table: Jameel’s models 3, 33, 48, 54 and 59 are EXTREMELY recommended at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods) while, LEFT of models 1, 2, 5, 16, 31, 35, 36, 47 and are partially HIGHER values reference to the BSM’s price, nevertheless, they also useful. Jameel’s models 7, 34, 41, 50, 52 and 57 are EXTREMELY higher values considering BSM but could be useful in other market conditions.
Table 4.4.4 Put Option Black – Scholes – Merton (1973), First and Second Jameel’s Proposed Models reference to BSM

<table>
<thead>
<tr>
<th>PUT</th>
<th>1ST PROPOSED MODEL CLASS (+)</th>
<th>1ST PROPOSED MODEL CLASS (-)</th>
<th>BLACK - SCHOLES</th>
<th>2ND PROPOSED MODEL CLASS (+)</th>
<th>2ND PROPOSED MODEL CLASS (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M I II TYPE 3</td>
<td>0.521943215</td>
<td>0.404074959</td>
<td>0.521477241</td>
<td>0.521477241</td>
<td>0.404540933</td>
</tr>
<tr>
<td>M I II TYPE 33</td>
<td>0.521943163</td>
<td>0.404075012</td>
<td>0.463009087</td>
<td>0.521477188</td>
<td>0.404540986</td>
</tr>
<tr>
<td>M I II TYPE 59</td>
<td>0.521943268</td>
<td>0.404074907</td>
<td>0.521943268</td>
<td>0.521477188</td>
<td>0.404540881</td>
</tr>
</tbody>
</table>

The above table summarizes the comparisons between FIRST and SECOND proposed Jameel’s models. Comparing the columns of 1st proposed model class (+) and 2nd proposed model class (+), they ultimately approximate one another with a very strong positive correlation, similarly, 1st proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between BSM Price. These conclude that both FIRST and SECOND proposed Jameel’s models are extremely recommended to be used at the times of Economic and Financial MELTDOWN (recoveries and recessions stress periods).

Note: All the tables and examples are extracted from Jamilu (2015), Asian Journal of Management Sciences, 03 (11), 09 – 19

Proposed Theorem (Bash – Eves – Kali’s Sacrifice):

Consider the following diagram: Basel II Distribution of Losses

Let DB be the distribution of losses under BASEL II accord described in the above diagram. Let \( f(M)' \) be all Jameel’s Advanced Stressed Stochastic and Deterministic Financial and Economic models obtained as the result of Jameel’s CONTRACTIONAL and EXPANSIONAL stress methods, let \( \alpha \) and \( (1 - \alpha) \) be Normal Markets Confidence and Significant levels respectively, let \( \overline{\alpha} \) be an infinitesimal positive constant then \( f(M)' \)'s Advanced Stressed Stochastic and Deterministic Financial and Economic models have increases Markets CONFIDENCE LEVEL by \( (+ \overline{\alpha}) \) and decreases Markets SIGNIFICANT LEVEL by \( (- \overline{\alpha}) \), meaning now, Markets Confidence and Significant levels have consequently become \( (\alpha + \overline{\alpha}) \) and \( (1 - (\alpha + \overline{\alpha})) \) respectively. Where, \( \overline{\alpha} \) is called JAMEEL’S CONSTANT.

Interpretation and Conclusion: Bash – Eves Sacrifice Theorem has increases MARKETS CONFIDENCE dramatically and reduces MARKETS RISKS drastically.
Proposed Eve’s Transition Diagram:

New Proposed Basel II Distribution of Losses

5. Conclusion

Based on the available results, there are very huge differences between the Normal Market Value and Jameel’s Advanced Stressed Economic and Financial Crises Values. These differences will definitely help in tracing the trajectories of the PAST Crises and in addressing FUTURE Economic and Financial Crises.

For the sake of practitioners, it is believe that the existing Quantitative Risk Management Models underestimates (overestimates) Default Risks especially at the times of Economic and Financial Crises to the extent in which Tim Harford (2012) published an article entitled ‘Black – Scholes: The Maths Formula linked to the Financial Crash’ where he stated that ‘…It has been argued that one formula known as Black – Scholes, along with its descendants, helped to blow up the financial world’. Many other articles have been published in respect to that. The models here presented will serve as the complimentary but not substitute of the Black – Scholes and its descendants because the models are more robust, holistic and extraordinary, providing better approximations, increasing the probabilities of high losses and above all have the ability to precisely traces the trajectories of the past and future economic and financial crises from the results and graphs shown, since they incorporated fat –tail effects.

Finally, for the sake of future research direction, the models can be improved further to capture more vital information using more macroeconomic indicators and models’ independent variables.

Nassim Nicholas Taleb et al (2009) stated that “Black Swan events are almost impossible to predict. Instead of perpetuating the illusion that we can anticipate the future, risk management should try to reduce the impact of the threats we don’t understand.”

CreditMetrics™ (1997) stated that “We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks. CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results.”

“If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one”. It is better off improving Credit Risk Models than not.

References


APPENDIX A

Proposed Jameel’s Models I:

The proposed models considering simple Logistic Regression Model are given by:

Type A:

\[
PD_{\text{Stressed}} = \frac{1}{1 + \exp(-\mu + \sum_{i=0}^{\xi} \beta X_i + \sigma \varphi \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right))}
\]

Type A* (Higher Probabilities):

\[
PD_{\text{Stressed}} = \frac{1}{1 + \mu_x \exp(-\sum_{i=0}^{\xi} \beta X_i + \sigma \varphi \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right))}
\]

Type B:

\[
PD_{\text{Stressed}} = \frac{1}{1 + \exp(-\sum_{i=0}^{\xi} \beta X_i + \varphi \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right))}
\]


Type B* (Higher Probabilities):

\[
PD_{\text{stressed}} = \frac{1}{1 + \mu_A \exp \left( \sum_{i=0}^{1} \beta_i X_i \right)} \pm f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type C:

\[
PD_{\text{stressed}} = \frac{1}{1 + \exp \left( \sum_{i=0}^{1} \beta_i X_i \right)} \pm \sigma_A f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type D:

\[
PD_{\text{stressed}} = \frac{1}{1 + \exp \left( \sum_{i=0}^{1} \beta_i X_i \right)} \pm f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Where, the Simple Logistic Regression Model (Logit) is given by:

\[
PD = \frac{1}{1 + \exp \left( \sum_{i=0}^{1} \beta_i X_i \right)}
\]

\[X = \left( X_1, X_2, \ldots, X_k \right)\] is a vector of explanatory variables (Macro-economic Indicators).

Proposed Jameel’s Models II:

The proposed models considering Merton’s (Probit) Model are given by:

Type A:

\[
PD_{\text{stressed}} = \Phi \left( \beta_0 + \mu_A \sum_{j=1}^{l} \beta_j X_j \right) \pm \sigma_A f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type A*:

\[
PD_{\text{stressed}} = \Phi \left[ \mu_A \left( \beta_0 + \sum_{j=1}^{l} \beta_j X_j \right) \right] \pm \sigma_A f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type B:

\[
PD_{\text{stressed}} = \Phi \left( \beta_0 + \mu_A \sum_{j=1}^{l} \beta_j X_j \right) \pm f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type B*:

\[
PD_{\text{stressed}} = \Phi \left[ \mu_A \left( \beta_0 + \sum_{j=1}^{l} \beta_j X_j \right) \right] \pm f \left( x; \mu_{\text{company}}, \sigma_{\text{company}}, \xi \right)
\]

Type C:
The proposed models considering **Black – Scholes – Merton (1973)** Default Probability Model are given by:

**TYPE A:** \( PD_{\text{Stressed}} = \Phi(-\mu J) \pm \sigma_A f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

**TYPE B:** \( PD_{\text{Stressed}} = \Phi(-\mu J) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

**TYPE C:** \( PD_{\text{Stressed}} = \Phi(-J) \pm \sigma_A f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \), **TYPED:** \( PD_{\text{Stressed}} = \Phi(-J) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

Where, \( PD = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j\right) \)

**APPENDIX B**

**Proposed Jameel’s Models VII:**

The proposed models considering **Merton (1974)** Recovery Rate Model are given by:

**TYPE A:** \( RR_{\text{Stressed}} = \frac{A_j}{D} \exp(\mu J) \frac{\Phi(-\mu_{A} d_{j})}{\Phi(-\mu_{A} d_{j})} \pm \sigma_R f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

**TYPE B:** \( RR_{\text{Stressed}} = \frac{A_j}{D} \exp(\mu J) \frac{\Phi(-\mu_{A} d_{j})}{\Phi(-\mu_{A} d_{j})} \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

**TYPE C:** \( RR_{\text{Stressed}} = \frac{A_j}{D} \exp(\mu J) \frac{\Phi(-d_{1})}{\Phi(-d_{1})} \pm \sigma_R f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \), **TYPED:** \( RR_{\text{Stressed}} = \frac{A_j}{D} \exp(\mu J) \frac{\Phi(-d_{1})}{\Phi(-d_{1})} \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

Where, \( RR = \frac{A_j}{D} \exp(\mu J) \frac{\Phi(-d_{1})}{\Phi(-d_{1})} \), \( d_{1} = \frac{\ln \left( \frac{A}{L} + \mu_{A} - \frac{\sigma^2_{\text{A}}}{2} (T-t) \right)}{\sigma_{\text{A}} \sqrt{T-t}} \) and \( d_{2} = d_{1} - \sigma_{\text{A}} \bar{T} \)

**Proposed Jameel’s Models IX:**

The proposed models considering **KMV – Merton** Default Probability Model are given by:

**TYPE A:** \( PD_{\text{Stressed}} = \Phi(-\mu A, DD) \pm \sigma_A f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \), **TYPE B:** \( PD_{\text{Stressed}} = \Phi(-\mu A, DD) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)

**TYPE C:** \( PD_{\text{Stressed}} = \Phi(-DD) \pm \sigma_A f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \), **TYPE D:** \( PD_{\text{Stressed}} = \Phi(-DD) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \)
Where, the Distance to Default can be calculated as:

\[
DD = \frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5 \sigma_v^2)T}{\sigma_v \sqrt{T}}
\]

Where \( \mu \) is an estimate of the expected annual return of firm’s assets. The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or EDF) is given by:

\[
\pi_{KMV} = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5 \sigma_v^2)T}{\sigma_v \sqrt{T}}\right)
\]

and

\[
\pi_{KMV} = \Phi\left(-DD\right)
\]

Existing Model (Naïve KMV – Merton Alternative)

Proposed Jameel’s Models X:

The proposed models considering NaïveKMV – Merton Default Probability Model are given by:

**TYPE A:**

\[
P_{D_{Naive}} = \Phi(-\mu_{Naive}DD) \pm \sigma_f (x, \mu_{Company}, \sigma_{Company}, \xi)
\]

**TYPE B:**

\[
P_{D_{Naive}} = \Phi(-\mu_{Naive}DD) \pm f (x, \mu_{Company}, \sigma_{Company}, \xi)
\]

**TYPE C:**

\[
P_{D_{Naive}} = \Phi(-\mu_{Naive}DD) \pm \sigma_f (x, \mu_{Company}, \sigma_{Company}, \xi)
\]

**TYPE D:**

\[
P_{D_{Naive}} = \Phi(-\mu_{Naive}DD) \pm f (x, \mu_{Company}, \sigma_{Company}, \xi)
\]

Where, the Distance to Default can be calculated as:

\[
DD = \frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5 \sigma_v^2)T}{\sigma_v \sqrt{T}}
\]

Where \( \mu \) is an estimate of the expected annual return of firm’s assets. The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or EDF) is given by:

\[
\pi_{KMV} = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5 \sigma_v^2)T}{\sigma_v \sqrt{T}}\right)
\]

\[
\pi_{KMV} = \Phi\left(-DD\right)
\]

with the substitutions that the volatility of each firm’s debt is given by: Naïve \( \sigma_{\mu} = 0.05 + 0.25 \sigma_{\mu} \)

Naïve \( \mu = r_{u-1} \)

By allowing Naïve estimate of \( \mu \) to depends on past returns, we incorporate the same information. The Naïve Distance to Default is given by:

\[
Naive DD = \frac{\ln\left(\frac{E + F}{F}\right) + \left(r_{u-1} - 0.5 Naive \sigma_v^2\right)T}{Naive \sigma_v \sqrt{T}}
\]

Naïve Probability estimate is given by:

\[
\pi_{Naive} = \Phi(-Naive DD)
\]


**TYPE A:**

\[
CPD_{Stressed} = \Phi(-\mu_A CDD) \pm \sigma_f (x, \mu_{Company}, \sigma_{Company}, \xi)
\]
TYPE B:

\[ CPD_{\text{Stressed}} = \Phi(-\mu, CDD) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \]

TYPE C:

\[ CPD_{\text{Stressed}} = \Phi(-CDD) \pm \sigma_{A} f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \]

TYPE D:

\[ CPD_{\text{Stressed}} = \Phi(-CDD) \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \]

Existing Model (Laplace/Gauss – Hermite Default Rate Approximation):
The Laplace/Gauss – Hermite approximation of the likelihood Generalized Linear Mixed Models estimation is given by:

Default Rate Model:

\[ PD_{k,t} = \Phi(\theta_{r(k)} + X_{t} \beta + Z_{t}) \]

With the following notation:

\( \Phi \) is the standard normal cumulative distribution function, \( r(k) \) is the rating of obligor \( k \), \( \theta_{r(k)} \) is an intercept for rating \( r(k) \), \( X_{t} \) is an \( 1 \times p \) vector with the coefficients modeling the impact of the macroeconomic variables on the PD, \( Z_{t} \sim \Phi(0, \sigma^{2}) \) is a latent factor with variance \( \sigma^{2} \). Latent factors can be correlated over time with correlation matrix \( \text{Corr}(Z_{1}, Z_{2}, \ldots, Z_{r}) = C \).

Migration Model:

\[ P_{\text{down},k,t} = \Phi(\theta_{r(k),d} + X_{t} \beta + Z_{t}) \]

\[ P_{\text{up},k,t} = 1 - \Phi(\theta_{r(k),d} + X_{t} \beta + Z_{t}) \]

Proposed Jameel’s Models 22:

Up Migration Default Rate Models:
The Up Migration Default Rate Models of a Company under stress are given by:

TYPE A:

\[ P_{\text{down},(k,t),\text{stressed}} = \Phi\left[ \mu_{A} \left( \theta_{r(k),d} + X_{t} \beta + Z_{t} \right) \right] \pm \sigma_{A} f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \]

TYPE B:

\[ P_{\text{down},(k,t),\text{stressed}} = \Phi\left[ \mu_{A} \left( \theta_{r(k),d} + X_{t} \beta + Z_{t} \right) \right] \pm f(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi) \]
TYPE C:

\[
P_{\text{down}(k,j)\text{stressed}} = \Phi\left(\theta_{r(k),d} + X, \beta + Z_i\right) \pm \sigma_A f\left(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi\right)
\]

TYPE D:

\[
P_{\text{down}(k,j)\text{stressed}} = \Phi\left(\theta_{r(k),d} + X, \beta + Z_i\right) \pm f\left(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi\right)
\]

**Down Migration Default Rate Models:**

For example, the Down Migration Default Rate Models of a Company under stress using M22 TYPE A are given by:

\[
P_{\text{up}(k,j)\text{stressed}} = 1 - \Phi\left[\mu_A \left(\theta_{r(k),d} + X, \beta + Z_i\right)\right] \pm \sigma_A f\left(x, \mu_{\text{company}}, \sigma_{\text{company}}, \xi\right)
\]

In similar way, we can find the Down Migration Default Rate Models of the remaining types.

**APPENDIX C**

**Proposed Jameel’s Models 22:**

Recall that the price of CALL OPTION is given by: \(C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}\) then

The proposed models considering Black – Scholes – Merton (1973) Model are given by:

**TYPE A \((\times)\):**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE A* \((\ast)\):**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE B \((\times)\):**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE B* \((\ast)\):**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE C:**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE D:**

\[
C(S,t)_{\text{stressed}} = S\left[\Phi\left(d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] - \left[\Phi\left(d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

The price of PUT OPTION is given by: \(P(S,t) = -\Phi\left(-d_1\right)S + \Phi\left(-d_2\right)Ke^{-r(T-t)}\) then

**TYPE A:**

\[
P(S,t)_{\text{stressed}} = -S\left[\Phi\left(-\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] + \left[\Phi\left(-\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]

**TYPE A*:**

\[
P(S,t)_{\text{stressed}} = -S\left[\Phi\left(-\mu_A d_1\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right] + \left[\Phi\left(-\mu_A d_2\right) \pm \sigma_A f\left(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi\right)\right]Ke^{r(T-t)}
\]
The proposed models considering Garman-Kohlhagen (1983) Foreign Exchange Rates Options Price are given by:

**TYPE A**: \( C(S_0, t)_{\text{Stressed}} = e^{-r(T-t)} \left[ F \Phi(d_1) - K \Phi(d_2) \right] \)

\[ P(S_0, t)_{\text{Stressed}} = e^{-r(T-t)} \left[ K \Phi(-d_1) - F \Phi(-d_2) \right] \]

**TYPE A*: \( C(S_0, t)_{\text{Stressed}} = e^{-r(T-t)} \left[ F \Phi(d_1) + \sigma_f \left( x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \tilde{\xi} \right) - K \Phi(d_2) - \sigma_f \left( x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \tilde{\xi} \right) \right] \)

\[ P(S_0, t)_{\text{Stressed}} = e^{-r(T-t)} \left[ K \Phi(-d_1) + \sigma_f \left( x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \tilde{\xi} \right) - F \Phi(-d_2) - \sigma_f \left( x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \tilde{\xi} \right) \right] \]

The PUT PRICE OPTIONS on foreign exchange rates is given by:

\[ P(S_0, t) = e^{-r(T-t)} \left( K \Phi(-d_2) - F \Phi(-d_1) \right) \]
TYPE B \( \times \) : \( P(S_{i,t})_{\text{Stressed}} = e^{-(r-i)} \left( K \left[ \Phi(-\mu_i d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right] - F \left( \Phi(-\mu_i d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right) \)

TYPE B* \( \star \) : \( P(S_{i,t})_{\text{Stressed}} = e^{-(r-i)} \left( K \left[ \Phi(-d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right] - F \left( \Phi(-d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right) \)

TYPE C: \( P(S_{i,t})_{\text{Stressed}} = e^{-(r-i)} \left( K \left[ \Phi(-d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right] - F \left( \Phi(-d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right) \)

TYPE D: \( P(S_{i,t})_{\text{Stressed}} = e^{-(r-i)} \left( K \left[ \Phi(-d_i) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right] - F \left( \Phi(-d_i) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right) \)

Proposed Jameel’s Models 25:

Recalled that CAPS Price is given by: \( \text{Cap}(t) = \sum_{i=1}^{M} \Delta \times D(t,T) \times \left[ F(t,T_i,T) \Phi(d_i) - E \times \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \right] \)

The proposed models considering Black (1976) for CAPS Price is given by:

TYPE A \( \times \) :

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(\mu_i d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(\mu_i d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

TYPE A* \( \star \) :

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(\mu_i d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

TYPE B \( \times \) :

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(\mu_i d_i) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(\mu_i d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

TYPE B* \( \star \) :

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(\mu_i d_i) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

TYPE C:

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

TYPE D:

\[ \text{Cap}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ F(t,T_i,T) \left( \Phi(d_i) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) - E \times \left( \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]

Proposed Jameel’s Models 26:

Recalled that FLOORS Price is given by: \( \text{Floor}(t) = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ -F(t,T_i,T) \Phi(-d_i) + E \times \Phi(-d_i - \sigma_i \sqrt{T_{i-1} - t}) \right] \)

The proposed models considering Black (1976) for FLOORS Price is given by:

TYPE A:

\[ \text{Floor}(t)_{\text{Stressed}} = \sum_{i=1}^{M} \Delta \times D(t,T_i) \times \left[ -F(t,T_i,T) \left( \Phi(-\mu_i d_i) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) + E \times \left( \Phi(\mu_i (-d_i + \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_i f \left( x, \mu_i \text{underlying}, \sigma_i \text{underlying}, \xi \right) \right) \right] \]
TYPE A*:

\[
Floor(t)_{\text{Stressed}} = \sum_{i=1}^{n} M \times \Delta \times D(t, T_i) \times \left[ -F(t, T_{i-1}, T_i) \left( \Phi(-\mu d_i) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) + E \times \left( \Phi\left( \mu_s \left( d - \sigma_i \sqrt{T_{i-1}} \right) \right) \pm \sigma_f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right]
\]

TYPE B:

\[
Floor(t)_{\text{Stressed}} = \sum_{i=1}^{n} M \times \Delta \times D(t, T_i) \times \left[ -F(t, T_{i-1}, T_i) \left( \Phi(-\mu d_i) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) + E \times \left( \Phi\left( \mu_s \left( d + \sigma_i \sqrt{T_{i-1}} \right) \right) \pm \sigma_f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right]
\]

TYPE B*:

\[
Floor(t)_{\text{Stressed}} = \sum_{i=1}^{n} M \times \Delta \times D(t, T_i) \times \left[ -F(t, T_{i-1}, T_i) \left( \Phi(-\mu d_i) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) + E \times \left( \Phi\left( \mu_s \left( d + \sigma_i \sqrt{T_{i-1}} \right) \right) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right]
\]

TYPE C:

\[
Floor(t)_{\text{Stressed}} = \sum_{i=1}^{n} M \times \Delta \times D(t, T_i) \times \left[ -F(t, T_{i-1}, T_i) \left( \Phi(-d_i) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) + E \times \left( \Phi\left( \mu_s \left( d + \sigma_i \sqrt{T_{i-1}} \right) \right) \pm \sigma_f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right]
\]

TYPE D:

\[
Floor(t)_{\text{Stressed}} = \sum_{i=1}^{n} M \times \Delta \times D(t, T_i) \times \left[ -F(t, T_{i-1}, T_i) \left( \Phi(-d_i) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) + E \times \left( \Phi\left( \mu_s \left( d + \sigma_i \sqrt{T_{i-1}} \right) \right) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right]
\]

Proposed Jameel's Models 27:

Recalled that the \textit{Payer Swaption} formula is given by:

\[
Swaption(t) = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \Phi(d) - F \times (d - \sigma \sqrt{T_{i-1}}) \right) \times D(t, T_i)
\]

The proposed models considering \textit{Black (1976) for Payer Swaption Prices} are given by:

\textbf{TYPE A}\( (\times)\): \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

\textbf{TYPE A}\( (\times)\): \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

\textbf{TYPE B}\( (\times)\): \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

\textbf{TYPE B}\( (\times)\): \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

\textbf{TYPE C}: \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm \sigma_s f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

\textbf{TYPE D}: \(Swaption(t)_{\text{Stressed}} = \sigma \times M \times \sum_{i=1}^{n} \left( F(t) \times \left( \Phi(\mu d) \pm f (x, \mu_{\text{underlying}} \sigma_{\text{underlying}} \xi) \right) \right) \times D(t, T_i)\)

The pricing formula for the \textit{Receiver Swaptions} is given by:
\[
\text{Swaption}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-d) + F \times \Phi(-d + \sigma \sqrt{T_i - t}) \right) D(t, T) \text{ then }
\]

TYPE A:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-\mu_i, d) \pm \sigma_i \times f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm \sigma_i \times f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

TYPE A*:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-\mu_i, d) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

TYPE B:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-\mu_i, d) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

TYPE B*:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-\mu_i, d) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

TYPE C:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-d) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

TYPE D:
\[
\text{Swaption}^{(\text{Stressed})}(t) = \sigma \times M \times \sum_{i=1}^{n} \left( -F_i(t) \times \Phi(-d) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) + F \times \Phi(-d + \sigma_i \sqrt{T_i - t}) \pm f \left( s, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi \right) \right) \times D(t, T)
\]

APPENDIX D (General Form)

First Proposed Jameel’s Models I:

The proposed models considering Black – Scholes – Merton (1973) Call Option Price are given by:

\[
C(S, t) = S \left[ \Phi \left( \mu_S \mu_k, d_1 \right) \pm \sigma_S \times f \left( s, \mu_S, \sigma_S, \xi \right) \pm \sigma_K \times f \left( k, \mu_K, \sigma_K, \pi \right) \right] - K \left[ \Phi \left( \mu_S \mu_k, d_2 \right) \pm \sigma_S \times f \left( s, \mu_S, \sigma_S, \xi \right) \pm \sigma_K \times f \left( k, \mu_K, \sigma_K, \pi \right) \right] e^{-(T-t)}
\]

The first type above is the general (initial) proposal not necessarily positive prices (but could also be negative values); however, using Jameel’s Contractional and Expansional Stress Methods, we can have the following possible COMBINATION of TERMS and SIGNS.

Combination of Terms and Signs:

Recall that \( C = n! / (n - r)! r! \) then we have \( C^0 + C^6 + C^3 + C^4 + C^5 + C^6 = 63 \) combination of terms using the general form above and \( C^0 + C^6 + C^3 + C^4 + C^5 + C^6 + C^8 + C^8 = 225 \) combination of Signs.

The proposed models considering Black – Scholes – Merton (1973) Put Option Price are given by:

\[
P(S, t) = -S \left[ \Phi \left( -\mu_S \mu_k, d_1 \right) \pm \sigma_S \times f \left( s, \mu_S, \sigma_S, \xi \right) \pm \sigma_K \times f \left( k, \mu_K, \sigma_K, \pi \right) \right] + K \left[ \Phi \left( -\mu_S \mu_k, d_2 \right) \pm \sigma_S \times f \left( s, \mu_S, \sigma_S, \xi \right) \pm \sigma_K \times f \left( k, \mu_K, \sigma_K, \pi \right) \right] e^{-(T-t)}
\]

Therefore, we have to further check 4 and 162 remaining combination of Terms and Signs respectively.
Where, \( P(S,t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)}. \)

**Second Proposed Jameel’s Models I:**

The proposed models considering Black – Scholes – Merton (1973) Call Option Price are given by:

\[
C(S,t)_{Stressed} = S\left[\Phi\left(\mu_{SK}d_1\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right] - K\left[\Phi\left(\mu_{SK}d_2\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right]e^{-r(T-t)}
\]

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both CALL and PUT options as in the case of first Proposed Jameel’s I Models shown in the table above.

Also, the GREEKS of the both CALL and PUT options can be found in similar ways and patterns.

Therefore, in this section, the author will treat the proposed Jameel’s models II to VIII as he treated the First and Second proposed Jameel’s models I above. Similarly in the case of Result and Discussion section.

**Proposed Jameel’s Models II:**

The proposed models considering Garman - Kohlhaven (1983) Foreign Exchange Rates Options Price are given by:

\[
C(S_0,t)_{Stressed} = \left[F\left[\Phi\left(\mu_{SK}d_1\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right] - K\left[\Phi\left(\mu_{SK}d_2\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right]\right]e^{-r(T-t)}
\]

And/or

\[
C(S_0,t)_{Stressed} = \left[F\left[\Phi\left(\mu_{SK}d_1\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right] - K\left[\Phi\left(\mu_{SK}d_2\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right]\right]e^{-r(T-t)}
\]

and that of PUT are given by:

\[
P(S_0,t)_{Stressed} = \left[-F\left[\Phi\left(-\mu_{SK}d_1\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right] + K\left[\Phi\left(-\mu_{SK}d_2\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right]\right]e^{-r(T-t)}
\]

And/or

\[
P(S_0,t)_{Stressed} = \left[-F\left[\Phi\left(-\mu_{SK}d_1\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right] + K\left[\Phi\left(-\mu_{SK}d_2\right)\pm \sigma_{SK} \cdot f \left(s, \mu_s, \sigma_s, \xi\right)\pm \sigma_{SK} \cdot f \left(k, \mu_k, \sigma_k, \pi\right)\right]\right]e^{-r(T-t)}
\]

Where, \( C(S_0,t) = e^{-r(T-t)} \left(F\Phi(d_1) - K\Phi(d_2)\right) \) and \( P(S_0,t) = e^{-r(T-t)} \left(K\Phi(-d_2) - F\Phi(-d_1)\right). \)

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both CALL and PUT options as in ALL the cases as shown in the tables above.

**Proposed Jameel’s Models III:**

The proposed models considering Black (1976) for CAPS Price are given by:
Cap(t)_{swapped} = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ F(t,T_{i+1},T_i) \left[ \Phi\left(\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] - K \left[ \Phi\left(\mu_k \mu_k \left( d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right]

And/or

Cap(t)_{swapped} = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ F(t,T_{i+1},T_i) \left[ \Phi\left(\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] - K \left[ \Phi\left(\mu_k \mu_k \left( d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right]

and that of FLOORLET are given by:

Floor(t)_{swapped} = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ -F(t,T_{i+1},T_i) \left[ \Phi\left(-\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] + K \left[ \Phi\left(\mu_k \mu_k \left(-d_i + \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right]

And/or

Floor(t)_{swapped} = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ -F(t,T_{i+1},T_i) \left[ \Phi\left(-\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] + K \left[ \Phi\left(\mu_k \mu_k \left(-d_i + \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right]

Where, \( Cap(t) = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ F(t,T_{i+1},T_i) \Phi(d_i) - E \times \Phi\left(d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right] \) and.

Floor(t) = \sum_{i=1}^{\Delta \times \Delta} D(t,T_i) \times \left[ -F(t,T_{i+1},T_i) \Phi(-d_i) + E \times \Phi\left(-d_i + \sigma_k \sqrt{T_{i+1} - T_i} \right) \right]

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both CAPS and FLOORS as in ALL the cases as shown in the tables above.

Proposed Jameel’s Models IV:
The proposed models considering Black (1976) for PAYER SWAPIONS Prices are given by:

Swaption(t)_{swapped} = \sigma \times \Delta \times \sum_{i=1}^{\Delta} \left[ F_i(t) \times \left[ \Phi\left(\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] - K \left[ \Phi\left(\mu_k \mu_k \left( d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right] \times D(t,T_i)

And/or

Swaption(t)_{swapped} = \sigma \times \Delta \times \sum_{i=1}^{\Delta} \left[ F_i(t) \times \left[ \Phi\left(\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] - K \left[ \Phi\left(\mu_k \mu_k \left( d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right] \times D(t,T_i)

and that of RECEIVER SWAPIONS Prices are given by:

Swaption(t)_{swapped} = \sigma \times \Delta \times \sum_{i=1}^{\Delta} \left[ -F_i(t) \times \left[ \Phi\left(-\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] + K \left[ \Phi\left(\mu_k \mu_k \left(-d_i + \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right] \times D(t,T_i)

And/or

Swaption(t)_{swapped} = \sigma \times \Delta \times \sum_{i=1}^{\Delta} \left[ -F_i(t) \times \left[ \Phi\left(-\mu_k \mu_k d_i \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right) \right] + K \left[ \Phi\left(\mu_k \mu_k \left(-d_i + \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \pm \sigma_k (s, \mu_k, \sigma_k, \xi) \pm \sigma_k f (k, \mu_k, \sigma_k, \pi) \right] \right] \times D(t,T_i)

Where, Payer Swaption is given by: \( Swaption(t) = \sigma \times \Delta \times \sum_{i=1}^{\Delta} \left( F_i(t) \times \Phi(d_i) - F \times \Phi\left(d_i - \sigma_k \sqrt{T_{i+1} - T_i} \right) \right) \times D(t,T_i) \)
Receiver Swaption is given by:

\[ Swaption(t) = \sigma \times M \times \sum_{i=1}^{d}( -F_i(t) \times \Phi(-d) + F_i(t) \times \Phi(-d + \sigma, \sqrt{t_0 - t})) \times D(t, T) \cdot \]

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both PAYER SWAPTION and RECEIVER SWAPTION as in ALL the cases as shown in the tables above.

RECOVERY RATES AND DEFAULT PROBABILITIES EXTENDED VERSIONS

Proposed Jameel’s Models V:
The proposed models considering Black – Scholes – Merton (1973) Default Probability Formula are given by:

\[ PD_{\text{stressed}} = \Phi(-\mu_s, \mu_k, J) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \tau) \]

And/or \[ PD_{\text{stressed}} = \Phi(-\mu_{sk}, J) \pm \sigma_{sk} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{sk} \cdot f(k, \mu_k, \sigma_k, \tau) \]

We have \( C_1^3 + C_2^3 + C_3^3 \) = 7 combination of terms and \( C_1^4 + C_2^4 + C_3^4 + C_4^4 \) = 15 combination of Signs of this nature.

Where, \( PD = \Phi \left( \frac{\ln \left( \frac{A}{L} + \mu_v - \frac{\sigma_v^2}{2}(T-t) \right)}{\sigma_v \sqrt{T-t}} \right) \)

Proposed Jameel’s Models VI:
The proposed models considering Black – Scholes – Merton (1973) Recovery Rate Formula are given by:

\[ RR_{\text{stressed}} = \frac{A}{D} \exp(\mu_v T) \cdot \Phi(-\mu_s, \mu_k, d_1) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \tau) \]

And/or \[ RR_{\text{stressed}} = \frac{A}{D} \exp(\mu_v T) \cdot \Phi(-\mu_{sk}, d_2) \pm \sigma_{sk} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{sk} \cdot f(k, \mu_k, \sigma_k, \tau) \]

We have \( C_1^3 + C_2^3 + C_3^3 \) = 7 combination of terms and \( C_1^4 + C_2^4 + C_3^4 + C_4^4 \) = 15 combination of Signs of this nature.

Where, \( RR = \frac{A}{D} \exp(\mu_v T) \cdot \Phi(-d_1) \)

Proposed Jameel’s Models VII:
The proposed models considering KMV – Merton are given by:

\[ PD_{\text{stressed}} = \Phi(-\mu_s, \mu_k, DD) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \tau) \]

And/or \[ PD_{\text{stressed}} = \Phi(-\mu_{sk}, DD) \pm \sigma_{sk} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{sk} \cdot f(k, \mu_k, \sigma_k, \tau) \]
We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_4^4 + C_5^4 + C_6^4 = 15$ combination of Signs of this nature. Where, 

\[
\pi_{KMV} - \Phi \left( \frac{\ln \left( \frac{V}{F} \right) + (\mu - 0.5 \sigma^2) T}{\sigma \sqrt{T}} \right)
\]

with 

\[
DD = \frac{\ln \left( \frac{V}{F} \right) + (\mu - 0.5 \sigma^2) T}{\sigma \sqrt{T}}
\]

\[
\pi_{KMV} = \Phi \left( -DD \right)
\]

Proposed Jameel’s Models VIII:

The proposed models considering NaiveKMV – Merton Alternative are given by:

\[
PD_{Stressed} = \Phi \left( -\mu_s, \mu_K, Naive DD \right) \pm \sigma_s f \left( s, \mu_s, \sigma_s, \xi \right) \pm \sigma_K f \left( k, \mu_K, \sigma_K, \pi \right)
\]

And/or

\[
PD_{Stressed} = \Phi \left( -\mu_s, Naive DD \right) \pm \sigma_s f \left( s, \mu_s, \sigma_s, \xi \right) \pm \sigma_K f \left( k, \mu_K, \sigma_K, \pi \right)
\]

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_4^4 + C_5^4 + C_6^4 + C_7^4 = 15$ combination of Signs of this nature. Where,

\[
Naive DD = \frac{\ln \left( \frac{E + F}{F} \right) + (r_{naive} - 0.5 \sigma_{naive}^2) T}{Naive \sigma_{naive} \sqrt{T}}
\]

with $\pi_{naive} = \Phi \left( -Naive DD \right)$. Naive $\sigma_d = 0.05 + 0.25 \sigma_s$ and Naive $\mu = r_{u-1}$

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